Magnetism and Magnetic Materials

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2025.10.20 天津

Outline

1. Static magnetic properties
    - Magnetic moment
    - Magnetic anisotropy and Exchange interaction
    - Magnetic ordering
2. Dynamic magnetic properties
    - Landau-Lifshitz-Gilbert (LLG) equation
    - Magnetic resonance
    - Magnetic excitation
3. Magnetic effect

Acknowledgement

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Textbooks

- Coey, J. M. D. (2010). *Magnetism and magnetic materials*. Cambridge University Press.
- Nolting, W. (2009). *Quantum theory of magnetism* (4th ed.). Springer.
- Blundell, S. (2001). *Magnetism in condensed matter*. Oxford University Press.
- Mario Reis. (2013). *Fundamentals of Magnetism*. Springer.

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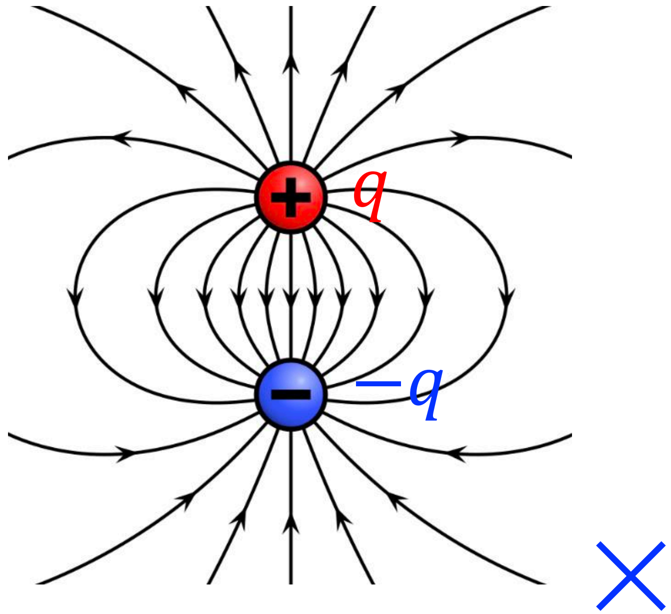
1.1 Magnetic moment



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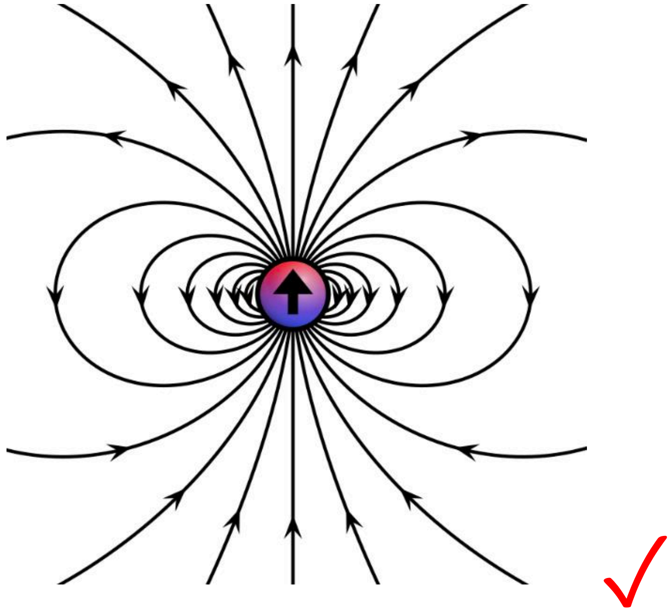


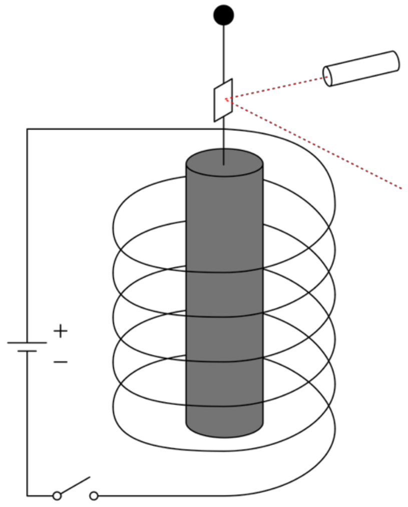
Magnetic dipole: $\mu = qd$

- $\nabla \cdot \vec{B} \neq 0$ (divergent)
- Magnetic line ends on charge q or $-q$

Current loop: $\mu = Is$

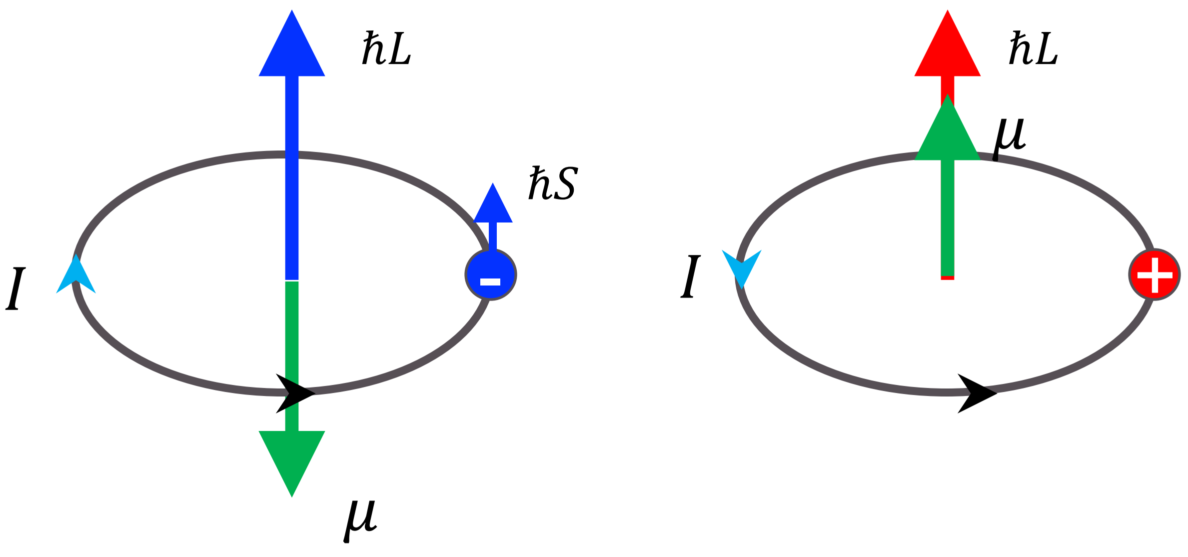
- $\nabla \cdot \vec{B} = 0$ (divergentless)
- Magnetic line never ends
- Magnetic moments are associated with the angular momentum

- Einstein-de Hall effect



- Barnett effect
- Dynamic response

1.1 Magnetic moment



$$\mu_e = \frac{-eL}{2m_e} \gg \mu_n = \frac{q_n L}{2m_n}$$

- $\mu = \gamma(\hbar L)$, γ: gyromagnetic ratio

$$I = q/\tau,\ \tau = 2\pi r/v$$

$$\mu = \pi r^2 I = \frac{\pi r^2 q}{\tau} = \frac{qrv}{2}$$

$$= q\frac{r \times p}{2m}$$

$$= \frac{qL}{2m}$$

Bohr quantization: $L = n\hbar$

- Bohr magneton: $\mu_B = -\frac{e\hbar}{2m_e}$

$$\mu_B = 9.274 \times 10^{-24}\,\frac{J}{T}$$

(SI) $= 5.788 \times 10^{-5}\,\frac{eV}{T}$

1.1 Magnetic moment

- Orbital angular momentum: $\hbar \hat{L} = \hat{r} \times \hat{P}$

$$[\hat{L}_a, \hat{L}_b] = i\epsilon_{abc}\hat{L}_c$$

$$\hat{L}^2|l,m\rangle = l(l+1)|l,m\rangle$$

$|l,m\rangle$: eigenstate of $\hat{L}^2$ and $\hat{L}_z$

$$\hat{L}_z|l,m\rangle = m|l,m\rangle$$

l:	0,	1,	2,	3,	4,	5…
	s	p	d	f	g	h…

m: $(-l, \ldots 0, \ldots l)$

- Spin angular momentum: $S = 1/2$

$$\hat{S}^2|\pm\rangle = 1/2(1+1/2)|\pm\rangle$$

$$[\hat{S}_a, \hat{S}_b] = i\epsilon_{abc}\hat{S}_c$$

$$\hat{S}_z|\pm\rangle = \pm 1/2|\pm\rangle$$

- Full occupied orbit: $\sum_i \vec{L}_i = 0, \sum_i \vec{S}_i = 0$

1.1 Magnetic moment

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- $L-S$ coupling

$$\vec{L}=\sum_i \vec{l}_i \qquad \vec{S}=\sum_i \vec{s}_i$$

Eigenstate: $\psi = |l, m_l, s, m_s\rangle$

L and S are both conserved

Hund's rule:

1. Maximize total S
2. Maximize total L
3. If less than half filled $(n \le \frac{N}{2})$, $J = |L-S|$

   If more than half filled $(n > \frac{N}{2})$, $J = |L+S|$

Good for 4f orbital, however, bad for 3d orbital and heavy atoms

- $J-J$ coupling

Spin-orbit coupling(SOC): $\lambda\vec{S}\cdot\vec{L}$

J is conserved

Eigenstate: $\psi = |J, m_j, l, s\rangle$

1.1 Magnetic moment

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$$\vec{\mu}_L = \mu_B \vec{L} \qquad \gamma_L = \mu_B/\hbar$$

$$\vec{\mu}_S = 2\mu_B \vec{S} \qquad \gamma_S = 2\mu_B/\hbar$$

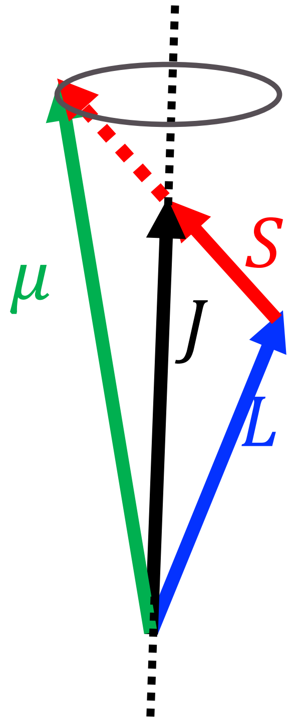
$$\vec{\mu} = \vec{\mu}_L + \vec{\mu}_S = \mu_B(\vec{L} + 2\vec{S})$$

$$\vec{J} = \vec{L} + \vec{S}$$

- Landé g-factor:

$$g \equiv \frac{\mu}{\mu_b J} = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}$$

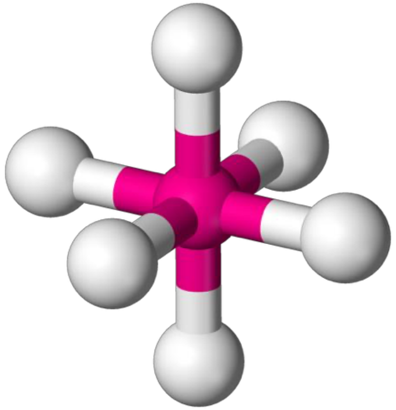
$$\langle \mu(t) \rangle = g\mu_B J$$

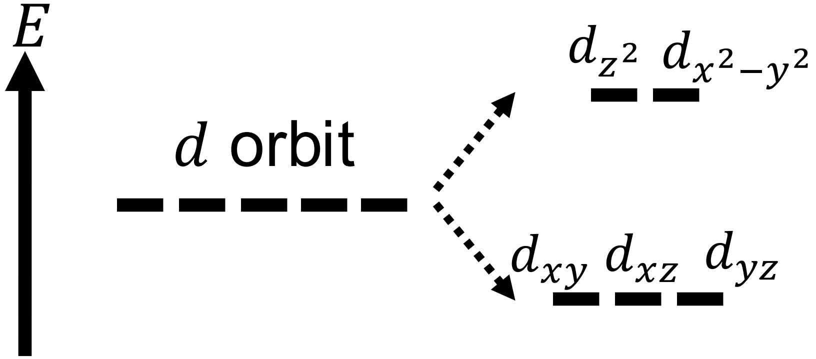


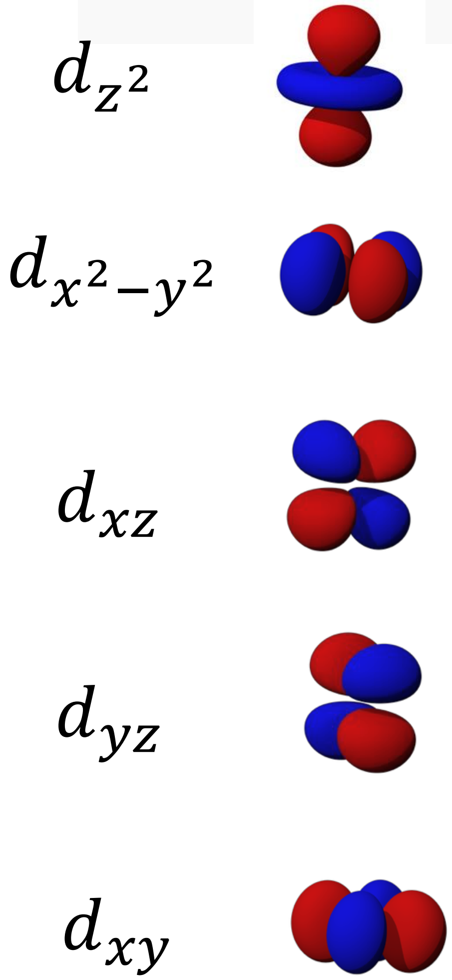
Deviation???

1.1 Magnetic moment-crystal field

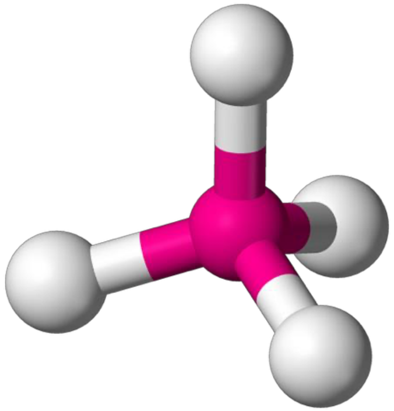
Octahedral

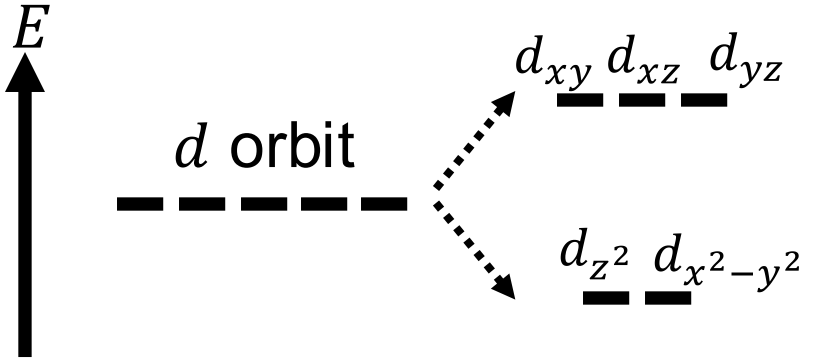






Tetrahedral

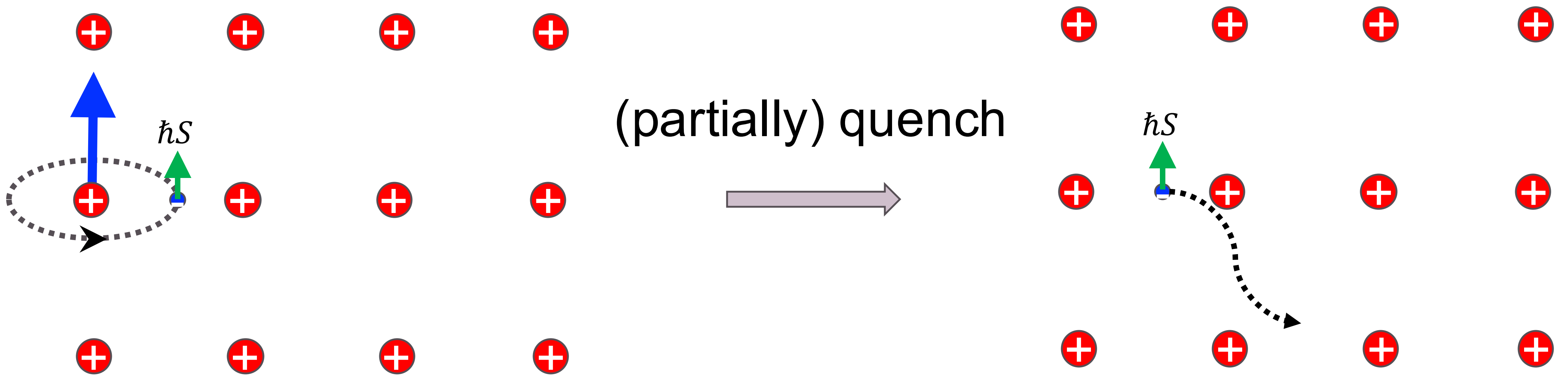




Jahn-Teller effect !

1.1 Magnetic moment-orbit quench

Crystal field　　　　　　Crystal field



Inner orbit without quench　　　　　　Outer orbit with quench

1.2 Magnetic anisotropy

- Magnetocrystalline anisotropy ⟸ Spin-orbit coupling: $H = \lambda\vec{\sigma}\cdot\vec{L}$

$$H_{ani} = K\left(\vec{e}\cdot\vec{S}\right)^2$$

Uniaxial

$K < 0$: easy axis; $K > 0$:hard axis

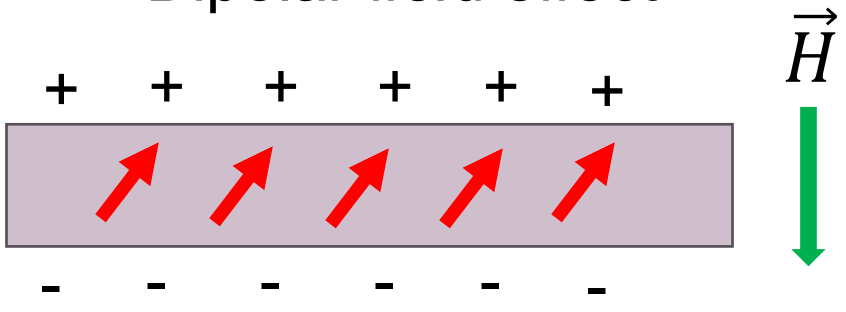
$$H_{ani} = K_1\left(\vec{e}_1\cdot\vec{S}\right)^2 + K_2\left(\vec{e}_2\cdot\vec{S}\right)^2$$

biaxial

Cubic anisotropy

- Shape anisotropy

Dipolar field effect



- Magnetoelastic anisotropy

- Exchange anisotropy

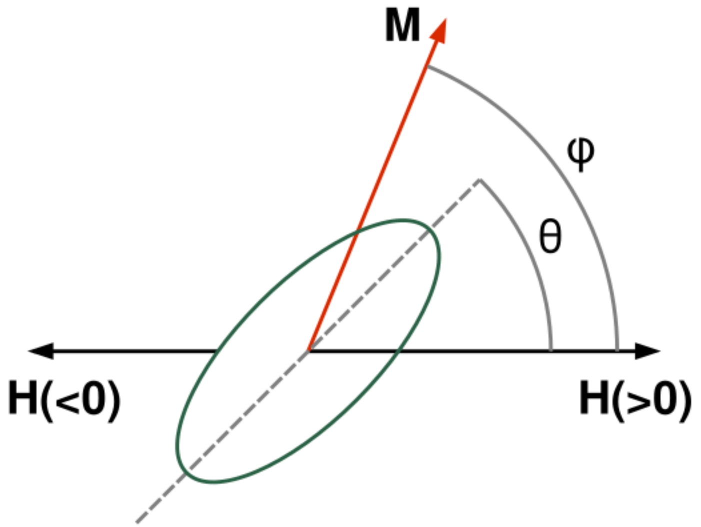
$$H = J\sum_{\langle ij\rangle}\vec{S}_i\cdot\vec{S}_j + K\sum_{\langle ij\rangle}S_i^zS_j^z$$

1.2 Mag anisotropy-Stoner-Wohlfarth model

Total energy

$$E = -\mu_0 \vec{H} \cdot \vec{M} - K\left(\vec{e} \cdot \vec{M}\right)^2$$

$$= -\mu_0 H M \cos\psi - K M^2 \cos(\psi - \theta)^2$$



Minimizing the total energy

$$\frac{\partial E}{\partial \psi} = 0 \Longrightarrow \mu_0 H M \sin\psi + K M^2 \sin(2(\psi - \theta)) = 0$$

$$\frac{\partial^2 E}{\partial \psi^2} > 0 \Longrightarrow \mu_0 H M \cos\psi + 2 K M^2 \cos\left(2(\psi - \theta)\right) > 0$$

1.2 Mag anisotropy-Stoner-Wohlfarth model

We obtain

$$h\sin\psi+\frac{1}{2}\sin(2(\psi-\theta))=0$$

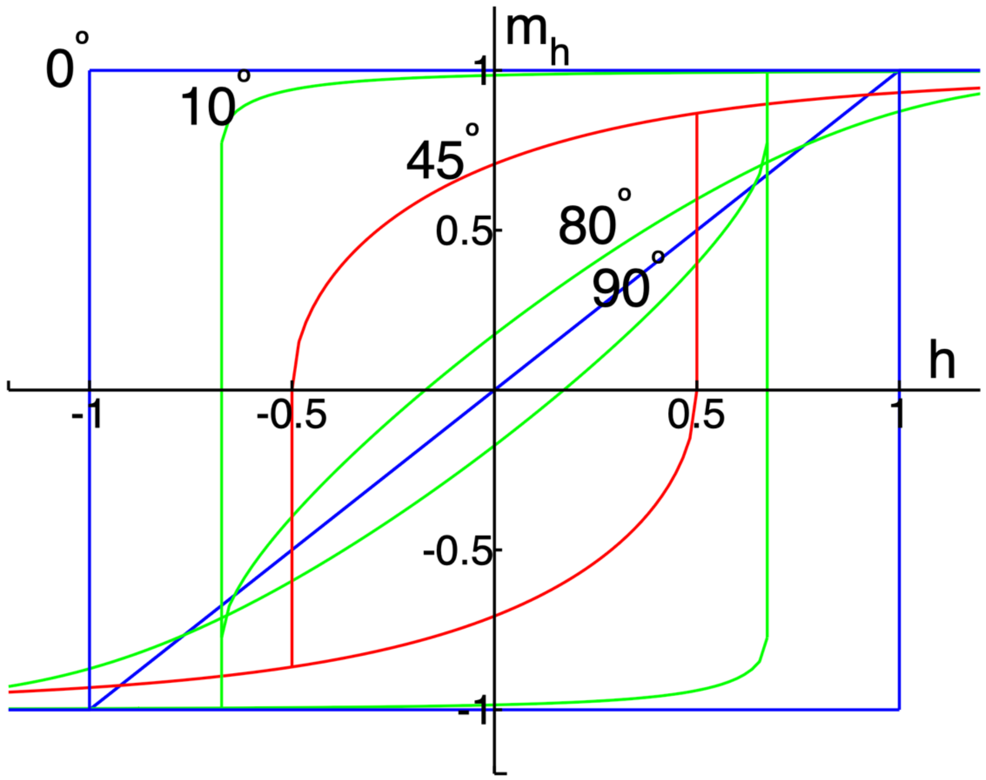
$$h\cos\psi+\cos\big(2(\psi-\theta)\big)>0$$

with $h=\mu_0 H/2KM$

$\theta=0$

$$\sin\psi=0 \Rightarrow \psi=0,\pi$$

$$h\cos\psi+1=0 \Rightarrow h_c=\pm 1$$



1.2 Mag anisotropy-exchange bias effect

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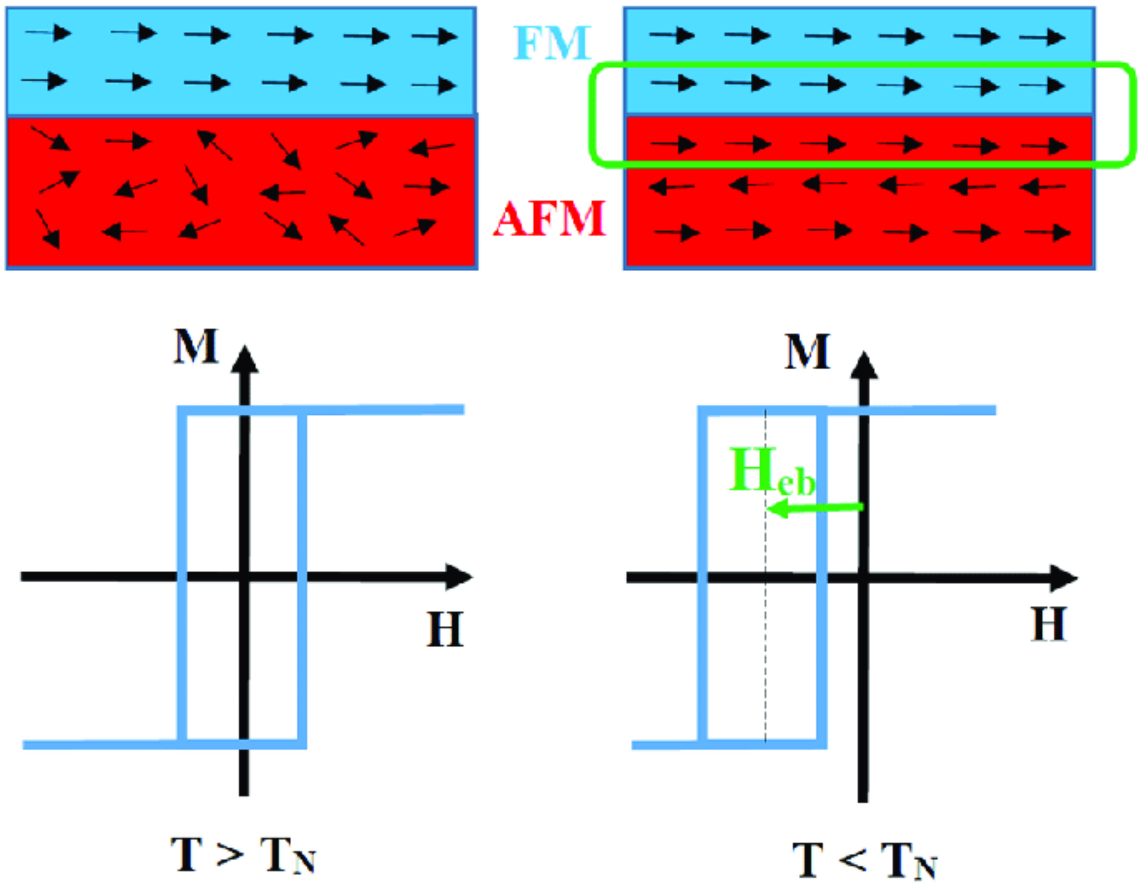
Total energy

$$E = \mu_0 \vec{H} \cdot \vec{M} + J\vec{m} \cdot \vec{M} - K(\vec{e} \cdot \vec{M})^2$$

$$= (\mu_0 H + Jm) M \cos\psi$$

$$-KM^2 \cos(\psi - \theta)^2$$



1.2 Mag anisotropy-exchange bias effect

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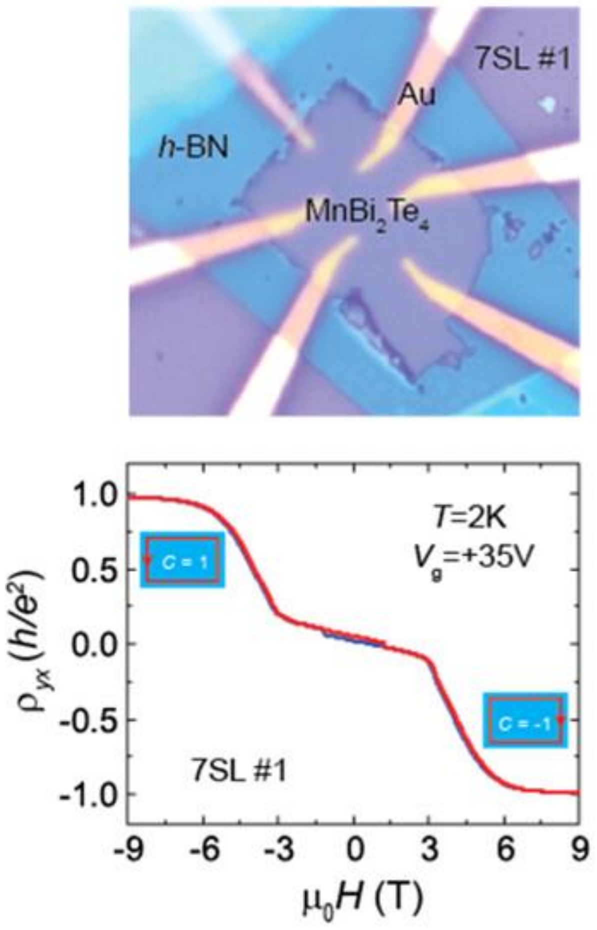

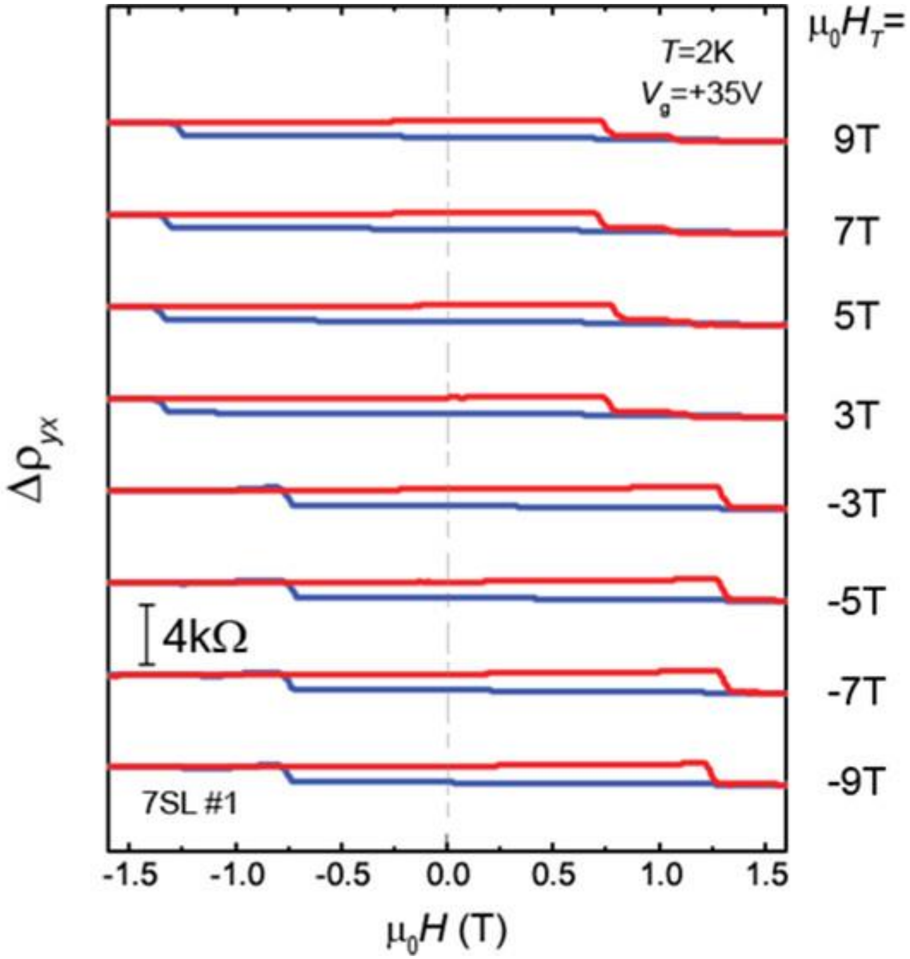

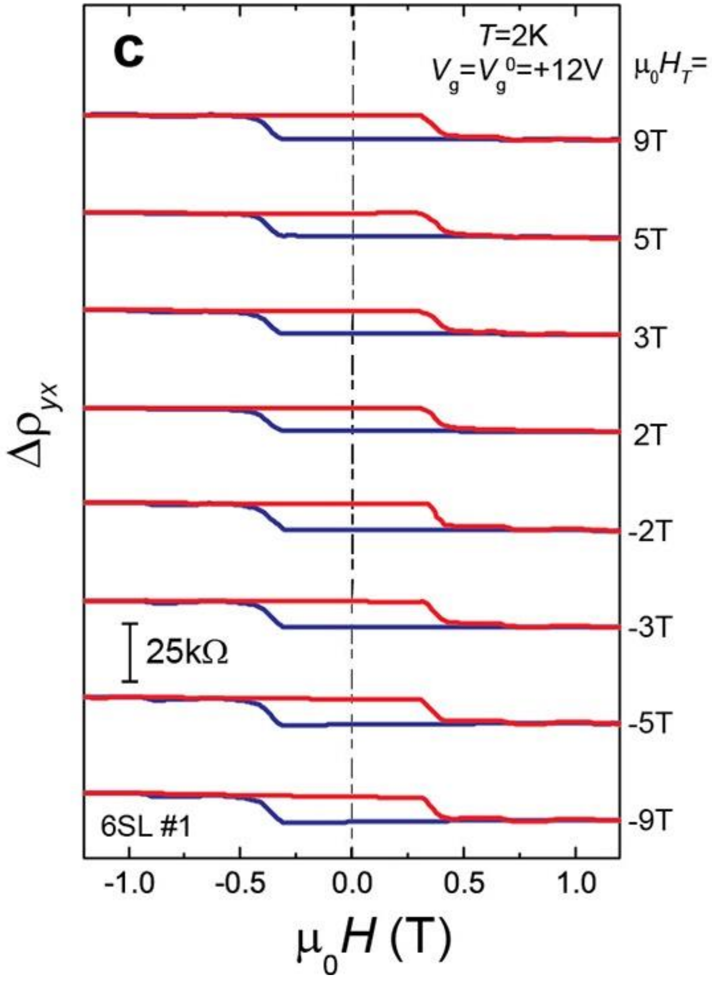

Bo Chen*, Xiaoda Liu*, **YHL***, *et. al.*, *Nano Letters* **24**, 8320–8326 (2024)

1.2 Mag anisotropy-exchange bias effect

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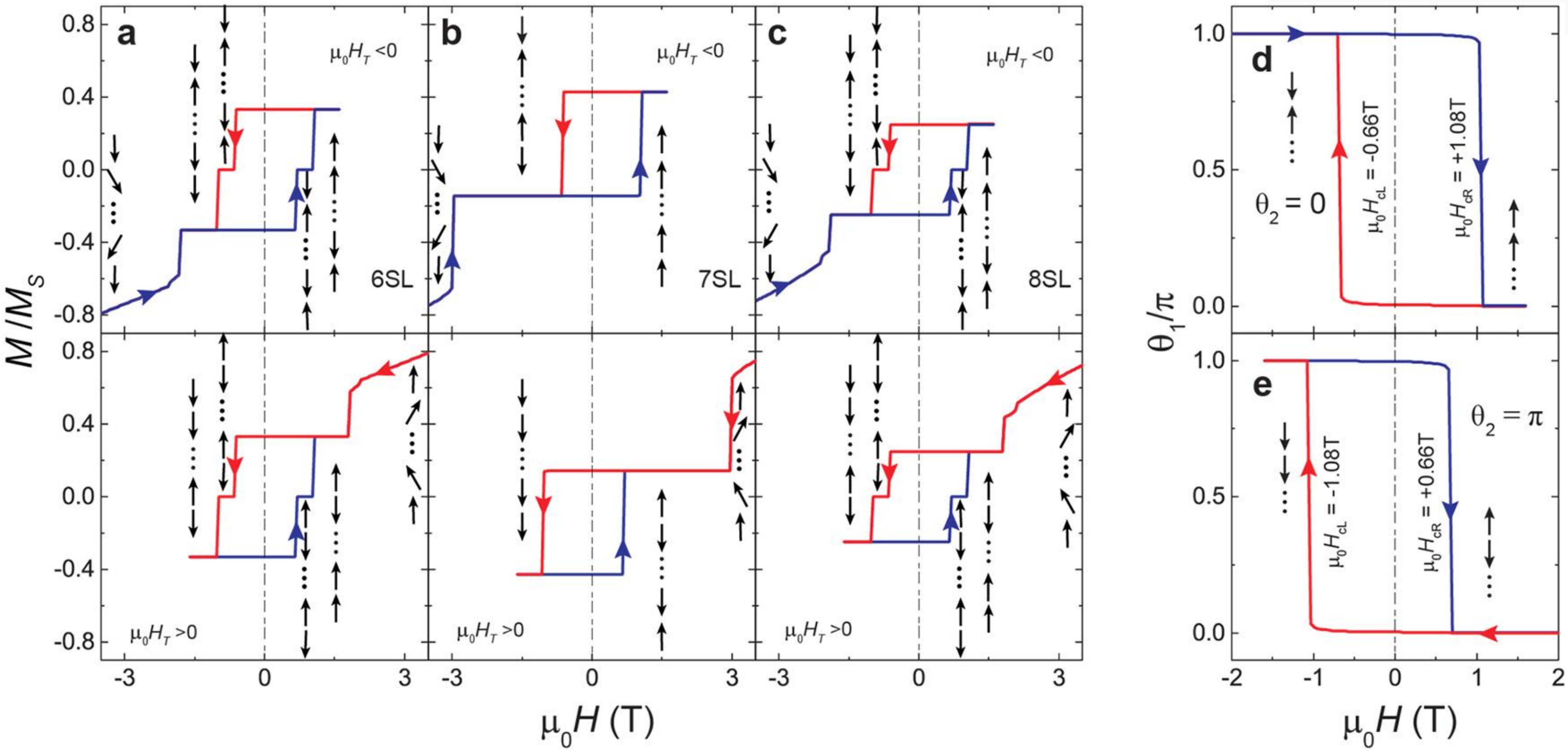
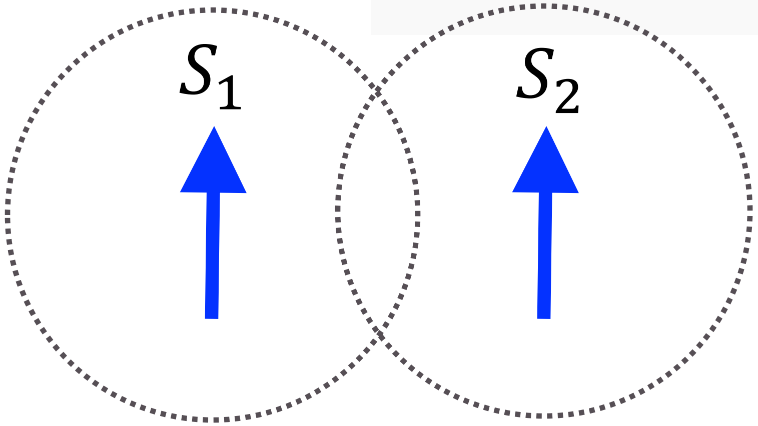


Bo Chen*, Xiaoda Liu*, **YHL***, *et. al.*, *Nano Letters* **24**, 8320–8326 (2024)

1.3 Exchange interaction

- Hamiltonian

$$H = \sum_{i=1,2} \frac{p_i^2}{2m} + V(r_1 - r_2)$$



Wavefunction: $\Psi = \psi(1,2)\chi(1,2)$

$$\psi(1,2) = \frac{\phi_1\phi_2 \pm \phi_2\phi_1}{\sqrt{2}} \qquad \chi(1,2) = |\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle \Leftrightarrow \chi(1,2) = |0,0\rangle, |1, m_s\rangle$$

$$\psi_1(1,2) = \frac{\phi_1\phi_2 + \phi_2\phi_1}{\sqrt{2}} \qquad \chi_1(1,2) = |0,0\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$$

$$\psi_2(1,2) = \frac{\phi_1\phi_2 - \phi_2\phi_1}{\sqrt{2}} \qquad \chi_2(1,2) = |1, m_s\rangle$$

1.3 Exchange interaction

- The wave function must be antisymmetric

$$\Psi_1 = \psi_1(1,2)\chi_1(1,2) \qquad \Psi_2 = \psi_2(1,2)\chi_2(1,2)$$

- The Coulomb's energy: $\Psi_i \mathrm{H} \Psi_i = \psi_i H \psi_i$

$$K = \iint dr_1 dr_2 |\phi_1(r_1)|^2 |\phi_2(r_2)|^2 V(r_1 - r_2)$$

$$J = \iint dr_1 dr_2 \phi_1(r_1)\phi_2(r_1) V(r_1 - r_2)\phi_2(r_2)\phi_1(r_2)$$

$$V_{sym}(r_1 - r_2) = K + J \qquad V_{anti}(r_1 - r_2) = K - J$$

$$\Psi_i \mathrm{H} \Psi_i = \psi_i H \psi_i = \chi_i H_S \chi_i \Rightarrow H_S = -J\vec{S}_1 \cdot \vec{S}_2$$

Direct exchange!!!

1.3 Exchange-indirect exchange

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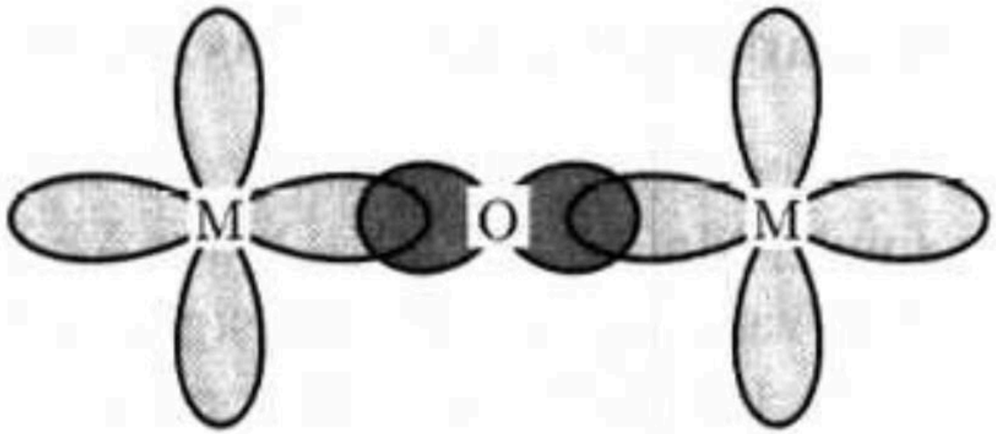
- Superexchange interaction



Two magnetic atoms are coupled through another nonmagnetic atom –such as oxygen

Mostly antiferromagnetic coupled---180°

Insulating materials

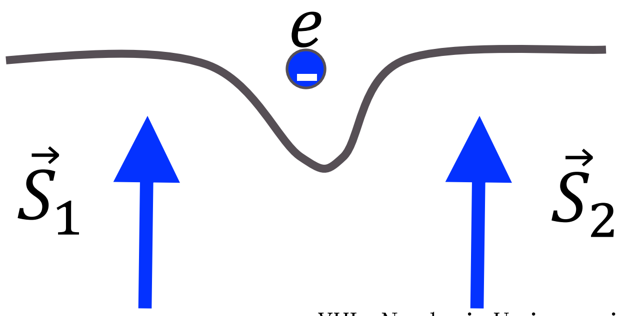
- Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction

Two magnetic atoms are coupled through conducting electrons

$$H = \lambda \sum_i \vec{\sigma} \cdot \vec{S}_i \delta(\vec{r} - \vec{R}_i)$$ *sd*-interaction

Interaction strength oscillates with distance r

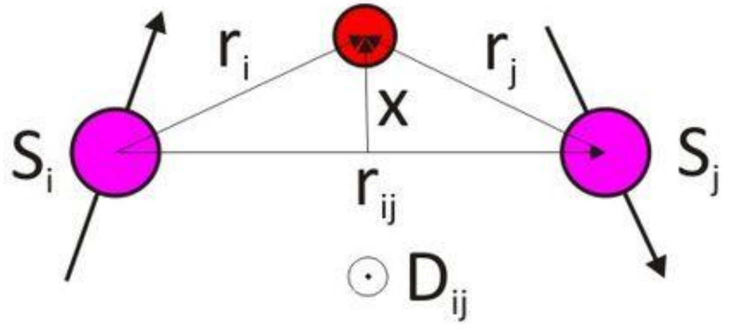
Conducting materials $J_{RKKY} \sim \cos(2k_F r)/r^3$



1.3 Exchange-asymmetric exchange

$$H_1 = \lambda_i \vec{\sigma} \cdot \vec{S}_i \delta(\vec{r} - \vec{R}_i) + \lambda_j \vec{\sigma} \cdot \vec{S}_j \delta(\vec{r} - \vec{R}_j) + l\vec{\sigma} \cdot \vec{L}$$

Spin-orbit coupling



Recall

$$\sum_{\sigma_1,\sigma_2,\sigma_3} \langle \sigma_1 | \vec{\sigma} \cdot \vec{S}_i | \sigma_2 \rangle \langle \sigma_2 | \vec{\sigma} | \sigma_3 \rangle \langle \sigma_3 | \vec{\sigma} \cdot \vec{S}_j | \sigma_1 \rangle = \mathrm{Tr}_\sigma (\vec{\sigma} \cdot \vec{S}_i) \vec{\sigma} (\vec{\sigma} \cdot \vec{S}_j) = -\frac{i}{4} \vec{S}_i \times \vec{S}_j$$

Applying the second order perturbation theory gives

$$H_{ani} = \vec{D} \cdot \vec{S}_i \times \vec{S}_j$$

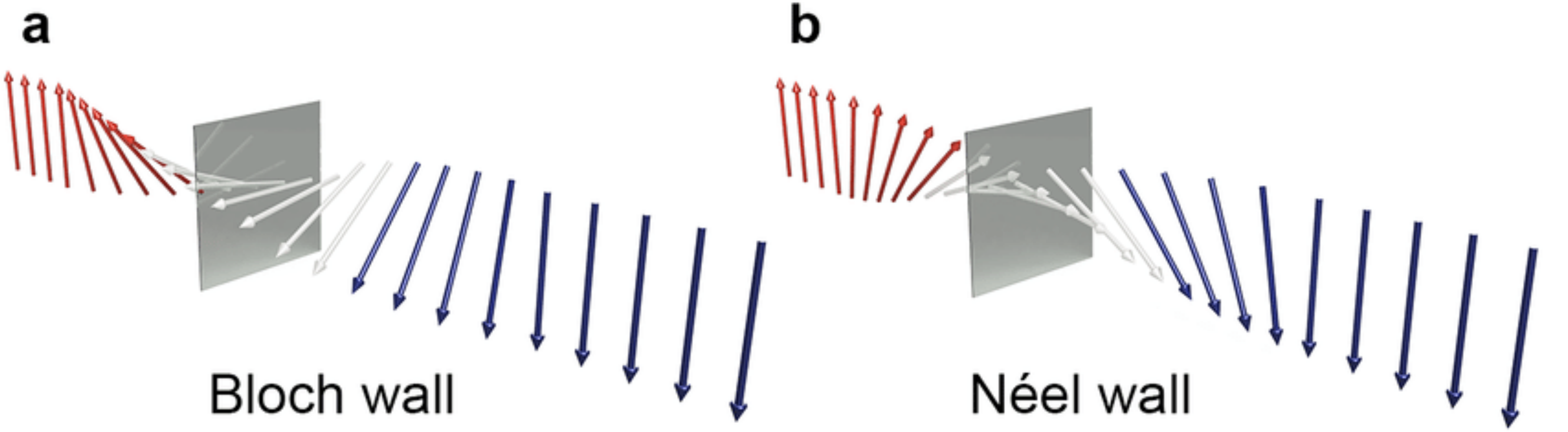
Dzyaloshinskii–Moriya interaction

1.3 Exchange interaction

- Magnetic dipolar interaction

Weak but long range!!!

$$H = \frac{\mu_0}{4\pi r^3}\left[\vec{\mu}_1 \cdot \vec{\mu}_2 - \frac{3}{r^2}(\vec{\mu}_1 \cdot \vec{r})(\vec{\mu}_2 \cdot \vec{r})\right] \quad \sim 10^{-23}\text{J, or } 0.1 \text{ meV}$$



1.4 Magnetic ordering

- Spontaneous symmetry breaking

Hamiltonian preserves the symmetry while the ground state does not!!!

$$H = J\sum_{ij} \vec{S}_i \cdot \vec{S}_j$$ Preserve SU(2) spin rotation symmetry

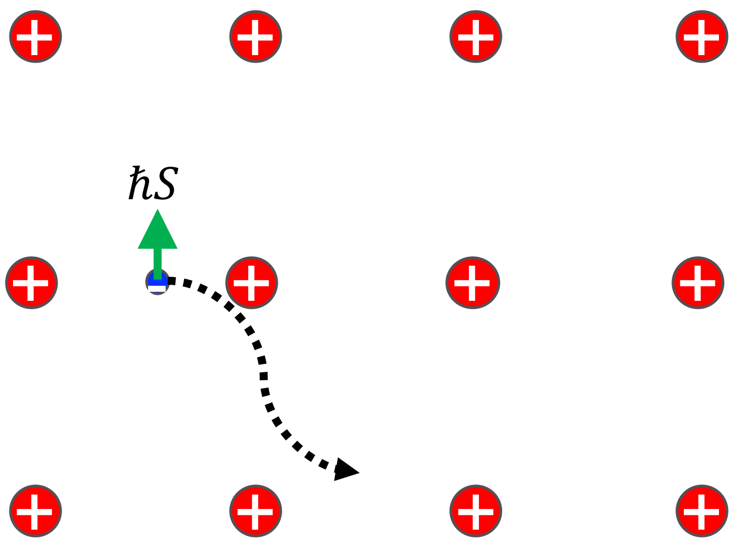
$$|\psi\rangle = |\uparrow_1\uparrow_2 \cdots\rangle$$ Breaks SU(2) spin rotation symmetry

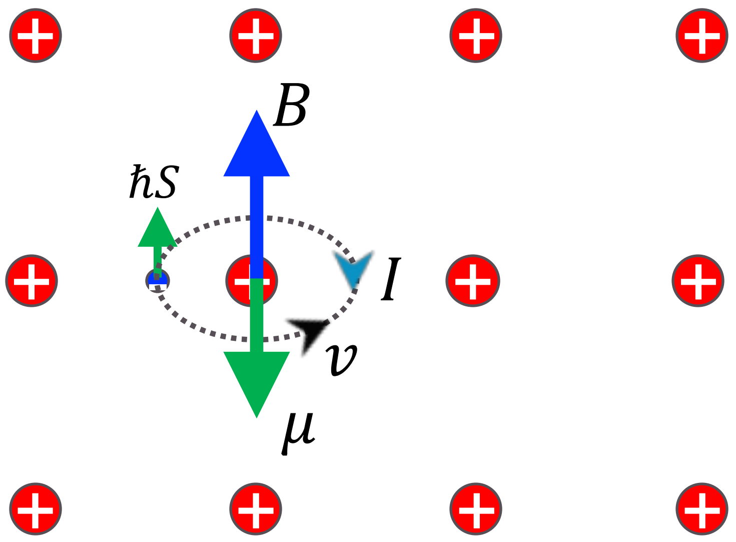
- Mermin-Wegner theorem

1. Short range interaction, 2. $d \leq 2, T > 0$, 3. continuous symmetry

⇨ **4. No spontaneous symmetry breaking**

1.4 Magnetic ordering-diamagnet





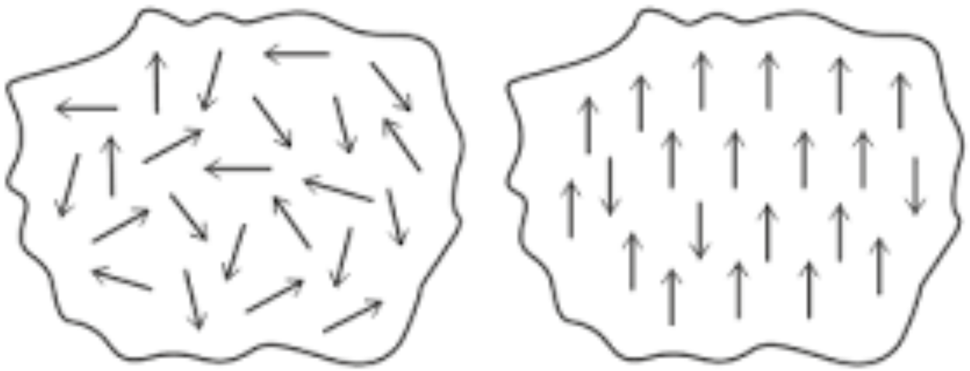
- Larmor diamagnet: $\chi = \frac{-ne^2\langle r^2\rangle}{6m\mu_0}$

What if B is vary large???

1.4 Magnetic ordering-paramagnet

- Independent spins

$\vec{B} = 0$ $\vec{B} \neq 0$



- Hamiltonian $H = g\mu_B \vec{S} \cdot \vec{B}$

$$E_{m_s} = g\mu_B B m_s$$

$$\psi_{m_s} = |s, m_s\rangle$$

- Boltzmann distribution function: $\rho(m_s) \propto \exp\{-E/k_B T\}$

$$M(B) = \frac{\sum_{m_s=-s,s} m_s \exp(-g\mu_B B m_s/k_B T)}{\sum_{m_s=-s,s} \exp(-g\mu_B B m_s/k_B T)}$$

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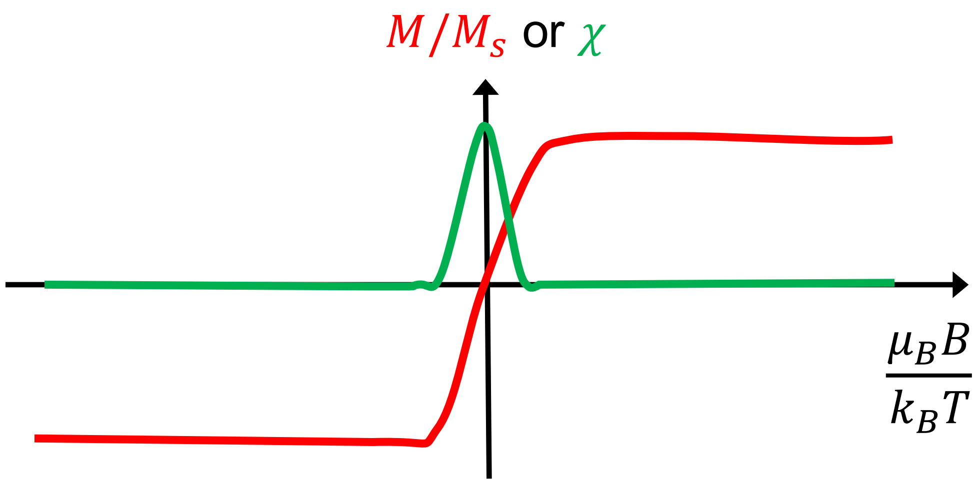
1.4 Magnetic ordering-paramagnet

- Spin $s = 1/2$

$$M(B) = \frac{\mu_B[\exp\left(-\frac{g\mu_B B}{2k_B T}\right) - \exp\left(\frac{g\mu_B B}{2k_B T}\right)]}{2[\exp\left(-\frac{g\mu_B B}{2k_B T}\right) + \exp\left(\frac{g\mu_B B}{2k_B T}\right)]} = \mu_B \tanh(\frac{g\mu_B B}{2k_B T})$$

$$M(B)/M_s = \tanh(\frac{\mu_B B}{k_B T})$$

$$\chi = \frac{\partial M(B)}{\partial B} = \frac{n\mu_0\mu_B^2}{k_B T}\cosh^{-2}(\mu_B B/k_B T)$$



What if large s? continuous s?

1.4 Magnetic ordering-ferromagnet

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- Hamiltonian

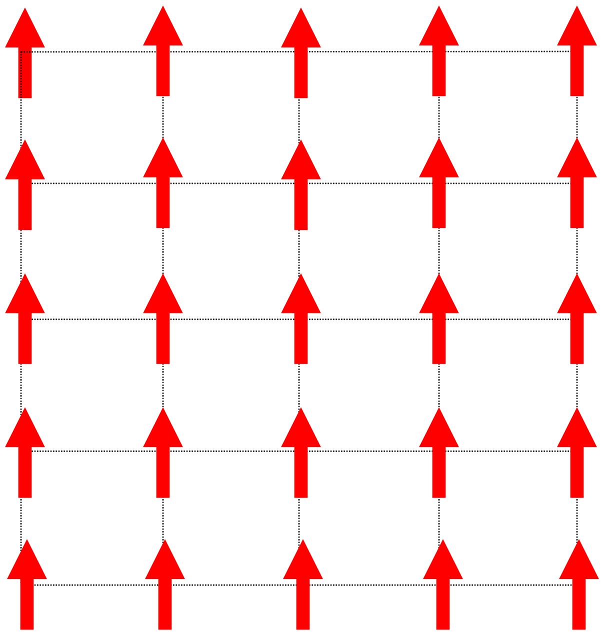
$$H = -J\sum_{\langle ij\rangle}\vec{S}_i\cdot\vec{S}_j - g\mu_B\sum_i \vec{B}\cdot\vec{S}_i$$

$$= -J\sum_{\langle ij\rangle}\vec{S}_i\cdot(\vec{S}_j - \langle\vec{S}_j\rangle + \langle\vec{S}_j\rangle) - g\mu_B\sum_i \vec{B}\cdot\vec{S}_i$$

$$= -J\sum_{\langle ij\rangle}\vec{S}_i\cdot(\langle\vec{S}_j\rangle + \delta\vec{S}_j) - g\mu_B\sum_i \vec{B}\cdot\vec{S}_i$$

$$\approx -J\sum_{\langle ij\rangle}\vec{S}_i\cdot\langle\vec{S}_j\rangle - g\mu_B\sum_i \vec{B}\cdot\vec{S}_i = \sum_i \vec{S}_i\Big(-g\mu_B\vec{B} - J\sum_{\langle j\rangle}\langle\vec{S}_j\rangle\Big)$$

$J > 0$

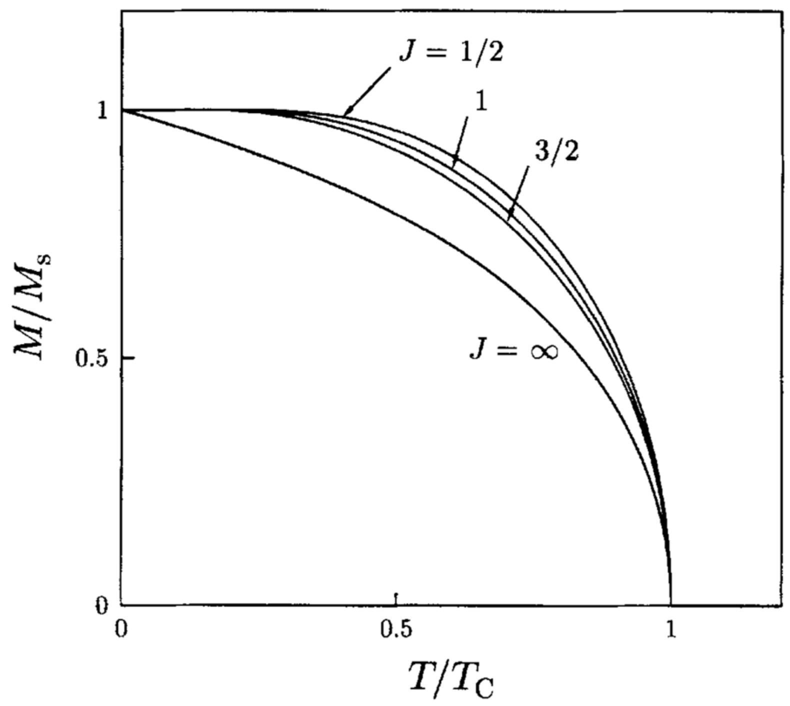


What if $d \leq 2$?

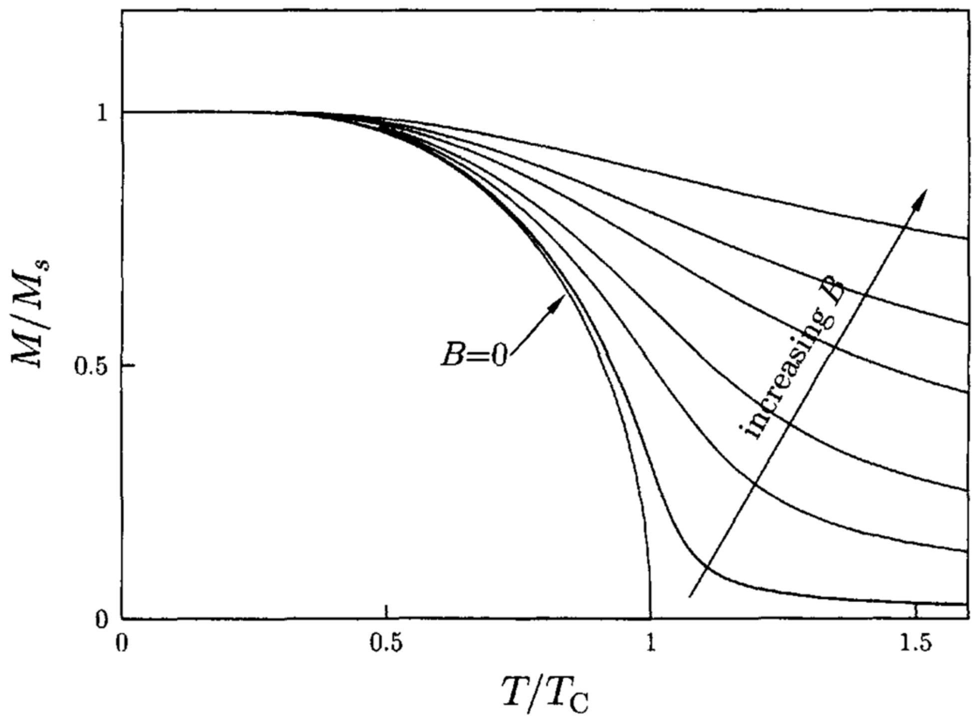
1.4 Magnetic ordering-ferromagnet

- Magnetization curve

At $B = 0$



$B \neq 0$



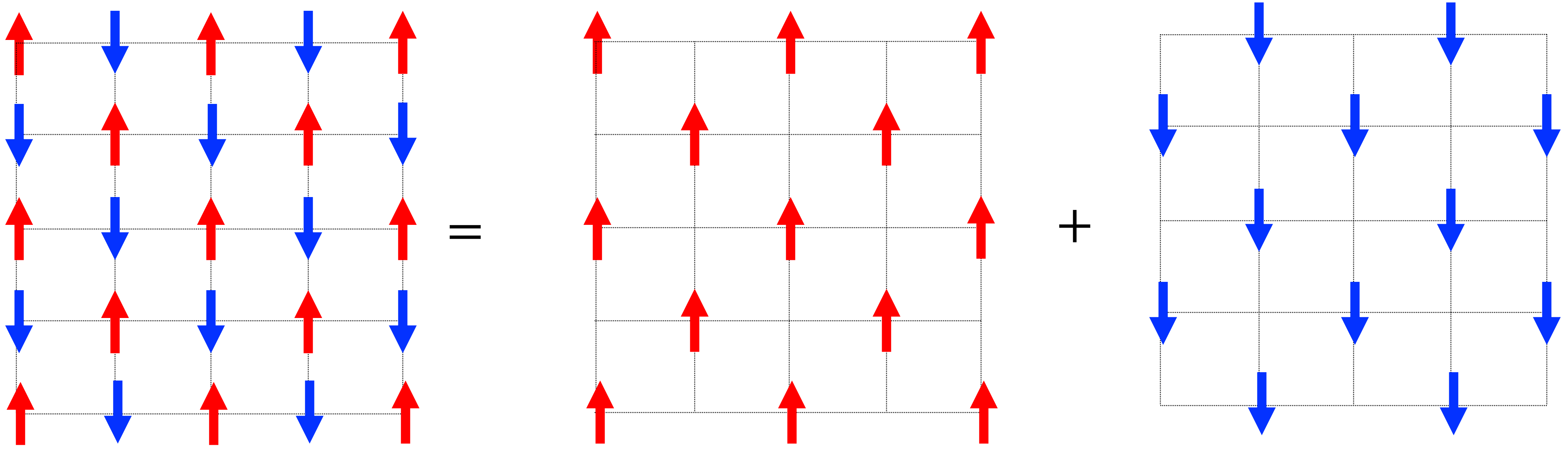
1.4 Magnetic ordering-antiferromagnet

- Hamiltonian

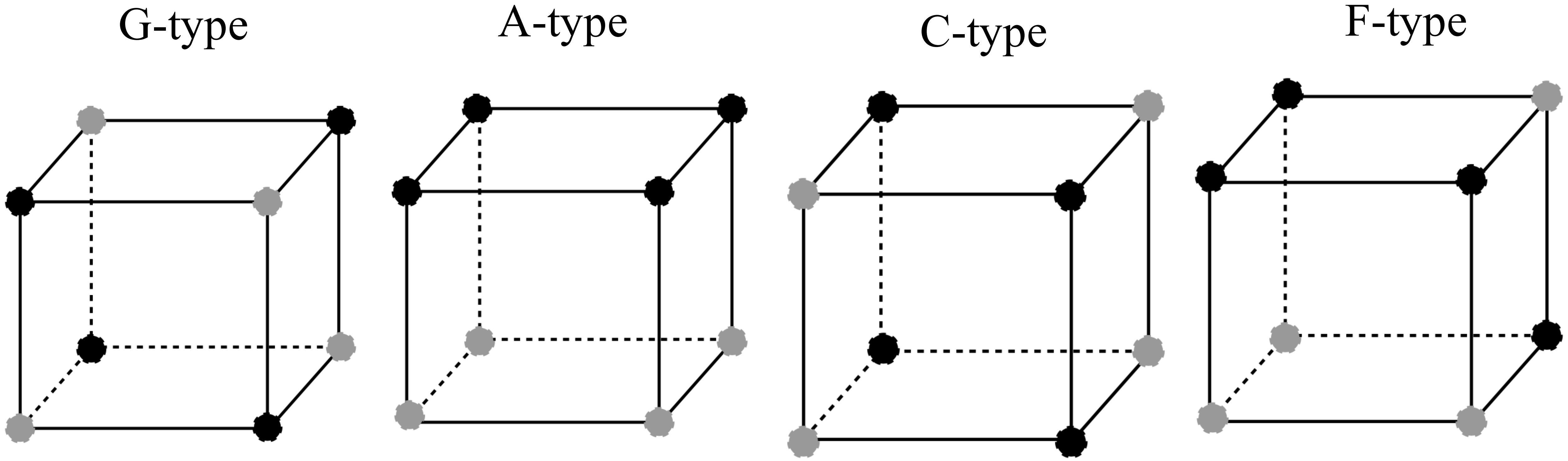
The solution to $d \leq 2$

$$H = J\sum_{\langle ij\rangle} \vec{S}_i \cdot \vec{S}_j - K\sum_i S_{iz}^2 - g\mu_B \sum_i \vec{B} \cdot \vec{S}_i$$

$J > 0$



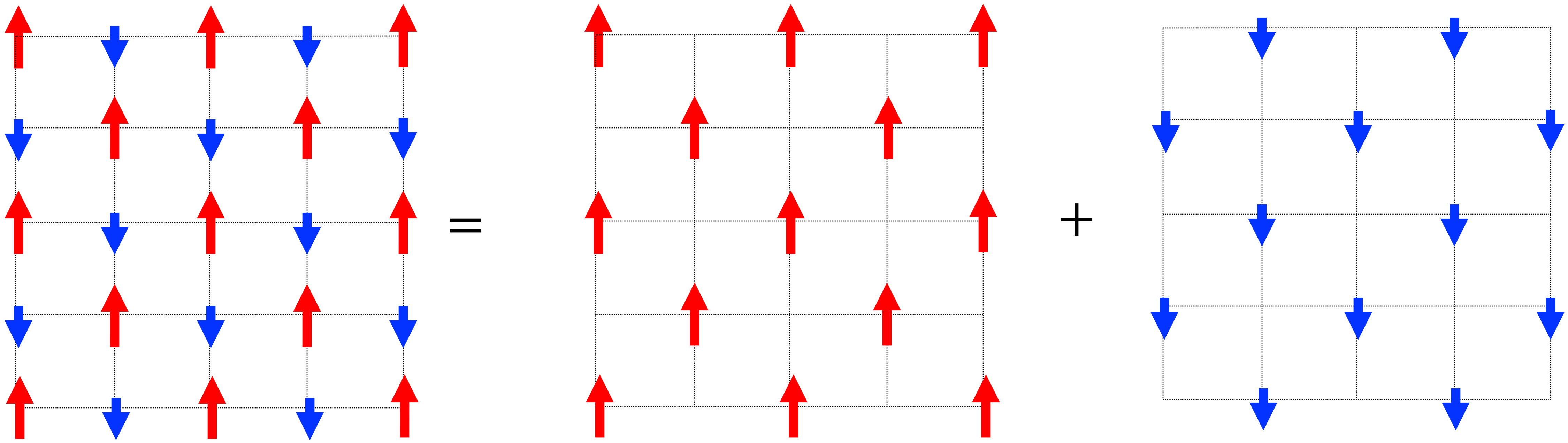
1.4 Magnetic ordering-antiferromagnet



1.4 Magnetic ordering-ferrimagnet

- Hamiltonian $J > 0,\ S_1 \neq S_2$

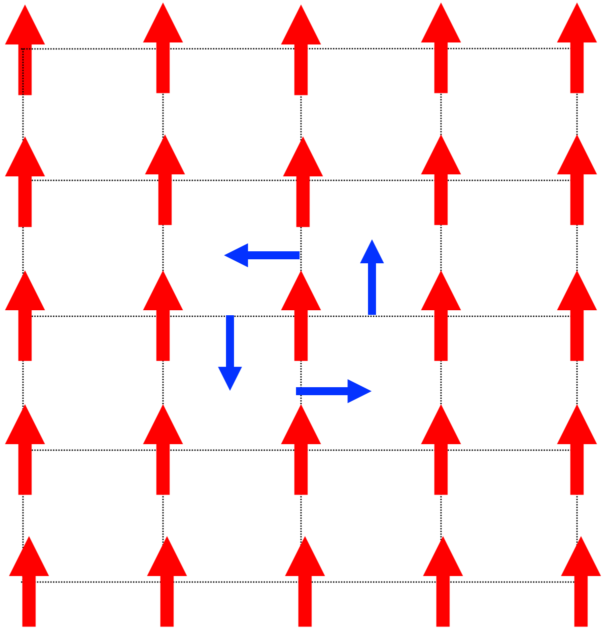
$$H = J\sum_{\langle ij\rangle}\vec{S}_i\cdot\vec{S}_j - K\sum_i S_{iz}^2 - g\mu_B\sum_i \vec{B}\cdot\vec{S}_i$$

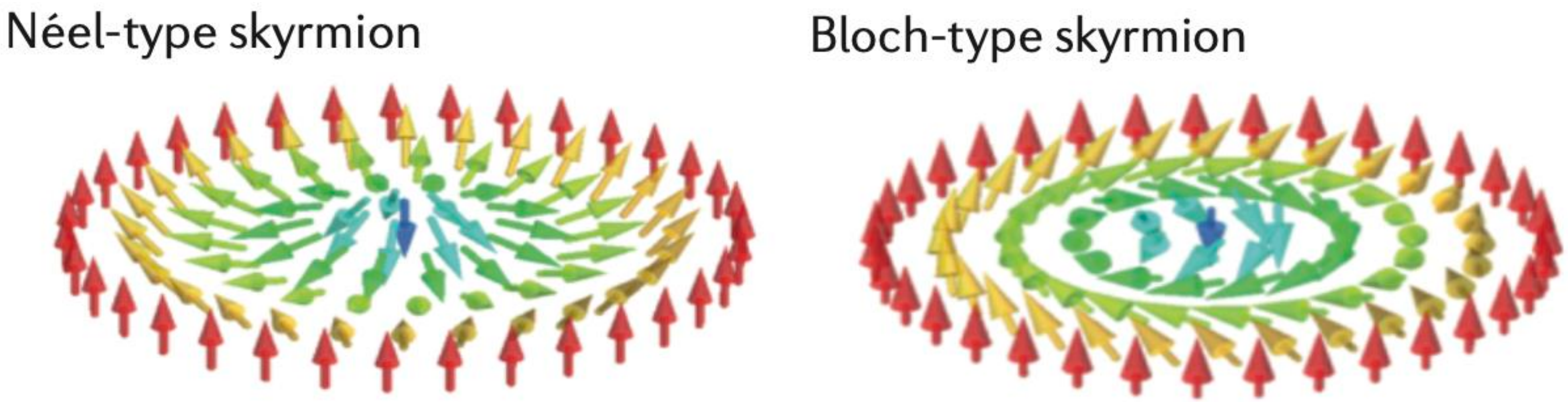


1.4 Magnetic ordering-noncolinear

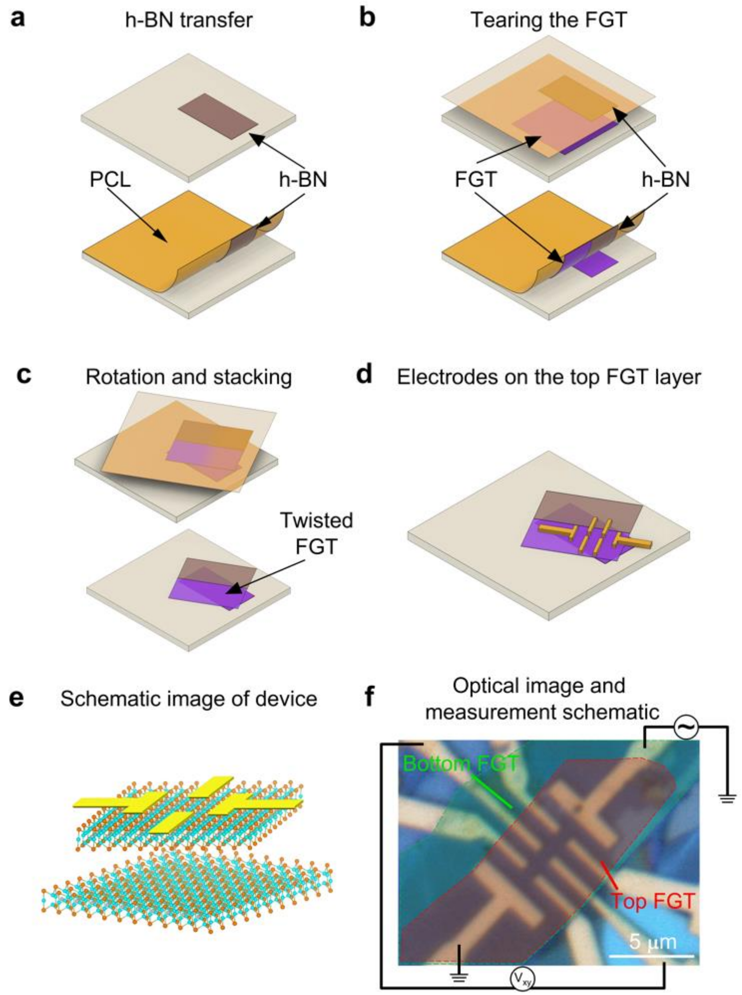
- Hamiltonian

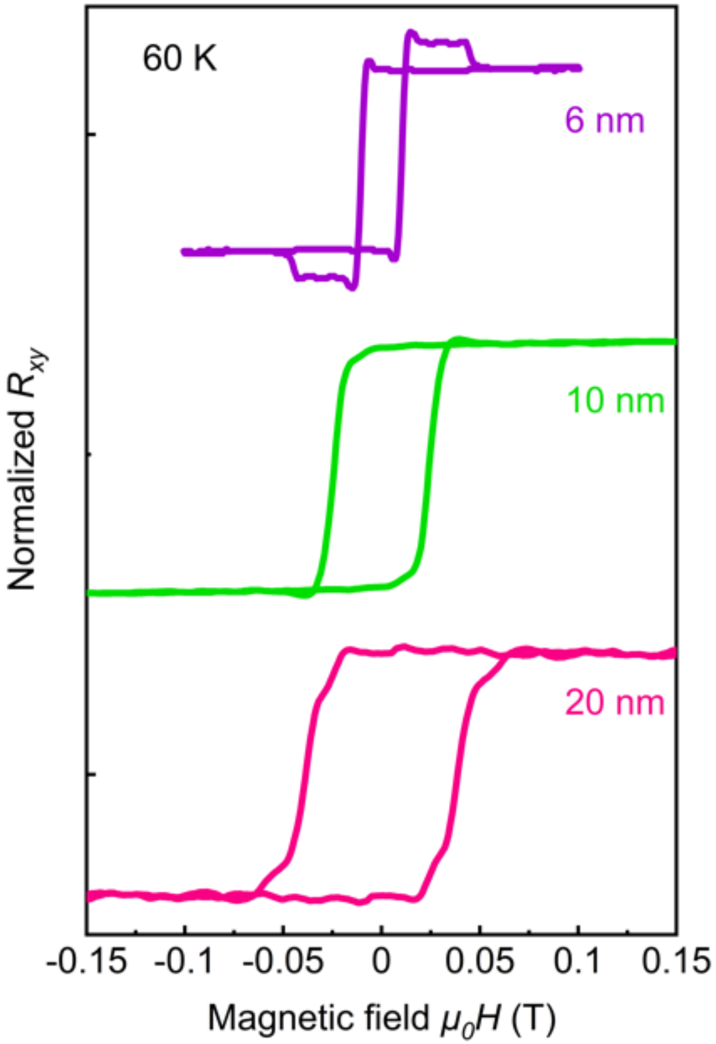
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + \sum_{\langle ij \rangle} (\vec{D} \times \vec{\epsilon}_{ij}) \cdot (\vec{S}_i \times \vec{S}_j) - K \sum_i S_{iz}^2 - g\mu_B \sum_i \vec{B} \cdot \vec{S}_i$$





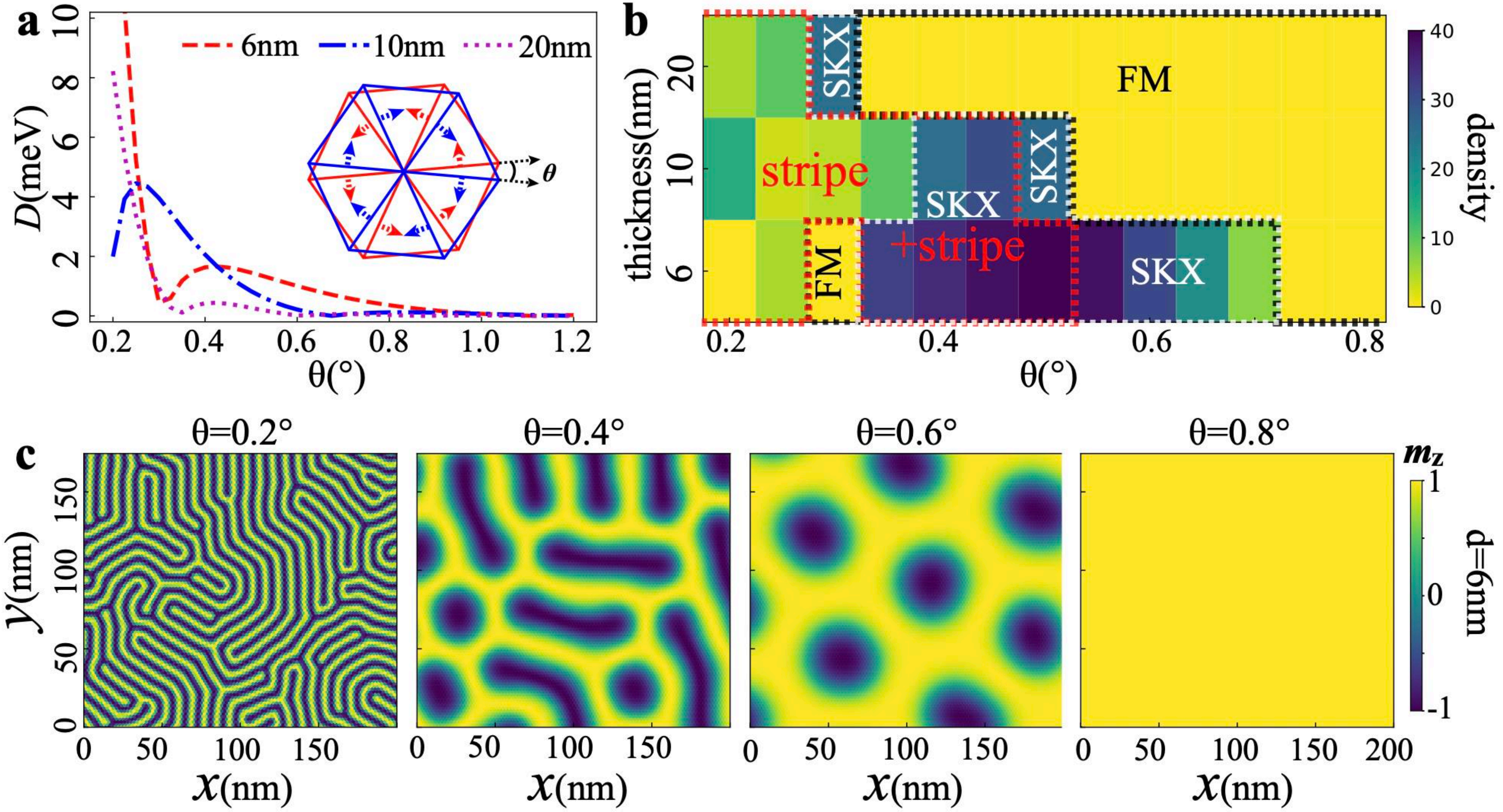
1.4 Magnetic ordering-noncolinear





Kim, Zhang, **YHL**, *et. al.*, Topological Hall effect in twisted Fe_3GeTe_2 metallic system, under review in *Nat. Commun.*

1.4 Magnetic ordering-noncolinear



Kim, Zhang, **YHL**, *et. al.*, Topological Hall effect in twisted Fe_3GeTe_2 metallic system, under review in *Nat. Commun.*

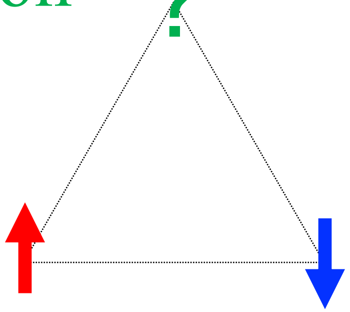
1.4 Magnetic ordering-disordered

- Hamiltonian

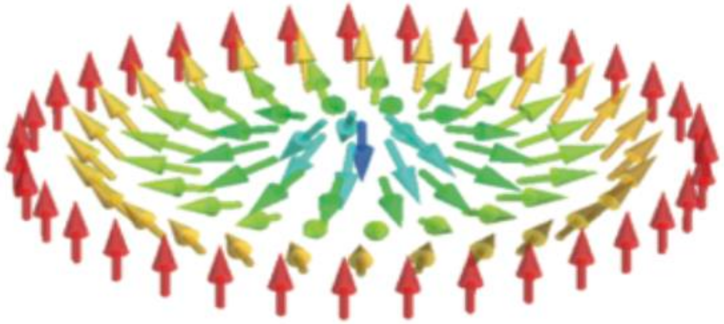
$J > 0$

$$H = J \sum_{\langle ij \rangle} S_i \cdot S_j - K \sum_i S_{iz}^2 - g\mu_B \sum_i B \cdot S_i$$

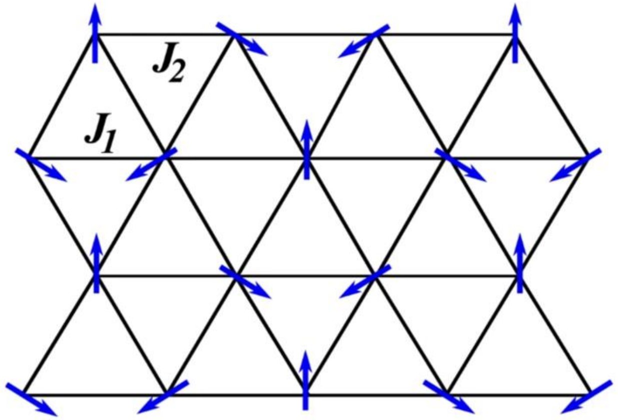
frustration ?



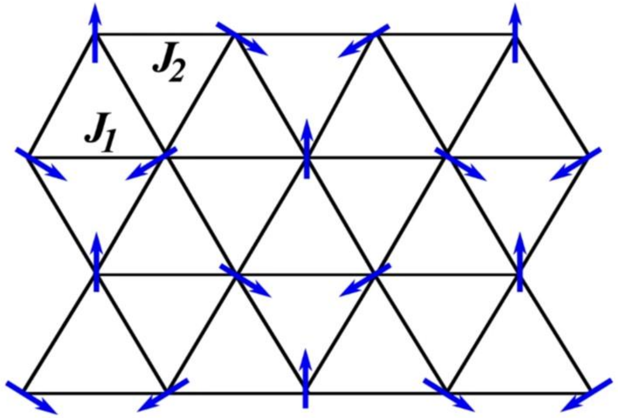
Skyrmion?



Spin liquid?



Spin glass?



How to determine the magnetic ground state???

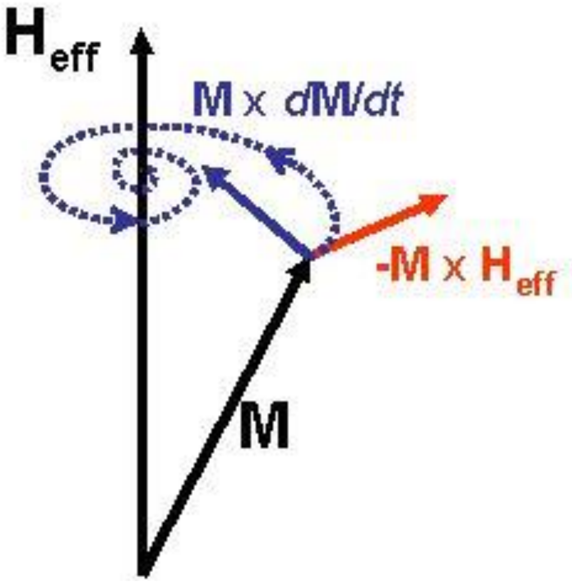
Outline

1. Static magnetic properties
   - Magnetic moment
   - Magnetic anisotropy and Exchange interaction
   - Magnetic ordering
2. Dynamic magnetic properties
   - Landau-Lifshitz-Gilbert (LLG) equation
   - Magnetic resonance
   - Magnetic excitation
3. Magnetic effect

2.1 LLG equation

Nankai University

$$\frac{d\vec{M}}{dt} = -\gamma\left(\vec{M}\times\vec{H}_{eff} - \eta\vec{M}\times\frac{d\vec{M}}{dt}\right) + \cdots \quad \text{(SOT, STT, T…)}$$



γ: gyromagnetic ratio η: damping factor

Effective field: $$\vec{H}_{eff} = \frac{\partial H}{\partial \vec{M}}$$

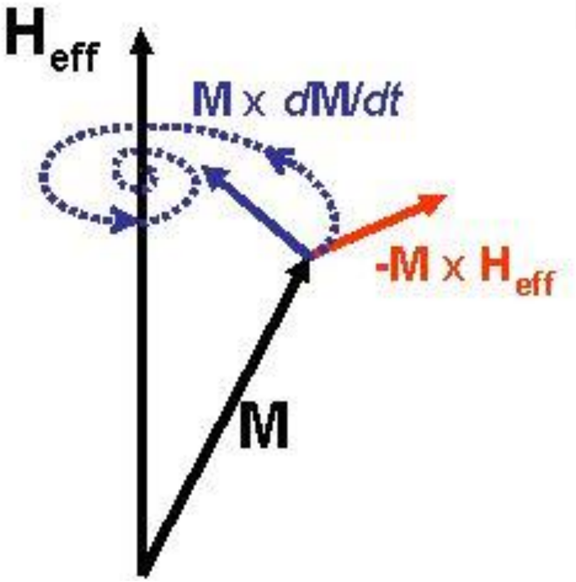
Hamiltonian

$$H = -J\sum_{\langle ij\rangle}\vec{S}_i\cdot\vec{S}_j - K\sum_i S_{iz}^2 - g\mu_B\sum_i \vec{B}\cdot\vec{S}_i + \cdots$$

$$= \int dr[Ja^2(\nabla m)^2 - K(m_z)^2 - g\mu_B(B\cdot m) + \cdots] \qquad \text{For FM}$$

2.2 Magnetic resonance-FMR

$$\frac{d\vec{M}}{dt} = -\gamma\left(\vec{M}\times\vec{H}_{eff} - \eta\vec{M}\times\frac{d\vec{M}}{dt}\right) + \cdots \quad \text{(SOT, STT, Tem…)}$$



$$\frac{d\vec{M}}{dt} = -\gamma\vec{M}\times\vec{H}_{eff} \quad \Longrightarrow \quad \begin{aligned} \frac{dM_x}{dt} &= \gamma M_y(\mu_0 H + KM_z) \\ \frac{dM_y}{dt} &= -\gamma M_x(\mu_0 H + KM_z) \\ \frac{dM_z}{dt} &\approx 0 \end{aligned}$$

$$M_x = m\cos\omega t\,, M_y = m\sin\omega t\,, M_z = 1 \Longrightarrow \ \omega = \gamma H_{eff}$$ Kittle formula

$$H_{eff} \sim 2KM \quad \Longrightarrow \quad f = \frac{\omega}{2\pi} \sim \text{GHz}$$

2.2 Magnetic resonance-AFMR

Hamiltonian

$$H = J\sum_{\langle ij\rangle}\vec{S}_i\cdot\vec{S}_j - K\sum_i S_{iz}^2 - g\mu_B\sum_i \vec{B}\cdot\vec{S}_i$$

$$= \int dr\{J(2m_1\cdot m_2)\ - K[(m_1^z)^2+(m_2^z)^2] - g\mu_B B\cdot(m_1+m_2)\}$$

$$= \int dr\{2Jm_1\cdot m_2 - K[(m_1^z)^2+(m_2^z)^2] - g\mu_B B\cdot(m_1+m_2)\}$$

Effective field

$$\vec{H}_{eff}^1 = \frac{\partial H}{\partial \vec{M}_1} = 2J\vec{M}_2 - 2K\vec{M}_1 - g\mu_B\vec{B} \qquad \vec{H}_{eff}^2 = \frac{\partial H}{\partial \vec{M}_2} = 2J\vec{M}_1 - 2K\vec{M}_2 - g\mu_B\vec{B}$$

2.2 Magnetic resonance-AFMR

$$\frac{d\vec{M}_1}{dt} = -\gamma \vec{M}_1 \times \vec{H}_{eff}^1$$

$$\frac{d\vec{M}_2}{dt} = -\gamma \vec{M}_2 \times \vec{H}_{eff}^2$$

$$\Longrightarrow \quad \frac{d}{dt}\begin{bmatrix} M_1^x \\ M_1^y \\ M_2^x \\ M_2^y \end{bmatrix} = -\gamma \begin{bmatrix} M_1^y H_{eff}^1 \\ -M_1^x H_{eff}^1 \\ M_2^y H_{eff}^2 \\ -M_1^x H_{eff}^2 \end{bmatrix}$$

Solve this eigenequation, we obtain:

$$f = \frac{\omega}{2\pi} = \gamma\sqrt{H_E(H_A \pm H_B)} \sim \text{THz}$$

AFMR is on the order of THz, which is about three orders of magnitude larger than FMR!!!

Fast dynamics !!!

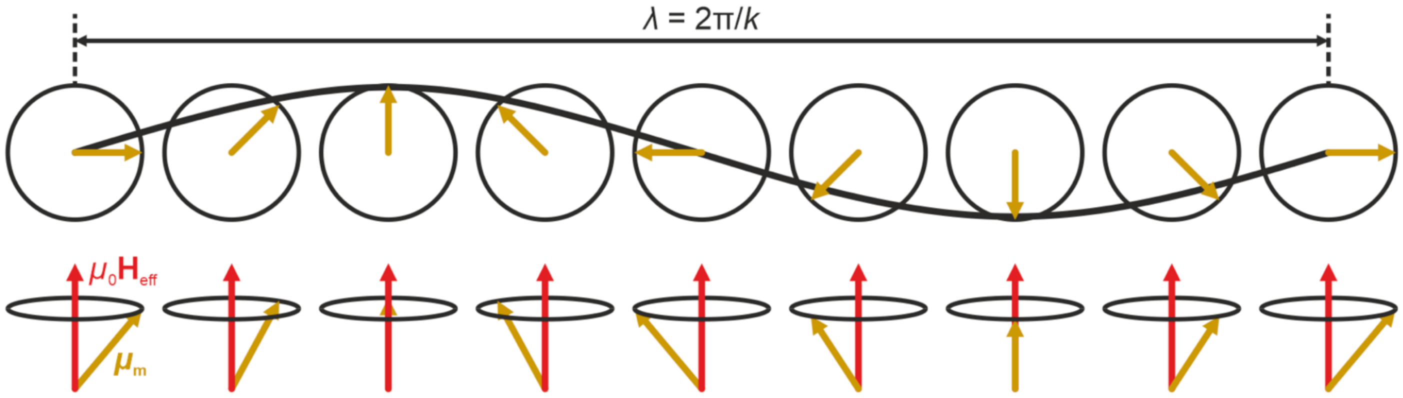
2.3 Magnetic excitation-spin wave

- Goldstone theorem

continuous symmetry is spontaneous breaking

gapless boson excitation!!!

$$T > 0$$



Hamiltonian

$$H = -J\sum_{\langle ij\rangle}\vec{S}_i\cdot\vec{S}_j$$

$$\frac{d\vec{S}_i}{dt} = -\frac{i}{\hbar}[\vec{S}_i, H] = \frac{2J}{\hbar}[\vec{S}_i, \vec{S}_j\times(\vec{S}_{j-1}+\vec{S}_{j+1})]$$

$$\hbar\frac{dS_j^x}{dt} = 2JS(2S_j^y - S_{j-1}^y - S_{j+1}^y)$$

$$\hbar\frac{dS_j^y}{dt} = -2JS(2S_j^x - S_{j-1}^x - S_{j+1}^x)$$

$$S_j^x = u\exp[i(jka-\omega_k t)]$$

$$S_j^y = v\exp[i(jka-\omega_k t)]$$

$$\epsilon_k = \hbar\omega_k = 4JS(1-\cos ka)$$

2.3 Magnetic excitation-HP transformation

Nankai University 南開大學

$$[S_\alpha, S_\beta] = i\epsilon_{\alpha\beta\gamma}\hbar S_\gamma$$

$$|s, s-n\rangle \mapsto \frac{1}{\sqrt{n!}}\left(a^\dagger\right)^n |0\rangle_B$$

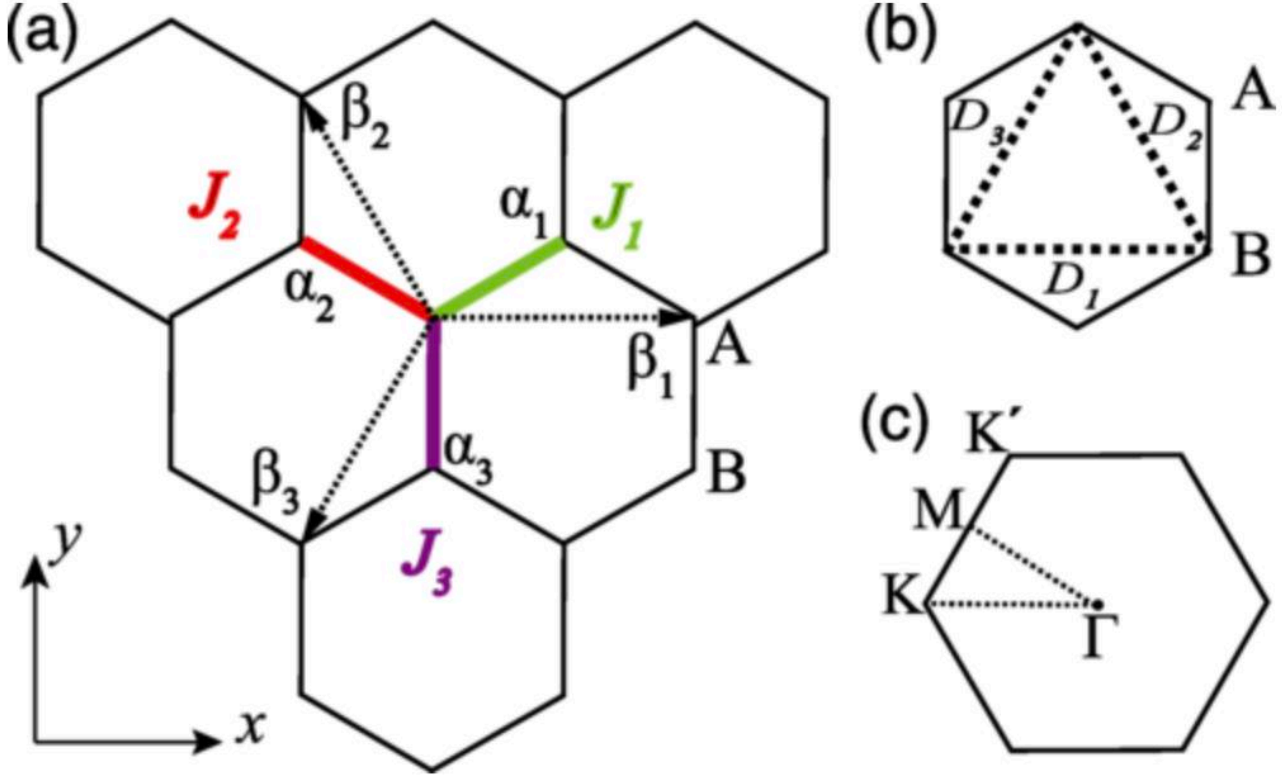
$$S^2\,|s, m_s\rangle = \hbar^2 s(s+1)\,|s, m_s\rangle\,,$$
$$S_z\,|s, m_s\rangle = \hbar m_s\,|s, m_s\rangle\,.$$

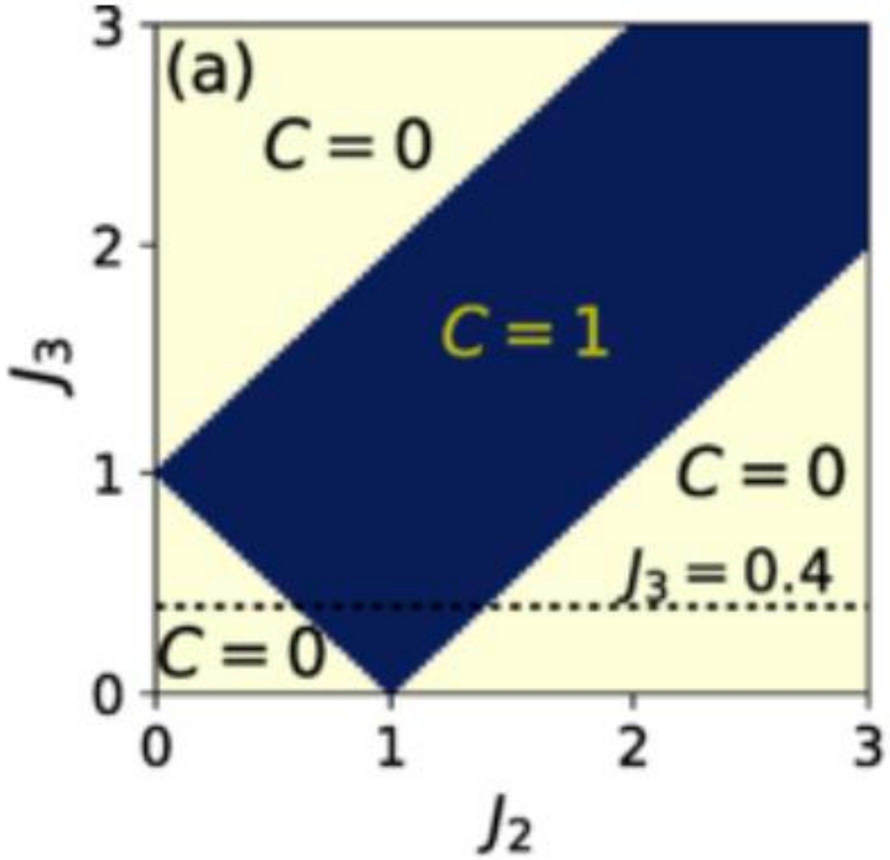
$$S_+ = S_x + iS_y,\ S_- = S_x - iS_y$$

Holstein-Primakoff Transformation

$$S_+ = \hbar\sqrt{2s}\sqrt{1-\frac{a^\dagger a}{2s}}\,a\,, \qquad S_- = \hbar\sqrt{2s}a^\dagger\sqrt{1-\frac{a^\dagger a}{2s}}\,, \qquad S_z = \hbar(s - a^\dagger a)\,.$$

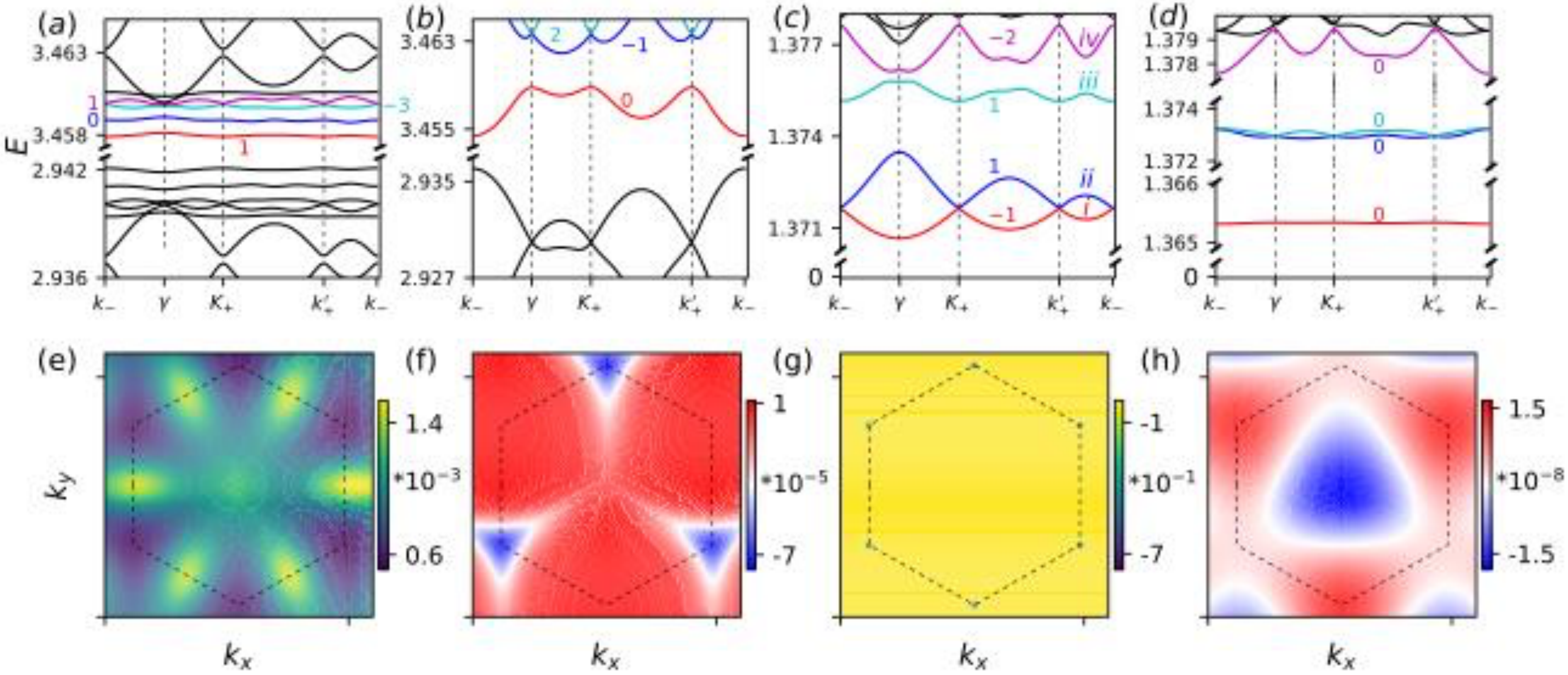
2.3 Magnetic excitation





YHL# and Ran Cheng#, *Physical Review B* **103**, 014407 (2021)

2.3 Magnetic excitation



YHL# and Ran Cheng#, *Physical Review B* **102**, 094404 (2020)

Outline

1. Static magnetic properties
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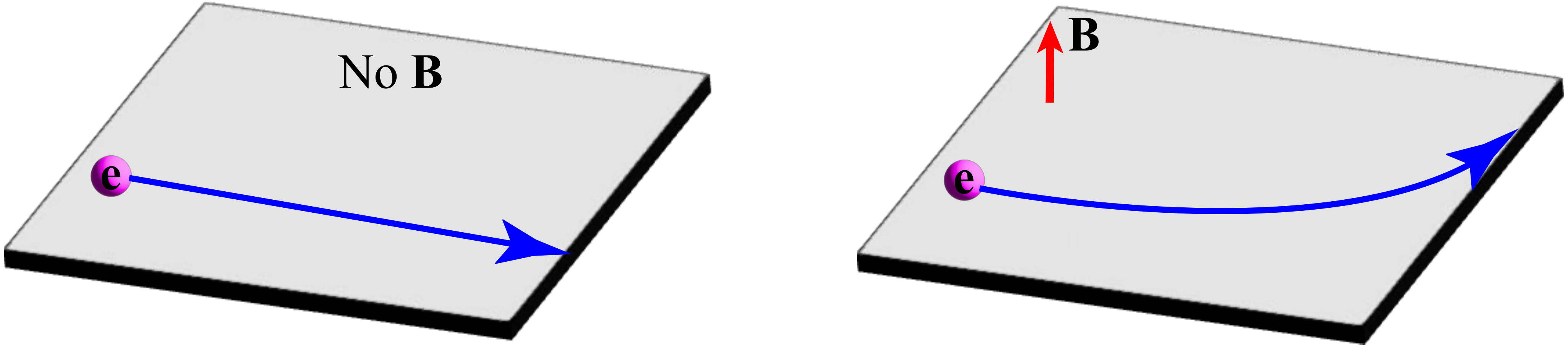
   - Landau-Lifshitz-Gilbert (LLG) equation
   - Magnetic resonance
   - Magnetic excitation
3. Magnetic effect

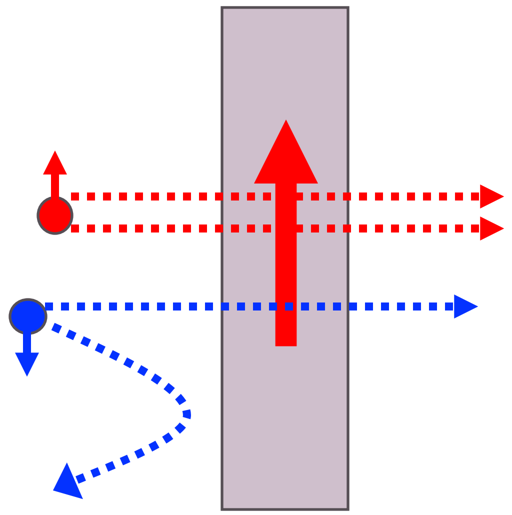
3.1 Magnetoresistance

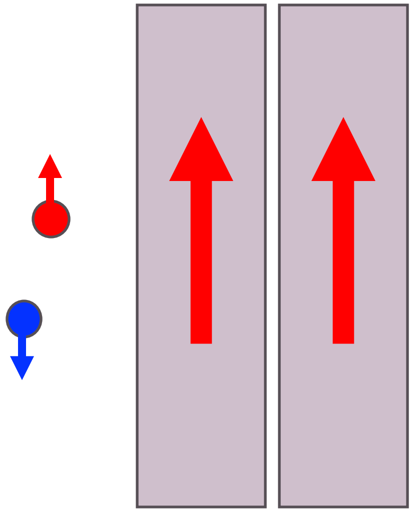


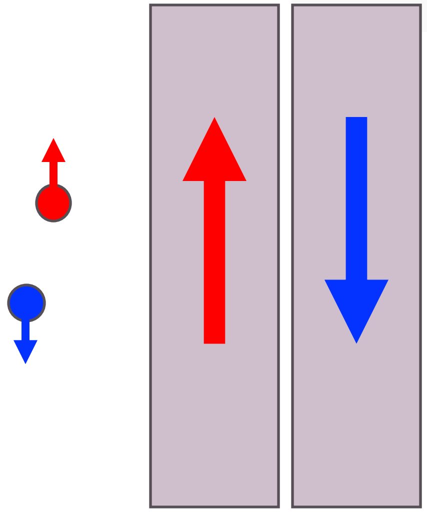
$$MR\% = \frac{R(\mu_0 \vec{B}) - R(0)}{R(0)} \times 100\%$$

Typically $< 2\%$

3.2 Giant Magnetoresistance







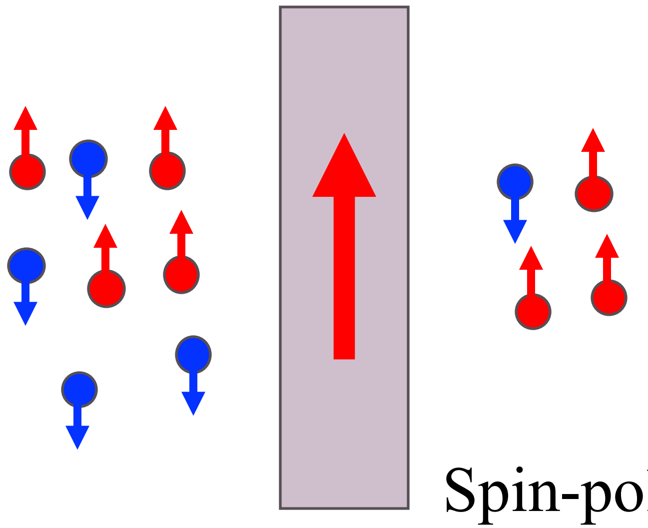
$$\frac{1}{R_L} = \frac{1}{2R_{\uparrow\uparrow}} + \frac{1}{2R_{\downarrow\downarrow}} \qquad \frac{1}{R_R} = \frac{1}{R_{\uparrow\uparrow} + R_{\uparrow\downarrow}} + \frac{1}{R_{\downarrow\uparrow} + R_{\downarrow\downarrow}}$$

$R_{\sigma\sigma} < R_{\sigma\bar{\sigma}}$

$R_L < R_R$ Typical value $MR\% \sim 10\% - 100\%$

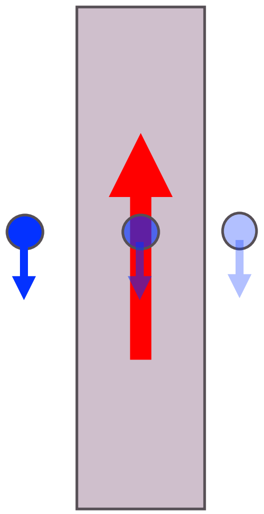
The Nobel Prize in Physics 2007 was awarded jointly to Albert Fert and Peter Grünberg "for the discovery of ***Giant Magnetoresistance***"

3.3 Spin pumping and spin torque

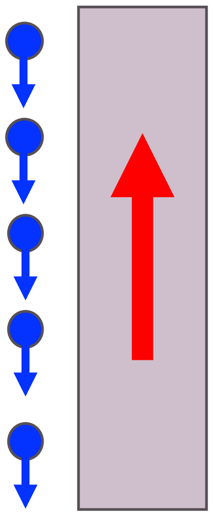


Spin-polarized

Unpolarized electron current



Spin transfer torque



Spin orbit torque

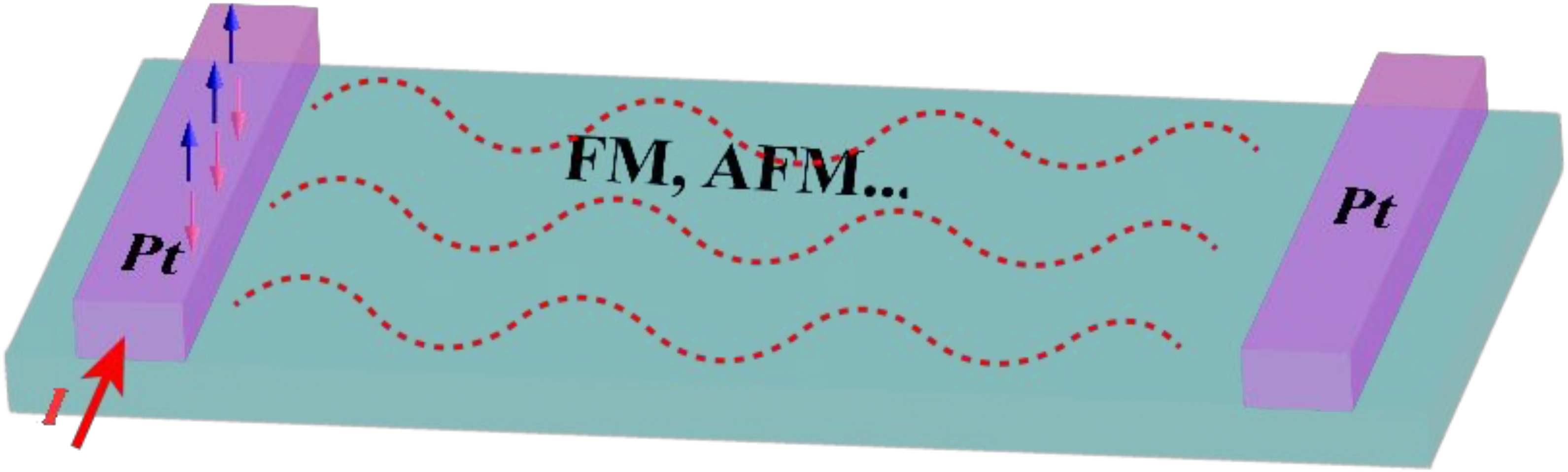
Spin pumping

$$H_{eff} = J\vec{s} \cdot \vec{S}$$

$$\tau_{field} = \frac{\partial \vec{S}}{\partial t} = c_1 \vec{s} \times \vec{S}$$

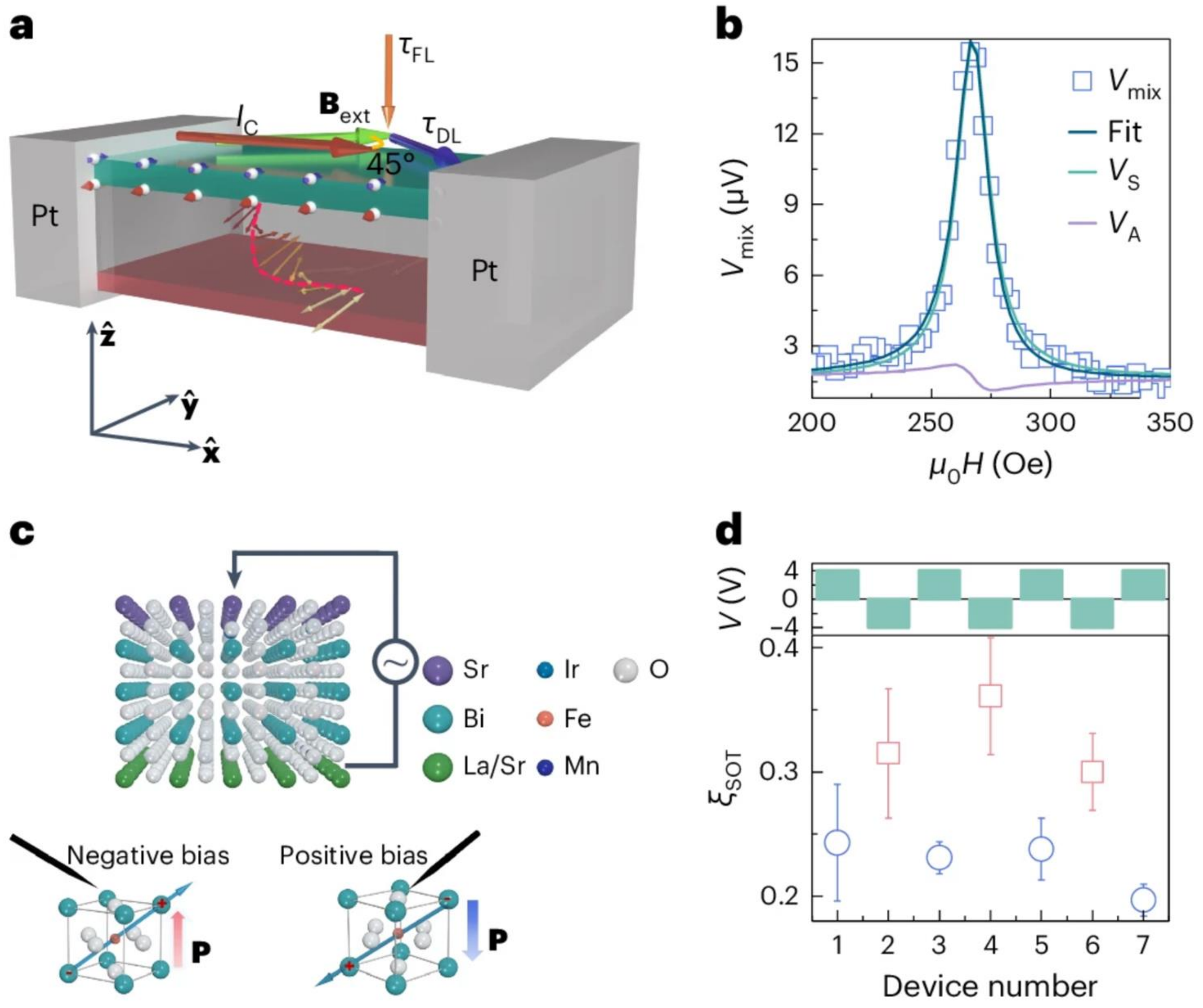
$$\tau_{damping} = c_2 (\vec{s} \times \vec{S}) \times \vec{S}$$

3.4 Spintronics



- Spin Hall effect and inverse spin Hall effect

3.4 Spintronics



$$J_{\rm s} = \frac{G_{\rm L}}{S}\int {\rm d}\mathbf{r}^2\,[\mathbf{L}(\mathbf{r})\cdot\mathbf{s}]^2 + \frac{G_{\rm m}}{S}\int {\rm d}\mathbf{r}^2\,[\mathbf{m}(\mathbf{r})\cdot\mathbf{s}]^2$$

$$\xi = \frac{J_{\rm s}(-\mathbf{P}) - J_{\rm s}(\mathbf{P})}{J_{\rm s}(-\mathbf{P}) + J_{\rm s}(\mathbf{P})}$$

Xiaoxi Huang*, Xianzhe Chen*, **YHL***, *et.al.*, *Nat. Mater.* **23**, 898–904 (2024)

Thanks for your attention