



南開大學
Nankai University

Magnetism and Magnetic Materials

栗宇航

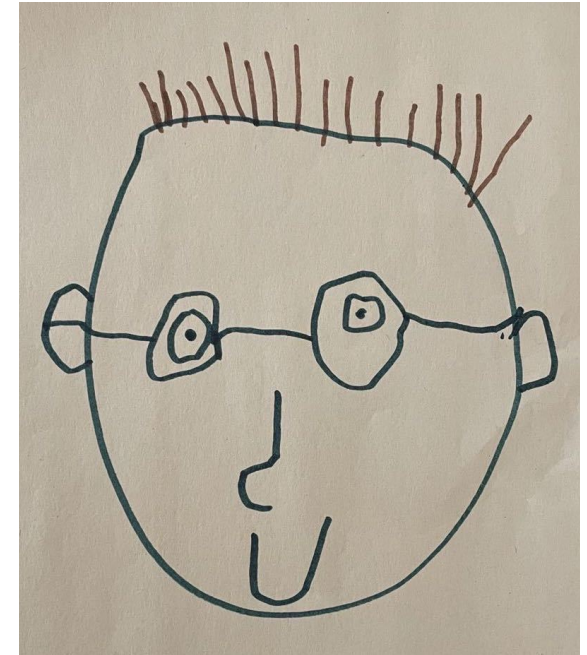
南开大学 物理科学学院

2026.10.20 天津

About me

Research interests:

Quantum transport in topological materials, magnetism, and spintronics. My research focuses on the theoretical understanding of quantum transport phenomena and spin dynamics in topological and magnetic materials, including axion insulators, magnetic skyrmion, and emergent spin textures.



A cartoon by Liz

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1. Static magnetic properties

- Magnetic moment
- Magnetic anisotropy and Exchange interaction
- Magnetic ordering

2. Dynamic magnetic properties

- Landau-Lifshitz-Gilbert (LLG) equation
- Magnetic resonance
- Magnetic excitation

3. Magnetic effect

Textbooks

- Coey, J. M. D. (2010). *Magnetism and magnetic materials*. Cambridge University Press.
- Nolting, W. (2009). *Quantum theory of magnetism* (4th ed.). Springer.
- Blundell, S. (2001). *Magnetism in condensed matter*. Oxford University Press.
- Mario Reis. (2013). *Fundamentals of Magnetism*. Springer.
- ...

References

- **Yu-Hang Li[#]** and Ran Cheng[#], “[Spin fluctuations in quantized transport of magnetic topological insulators](#),” *Physical Review Letters* **126**, 026601 (2021)
- Shuai Li, Ming Gong, **Yu-Hang Li[#]**, Hua Jiang[#], and X. C. Xie, “[High spin axion insulator](#),” *Nature Communications* **15**, 1 (2024)
- **Yu-Hang Li** and Ran Cheng[#], “[Magnonic Su-Schrieffer-Heeger model in honeycomb ferromagnets](#),” *Physical Review B* **103**, 014407 (2021).
- **Yu-Hang Li[#]** and Ran Cheng[#], “[Moiré magnons in twisted bilayer magnets with collinear order](#),” *Physical Review B* **102**, 094404 (2020)
- Bo Chen*, Xiaoda Liu*, **Yu-Hang Li***, *et. al.*, “[Even–Odd Layer-Dependent Exchange Bias Effect in MnBi₂Te₄ Chern Insulator Devices](#),” *Nano Letters* **24**, 8320–8326 (2024)

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1. Static magnetic properties

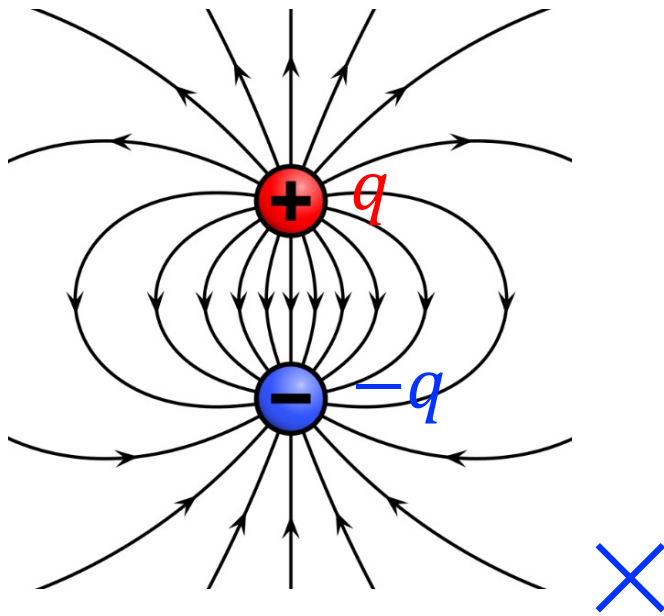
- Magnetic moment
- Magnetic anisotropy and Exchange interaction
- Magnetic ordering

2. Dynamic magnetic properties

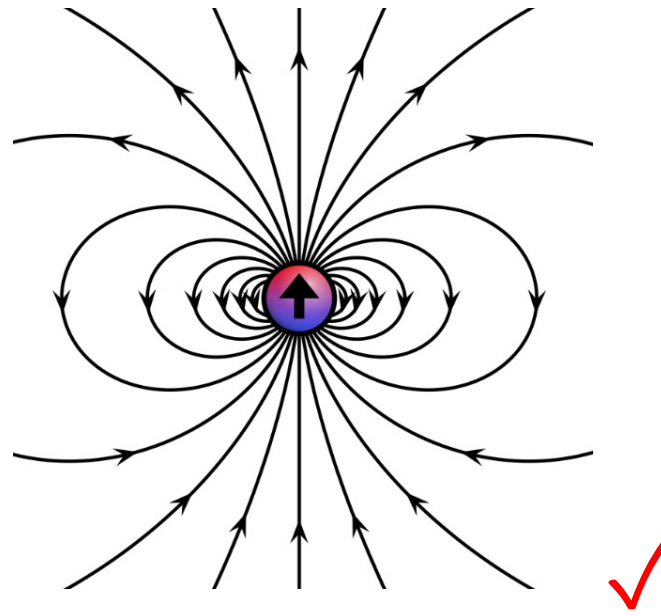
- Landau-Lifshitz-Gilbert (LLG) equation
- Magnetic resonance
- Magnetic excitation

3. Magnetic effect

1.1 Magnetic moment



VS



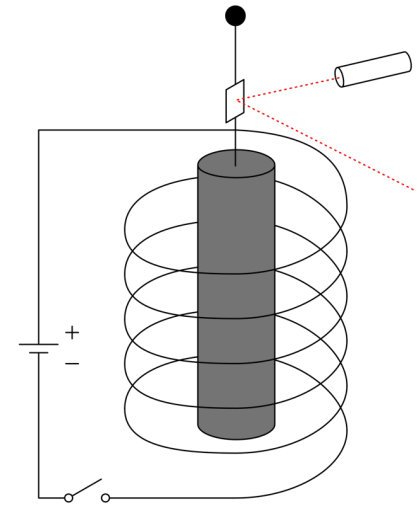
Magnetic dipole: $\mu = qd$

- $\nabla \cdot \vec{B} \neq 0$ (divergent)
- Magnetic line ends on charge q or $-q$

Current loop: $\mu = Is$

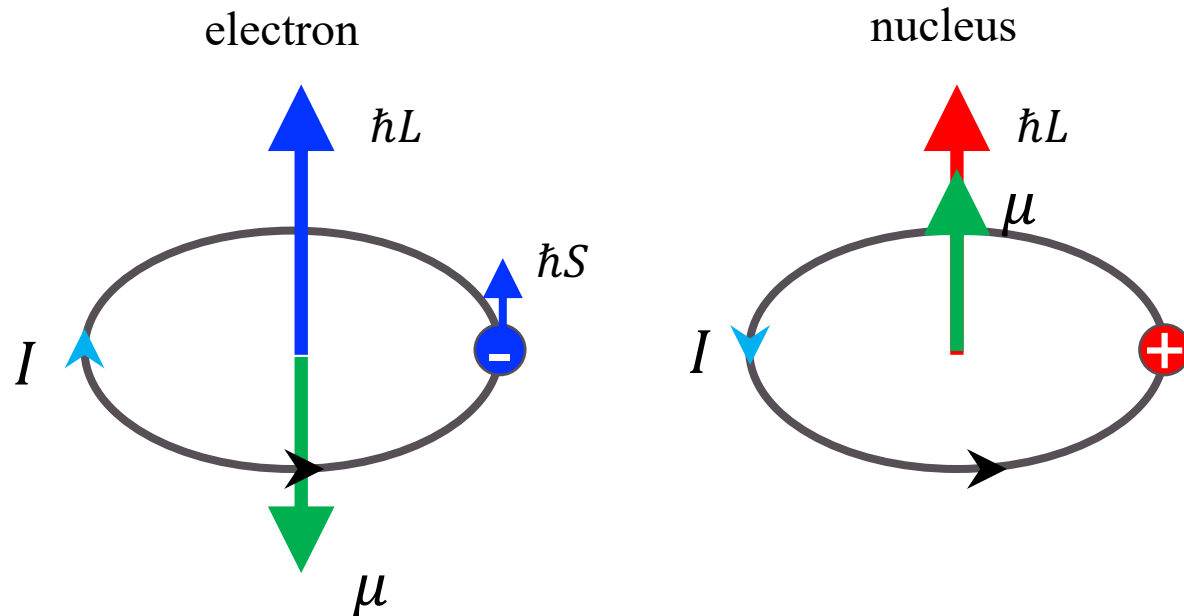
- $\nabla \cdot \vec{B} = 0$ (divergentless)
- Magnetic line never ends
- Magnetic moments are associated with the angular momentum

- Einstein-de Hall effect



- Barnett effect
- Dynamic response

1.1 Magnetic moment



$$\mu_e = \frac{-eL}{2m_e}$$

$\gg$

$$\mu_n = \frac{q_n L}{2m_n}$$

- $\mu = \gamma(\hbar L)$, γ : gyromagnetic ratio

$$I = q/\tau, \tau = 2\pi r/v$$

$$\begin{aligned} \mu &= \pi r^2 I = \frac{\pi r^2 q}{\tau} = \frac{qrv}{2} \\ &= q \frac{r \times p}{2m} = \frac{qL}{2m} \end{aligned}$$

Bohr quantization: $L = n\hbar$

- Bohr magneton: $\mu_B = -\frac{e\hbar}{2m_e}$

$$\begin{aligned} \mu_B &= 9.274 \times 10^{-24} \frac{\text{J}}{\text{T}} \text{ (SI)} \\ &= 5.788 \times 10^{-5} \frac{\text{eV}}{\text{T}} \end{aligned}$$

1.1 Magnetic moment

- Orbital angular momentum: $\hbar \hat{L} = \hat{r} \times \hat{P}$ $[\hat{L}_a, \hat{L}_b] = i\epsilon_{abc}\hat{L}_c$

$$\hat{L}^2 |l, m\rangle = l(l+1) |l, m\rangle$$

$|l, m\rangle$: eigenstate of $\hat{L}^2$ and $\hat{L}_z$

$$\hat{L}_z |l, m\rangle = m |l, m\rangle$$

l :	0,	1,	2,	3,	4,	5...	m :	$(-l, \dots 0, \dots l)$
	s	p	d	f	g	$h...$		

- Spin angular momentum: $S = 1/2$

$$\hat{S}^2 |\pm\rangle = 1/2(1 + 1/2) |\pm\rangle$$
 $[\hat{S}_a, \hat{S}_b] = i\epsilon_{abc}\hat{S}_c$

$$\hat{S}_z |\pm\rangle = \pm 1/2 |\pm\rangle$$

- Full occupied orbit: $\sum_i \vec{L}_i = 0, \sum_i \vec{S}_i = 0$

1.1 Magnetic moment

- $L - S$ coupling

$$\vec{L} = \sum_i \vec{l}_i \quad \vec{S} = \sum_i \vec{s}_i$$

Eigenstate: $\psi = |l, m_l, s, m_s\rangle$

L and S are both conserved

- $J - J$ coupling

Spin-orbit coupling(SOC): $\lambda \vec{S} \cdot \vec{L}$

J is conserved

Hund's rule:

1. Maximize total S
2. Maximize total L
3. If less than half filled ($n \leq \frac{N}{2}$), $J = |L - S|$
If more than half filled ($n > \frac{N}{2}$), $J = |L + S|$

Good for 4f orbital, however, bad for 3d orbital and heavy atoms

Eigenstate: $\psi = |J, m_j, l, s\rangle$

1.1 Magnetic moment

$$\vec{\mu}_L = \mu_B \vec{L} \quad \gamma_L = \mu_B / \hbar$$

$$\vec{\mu}_S = 2\mu_B \vec{S} \quad \gamma_S = 2\mu_B / \hbar$$

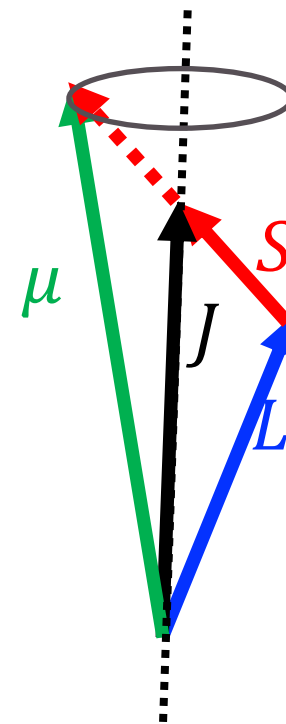
$$\vec{\mu} = \vec{\mu}_L + \vec{\mu}_S = \mu_B (\vec{L} + 2\vec{S})$$

$$\vec{J} = \vec{L} + \vec{S}$$

- Landé g -factor:

$$g \equiv \frac{\mu}{\mu_B J} = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}$$

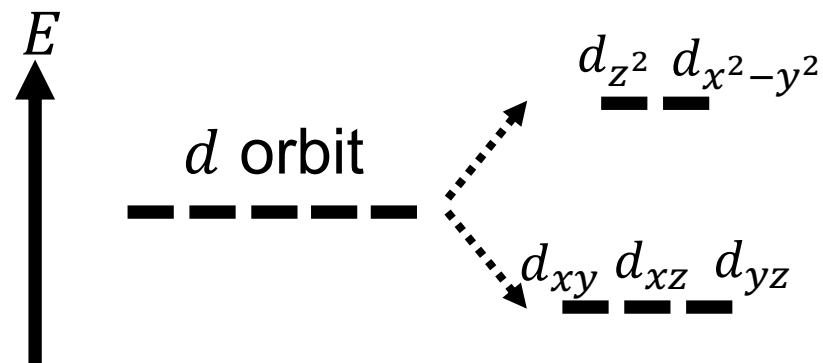
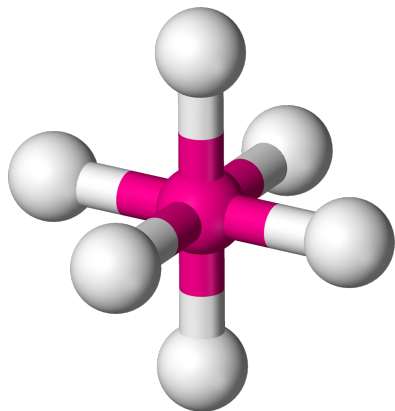
$$\langle \mu(t) \rangle = g\mu_B J$$



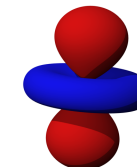
Deviation???

1.1 Magnetic moment-crystal field

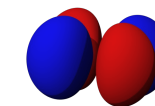
Octahedral



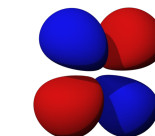
d_{z^2}



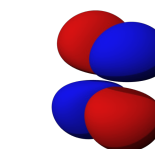
$d_{x^2-y^2}$



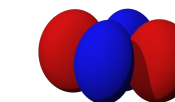
d_{xz}



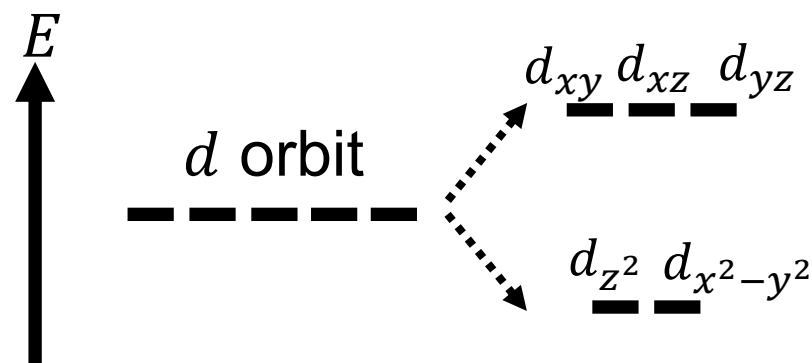
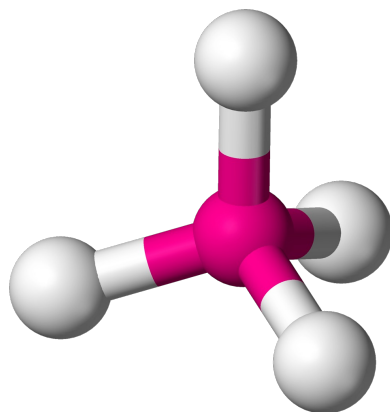
d_{yz}



d_{xy}



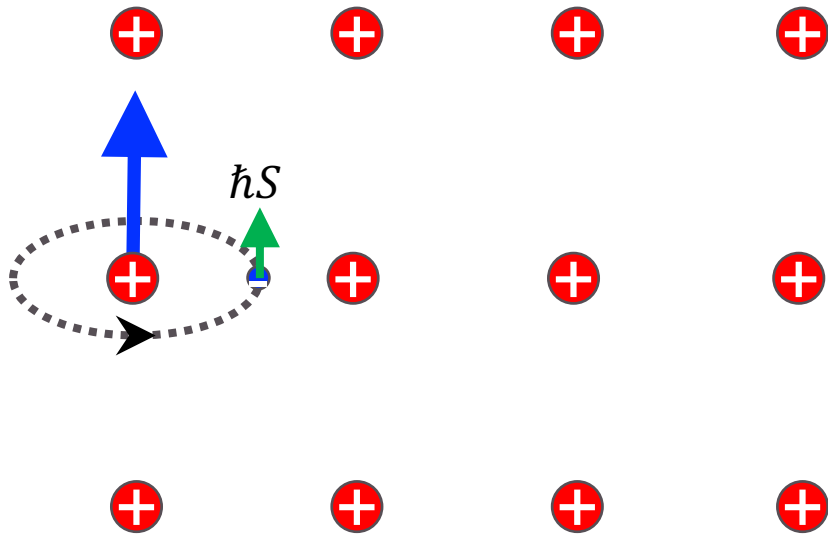
Tetrahedral



Jahn-Teller effect !

1.1 Magnetic moment-orbit quench

Crystal field

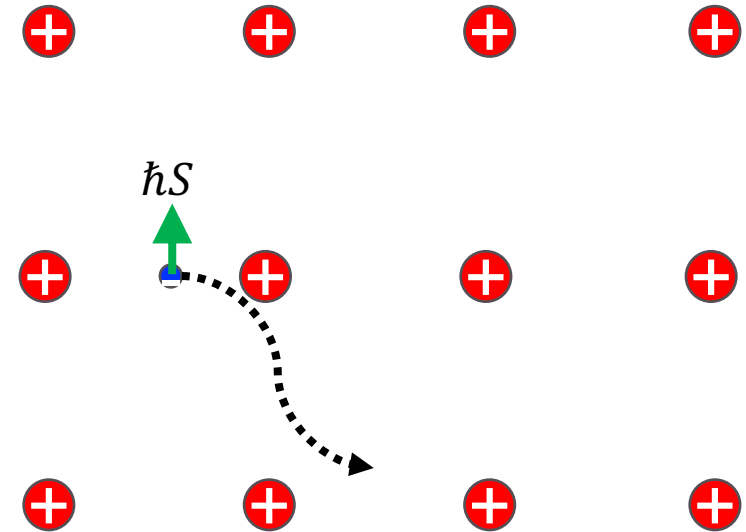


Inner orbit without quench

(partially) quench



Crystal field



Outer orbit with quench

1.2 Magnetic anisotropy

- Magnetocrystalline anisotropy ← Spin-orbit coupling: $H = \lambda \vec{\sigma} \cdot \vec{L}$

$$H_{ani} = K(\vec{e} \cdot \vec{S})^2$$

Uniaxial

$K < 0$: easy axis; $K > 0$: hard axis

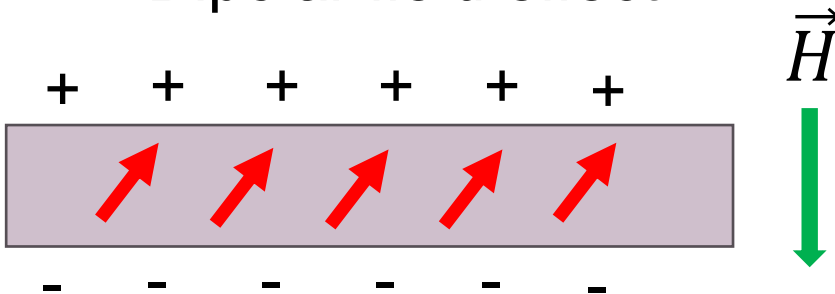
$$H_{ani} = K_1(\vec{e}_1 \cdot \vec{S})^2 + K_2(\vec{e}_2 \cdot \vec{S})^2$$

biaxial

Cubic anisotropy

- Shape anisotropy

Dipolar field effect



- Magnetoelastic anisotropy

- Exchange anisotropy

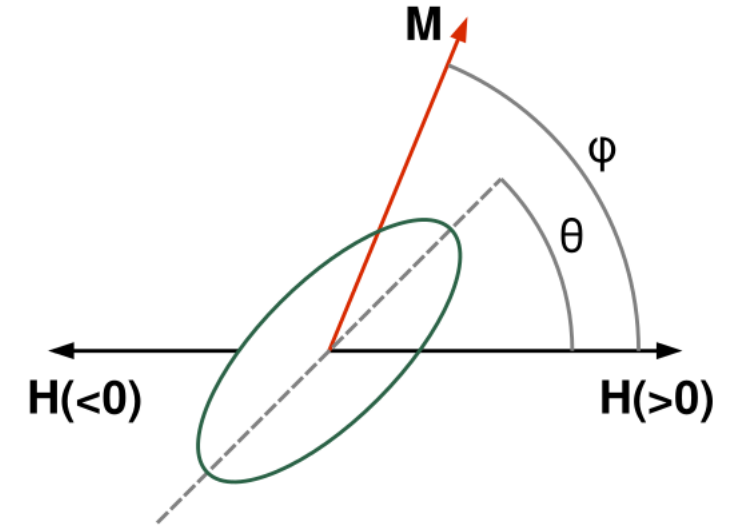
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\langle ij \rangle} S_i^z S_j^z$$

1.2 Mag anisotropy-Stoner-Wohlfarth model



Total energy

$$\begin{aligned} E &= -\mu_0 \vec{H} \cdot \vec{M} - K(\vec{e} \cdot \vec{M})^2 \\ &= -\mu_0 H M \cos \psi - K M^2 \cos(\psi - \theta)^2 \end{aligned}$$



Minimizing the total energy

$$\frac{\partial E}{\partial \psi} = 0 \quad \longrightarrow \quad \mu_0 H M \sin \psi + K M^2 \sin(2(\psi - \theta)) = 0$$

$$\frac{\partial^2 E}{\partial \psi^2} > 0 \quad \longrightarrow \quad \mu_0 H M \cos \psi + 2 K M^2 \cos(2(\psi - \theta)) > 0$$

1.2 Mag anisotropy-Stoner-Wohlfarth model



We obtain

$$h \sin \psi + \frac{1}{2} \sin(2(\psi - \theta)) = 0$$

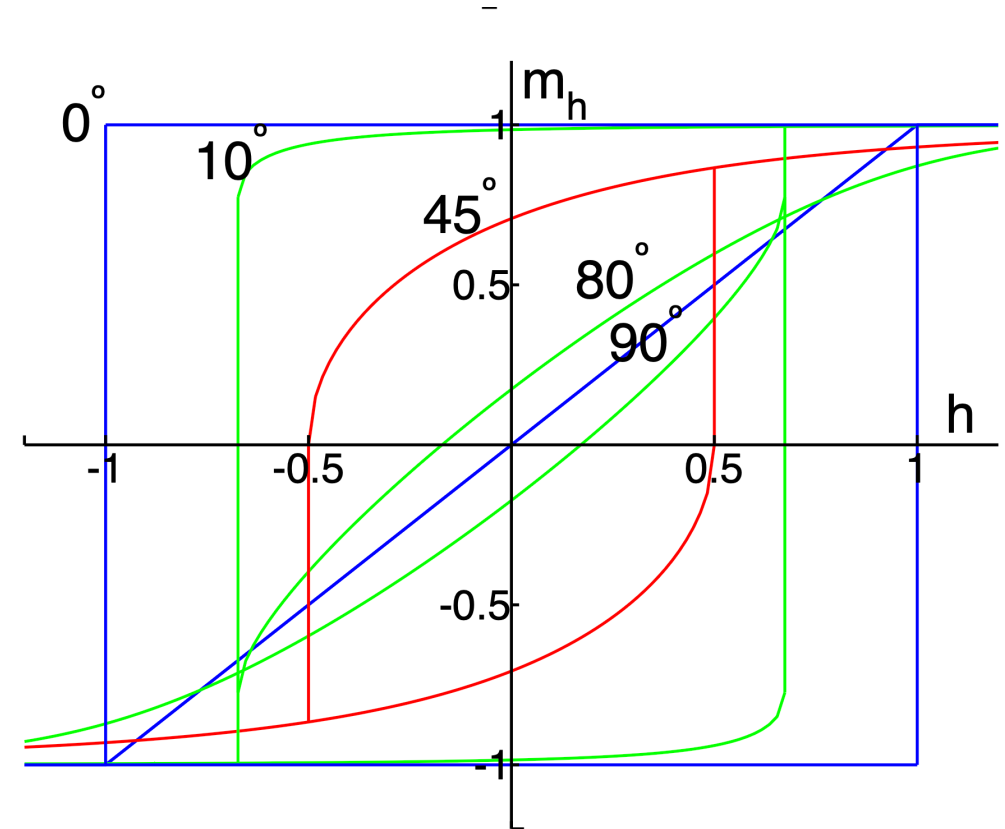
$$h \cos \psi + \cos(2(\psi - \theta)) > 0$$

with $h = \mu_0 H / 2KM$

$$\theta = 0$$

$$\sin \psi = 0 \longrightarrow \psi = 0, \pi$$

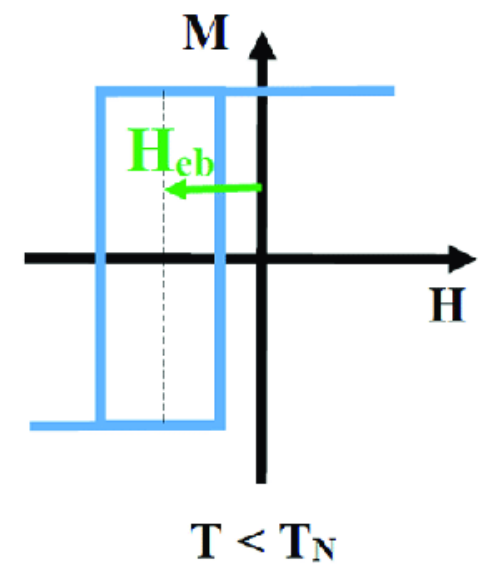
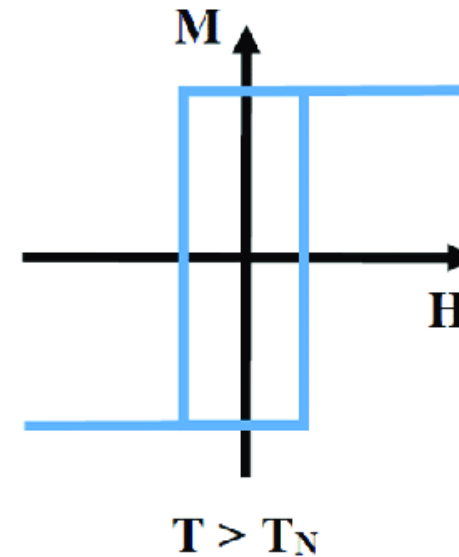
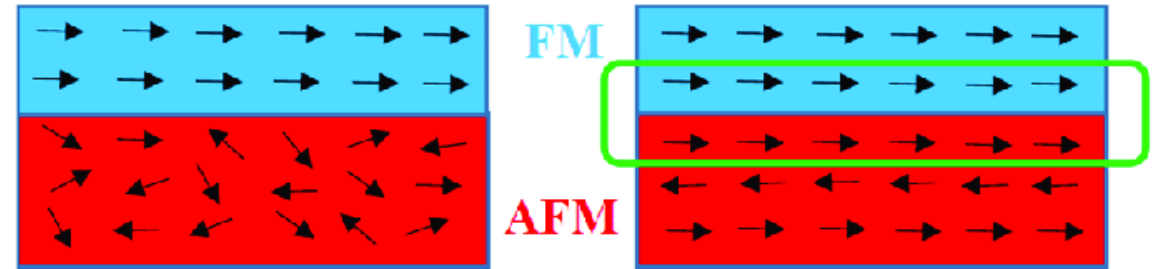
$$h \cos \psi + 1 = 0 \longrightarrow h_c = \pm 1$$



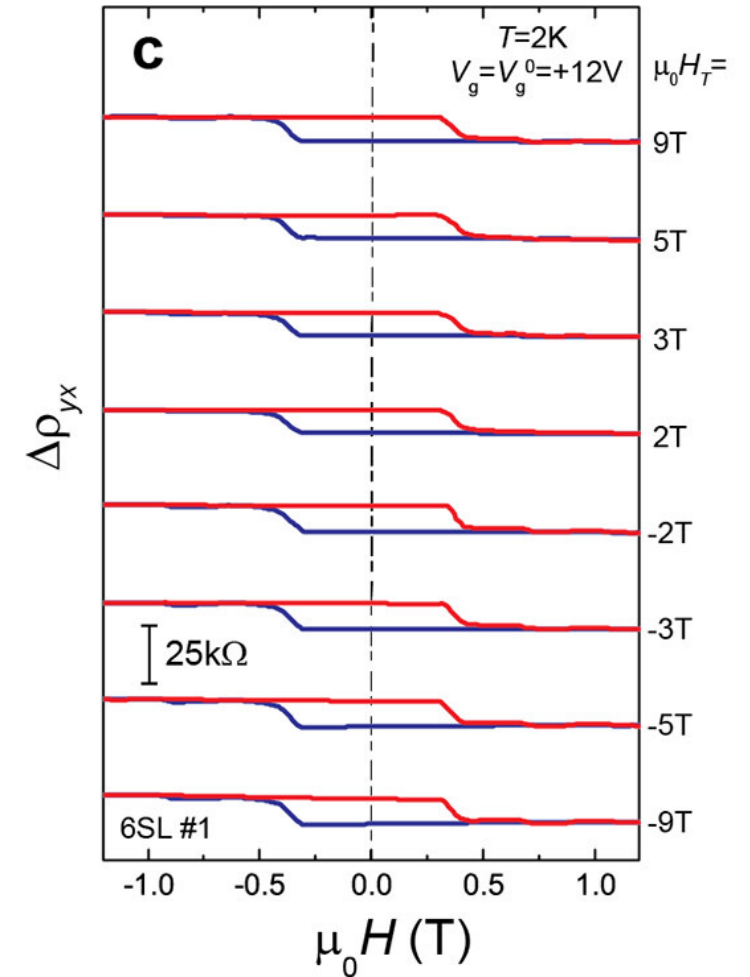
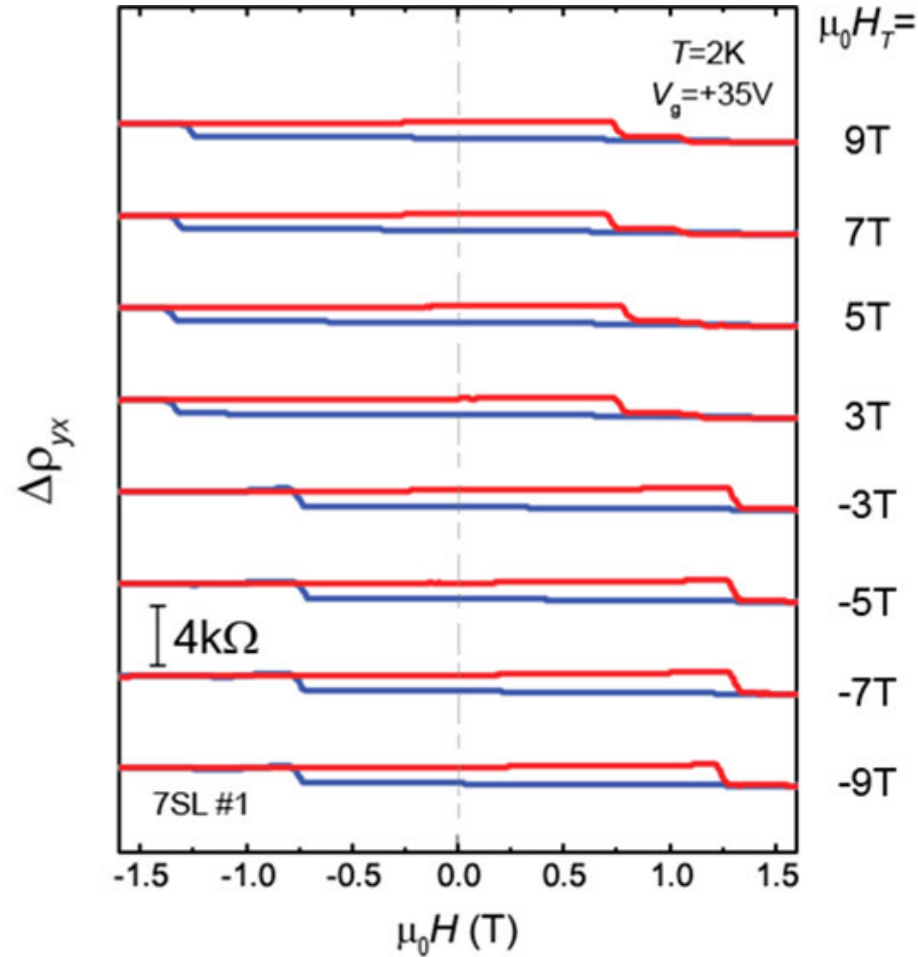
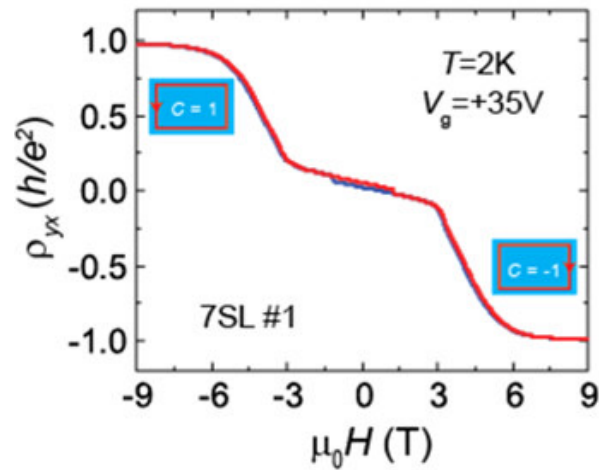
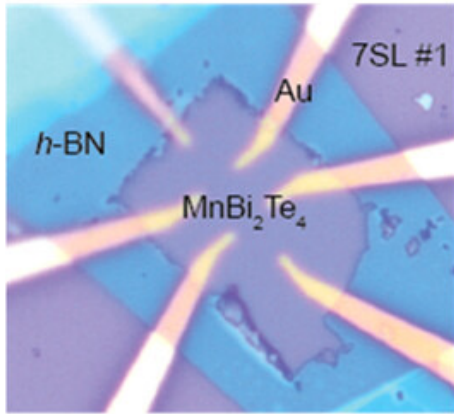
1.2 Mag anisotropy-exchange bias effect

Total energy

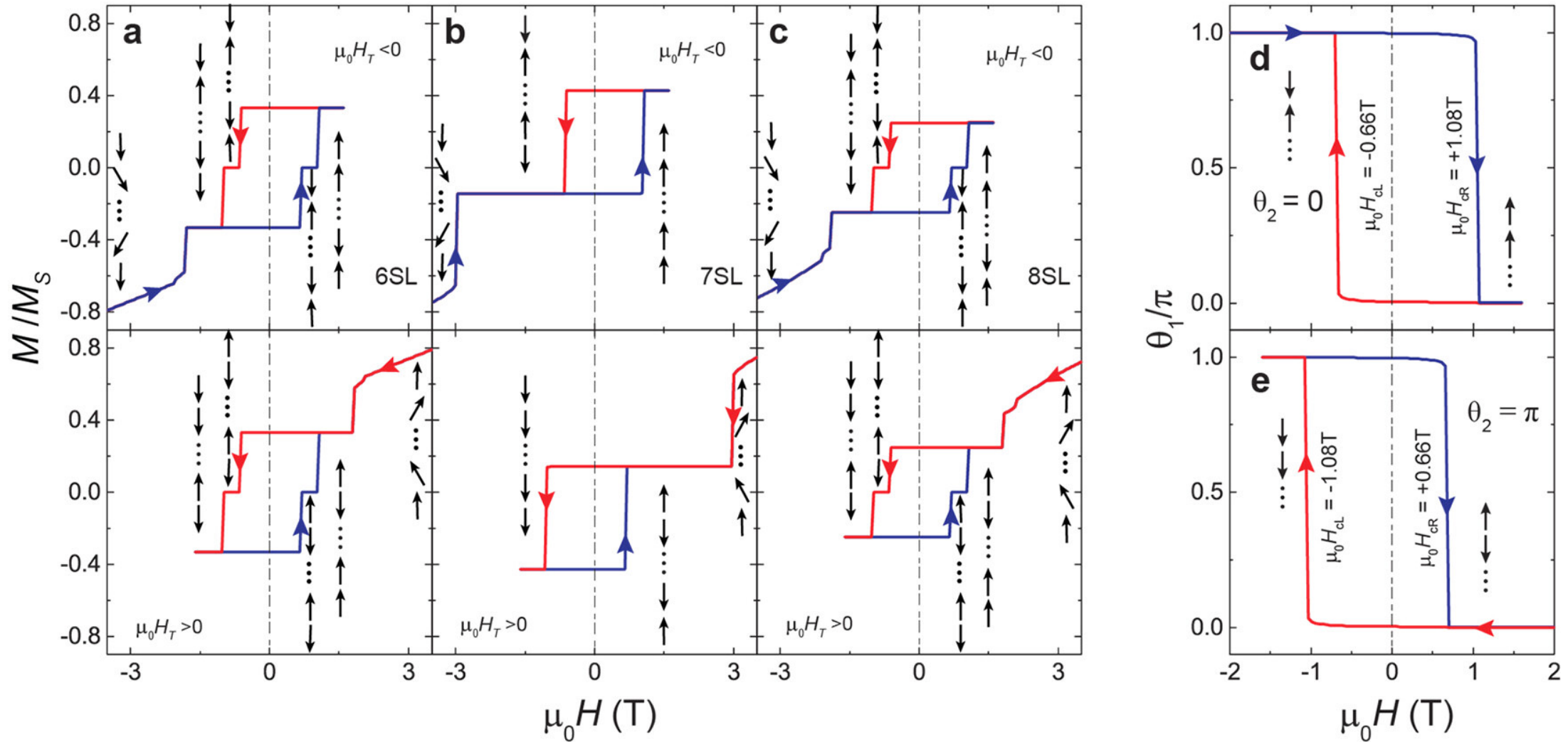
$$\begin{aligned} E &= \mu_0 \vec{H} \cdot \vec{M} + J \vec{m} \cdot \vec{M} - K(\vec{e} \cdot \vec{M})^2 \\ &= (\mu_0 H + Jm)M \cos \psi \\ &\quad - KM^2 \cos(\psi - \theta)^2 \end{aligned}$$



1.2 Mag anisotropy-exchange bias effect



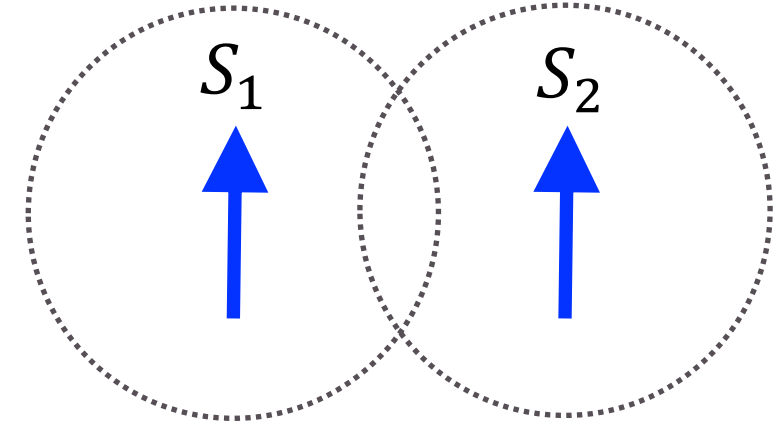
1.2 Mag anisotropy-exchange bias effect



1.3 Exchange interaction

- Hamiltonian

$$H = \sum_{i=1,2} \frac{p_i^2}{2m} + V(r_1 - r_2)$$



Wavefunction: $\Psi = \psi(1,2)\chi(1,2)$

$$\psi(1,2) = \frac{\phi_1\phi_2 \pm \phi_2\phi_1}{\sqrt{2}}$$

$$\chi(1,2) = |\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle \longleftrightarrow \chi(1,2) = |0,0\rangle, |1, m_s\rangle$$

$$\psi_1(1,2) = \frac{\phi_1\phi_2 + \phi_2\phi_1}{\sqrt{2}}$$

$$\chi_1(1,2) = |0,0\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$$

$$\psi_2(1,2) = \frac{\phi_1\phi_2 - \phi_2\phi_1}{\sqrt{2}}$$

$$\chi_2(1,2) = |1, m_s\rangle$$

1.3 Exchange interaction

- The wave function must be antisymmetric

$$\Psi_1 = \psi_1(1,2)\chi_1(1,2) \quad \Psi_2 = \psi_2(1,2)\chi_2(1,2)$$

- The Coulomb's energy: $\Psi_i H \Psi_i = \psi_i H \psi_i$

$$K = \iint dr_1 dr_2 |\phi_1(r_1)|^2 |\phi_2(r_2)|^2 V(r_1 - r_2)$$

$$J = \iint dr_1 dr_2 \phi_1(r_1) \phi_2(r_1) V(r_1 - r_2) \phi_2(r_2) \phi_1(r_2)$$

$$V_{sym}(r_1 - r_2) = K + J \quad V_{anti}(r_1 - r_2) = K - J$$

$$\Psi_i H \Psi_i = \psi_i H \psi_i = \chi_i H_S \chi_i \Rightarrow H_S = -J \vec{S}_1 \cdot \vec{S}_2 \quad \text{Direct exchange!!!}$$

1.3 Exchange-indirect exchange

- Superexchange interaction

Two magnetic atoms are coupled through another nonmagnetic atom –such as oxygen

Mostly antiferromagnetic coupled---180°

Insulating materials



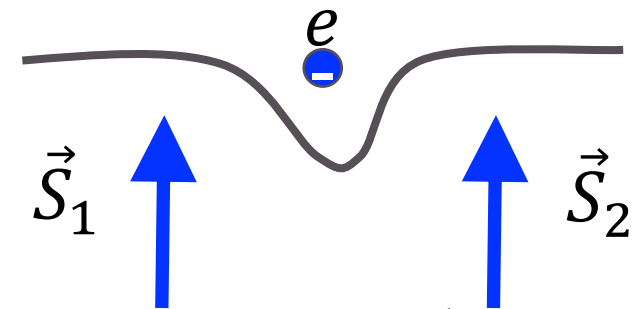
- Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction

Two magnetic atoms are coupled through conducting electrons

$$H = \lambda \sum_i \vec{\sigma} \cdot \vec{S}_i \delta(\vec{r} - \vec{R}_i) \quad \text{sd-interaction}$$

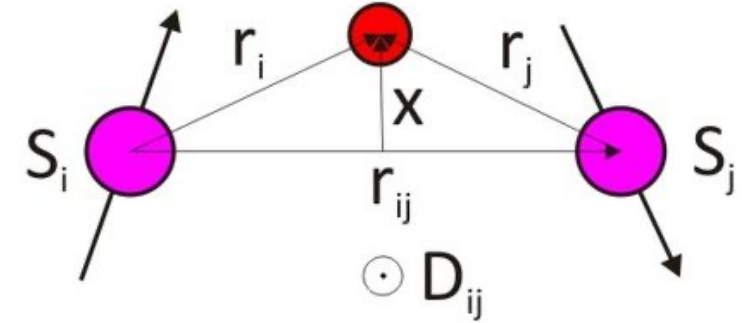
Interaction strength oscillates with distance r

Conducting materials $J_{RKKY} \sim \cos(2k_F r)/r^3$



1.3 Exchange-asymmetric exchange

$$H_1 = \lambda_i \vec{\sigma} \cdot \vec{S}_i \delta(\vec{r} - \vec{R}_i) + \lambda_j \vec{\sigma} \cdot \vec{S}_j \delta(\vec{r} - \vec{R}_j) + \underbrace{l \vec{\sigma} \cdot \vec{L}}_{\text{Spin-orbit coupling}}$$



Recall

$$\sum_{\sigma_1, \sigma_2, \sigma_3} \langle \sigma_1 | \vec{\sigma} \cdot \vec{S}_i | \sigma_2 \rangle \langle \sigma_2 | \vec{\sigma} | \sigma_3 \rangle \langle \sigma_3 | \vec{\sigma} \cdot \vec{S}_j | \sigma_1 \rangle = \text{Tr}_{\sigma} (\vec{\sigma} \cdot \vec{S}_i) \vec{\sigma} (\vec{\sigma} \cdot \vec{S}_j) = -\frac{i}{4} \vec{S}_i \times \vec{S}_j$$

Applying the second order perturbation theory
gives

$$H_{ani} = \vec{D} \cdot \vec{S}_i \times \vec{S}_j$$

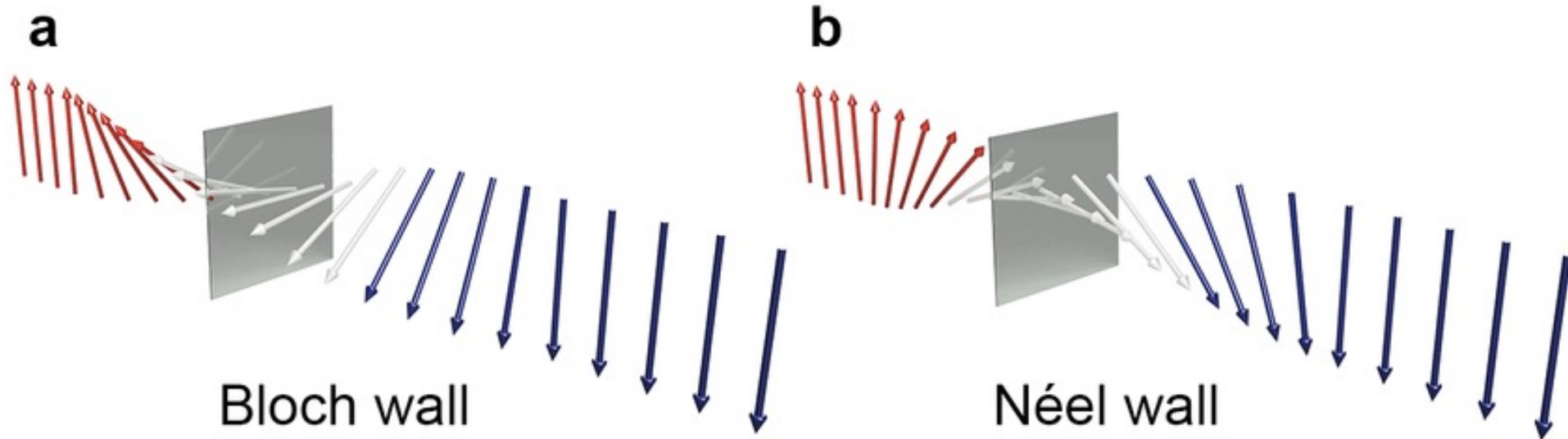
Dzyaloshinskii–Moriya interaction

1.3 Exchange interaction

- Magnetic dipolar interaction

Weak but long range!!!

$$H = \frac{\mu_0}{4\pi r^3} [\vec{\mu}_1 \cdot \vec{\mu}_2 - \frac{3}{r^2} (\vec{\mu}_1 \cdot \vec{r})(\vec{\mu}_2 \cdot \vec{r})] \sim 10^{-23} \text{J, or } 0.1 \text{ meV}$$



1.4 Magnetic ordering

- Spontaneous symmetry breaking

Hamiltonian preserves the symmetry while the ground state does not!!!

$$H = J \sum_{ij} \vec{S}_i \cdot \vec{S}_j$$

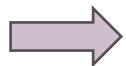
Preserve SU(2) spin rotation symmetry

$$|\psi\rangle = |\uparrow_1 \uparrow_2 \cdots\rangle$$

Breaks SU(2) spin rotation symmetry

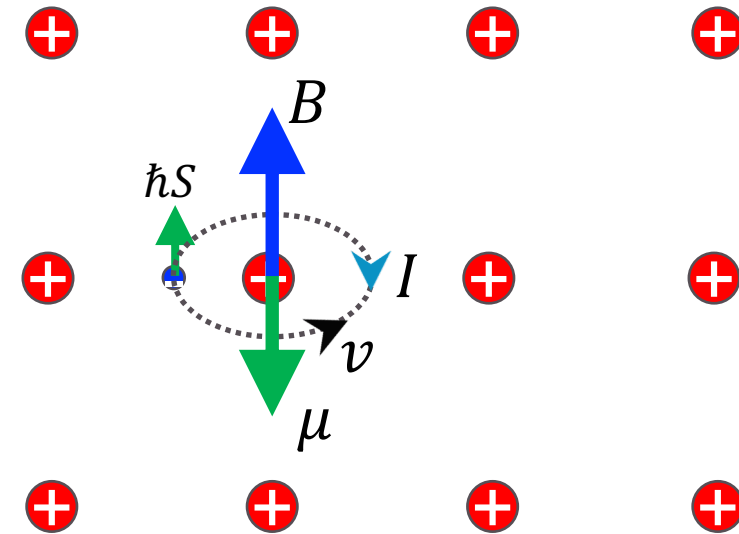
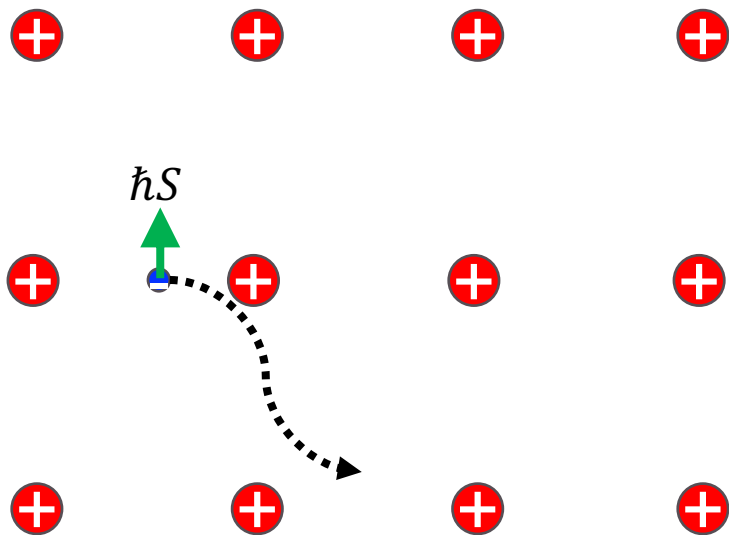
- Mermin-Wegner theorem

1. Short range interaction, 2. $d \leq 2$, $T > 0$, 3. continuous symmetry



4. No spontaneous symmetry breaking

1.4 Magnetic ordering-diamagnet



- Larmor diamagnet: $\chi = \frac{-ne^2\langle r^2 \rangle}{6m\mu_0}$

What if B is vary large???

1.4 Magnetic ordering-paramagnet

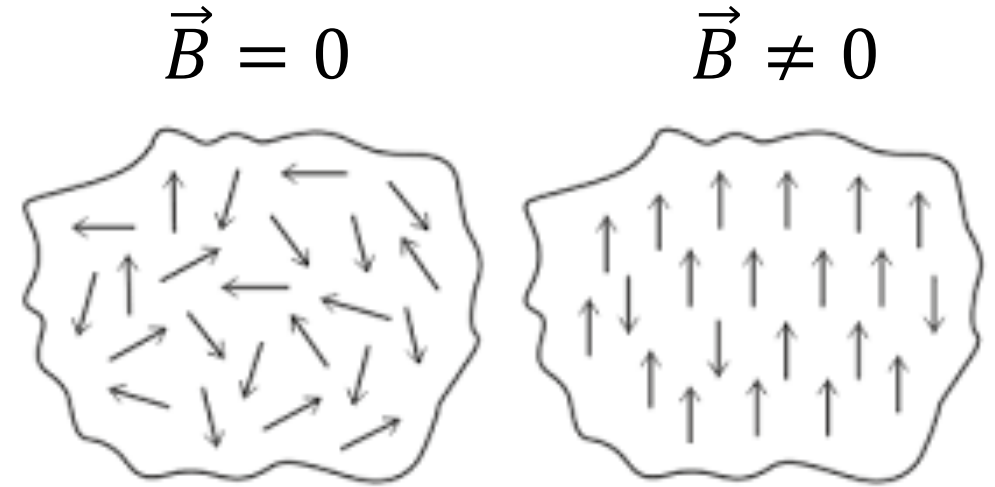
- Independent spins
- Hamiltonian $H = g\mu_B \vec{S} \cdot \vec{B}$

$$E_{m_s} = g\mu_B B m_s$$

$$\psi_{m_s} = |s, m_s\rangle$$

- Boltzmann distribution function: $\rho(m_s) \propto \exp\{-E/k_B T\}$

$$M(B) = \frac{\sum_{m_s=-s,s} m_s \exp(-g\mu_B B m_s / k_B T)}{\sum_{m_s=-s,s} \exp(-g\mu_B B m_s / k_B T)}$$



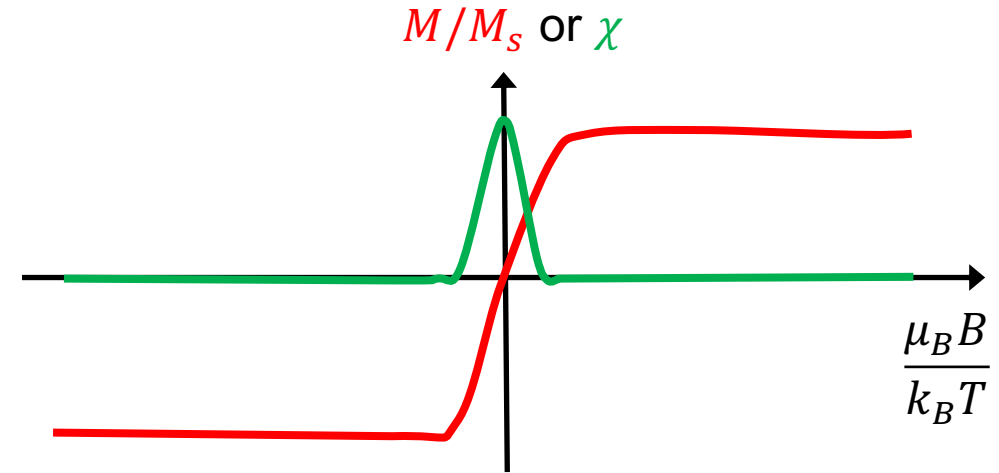
1.4 Magnetic ordering-paramagnet

- Spin $s = 1/2$

$$M(B) = \frac{\mu_B [\exp(-\frac{g\mu_B B}{2k_B T}) - \exp(\frac{g\mu_B B}{2k_B T})]}{2[\exp(-\frac{g\mu_B B}{2k_B T}) + \exp(\frac{g\mu_B B}{2k_B T})]} = \mu_B \tanh(\frac{g\mu_B B}{2k_B T})$$

$$M(B)/M_s = \tanh(\frac{\mu_B B}{k_B T})$$

$$\chi = \frac{\partial M(B)}{\partial B} = \frac{n\mu_0\mu_B^2}{k_B T} \cosh^{-2}(\mu_B B/k_B T)$$



What if large s ? continuous s ?

1.4 Magnetic ordering-ferromagnet

- Hamiltonian

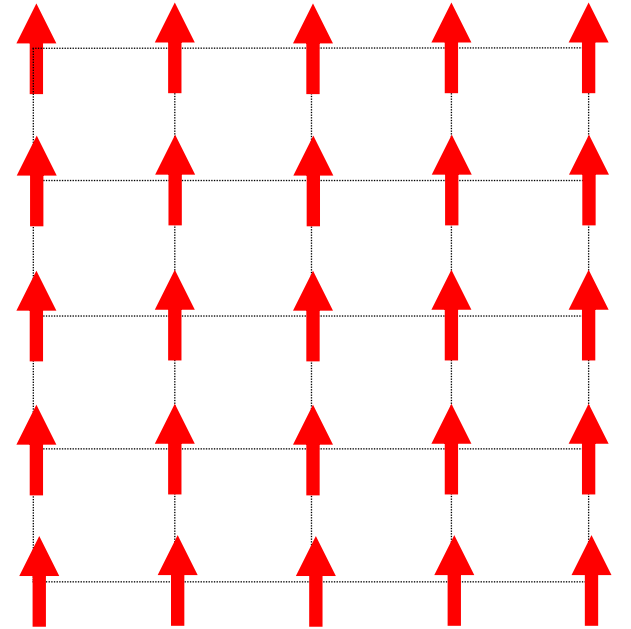
$$H = -J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j - g\mu_B \sum_i \vec{B} \cdot \vec{S}_i$$

$$J > 0$$

$$= -J \sum_{\langle ij \rangle} \vec{S}_i \cdot (\vec{S}_j - \langle \vec{S}_j \rangle + \langle \vec{S}_j \rangle) - g\mu_B \sum_i \vec{B} \cdot \vec{S}_i$$

$$= -J \sum_{\langle ij \rangle} \vec{S}_i \cdot (\langle \vec{S}_j \rangle + \delta \vec{S}_j) - g\mu_B \sum_i \vec{B} \cdot \vec{S}_i$$

$$\approx -J \sum_{\langle ij \rangle} \vec{S}_i \cdot \langle \vec{S}_j \rangle - g\mu_B \sum_i \vec{B} \cdot \vec{S}_i = \sum_i \vec{S}_i (-g\mu_B \vec{B} - J \sum_{\langle j \rangle} \langle \vec{S}_j \rangle)$$

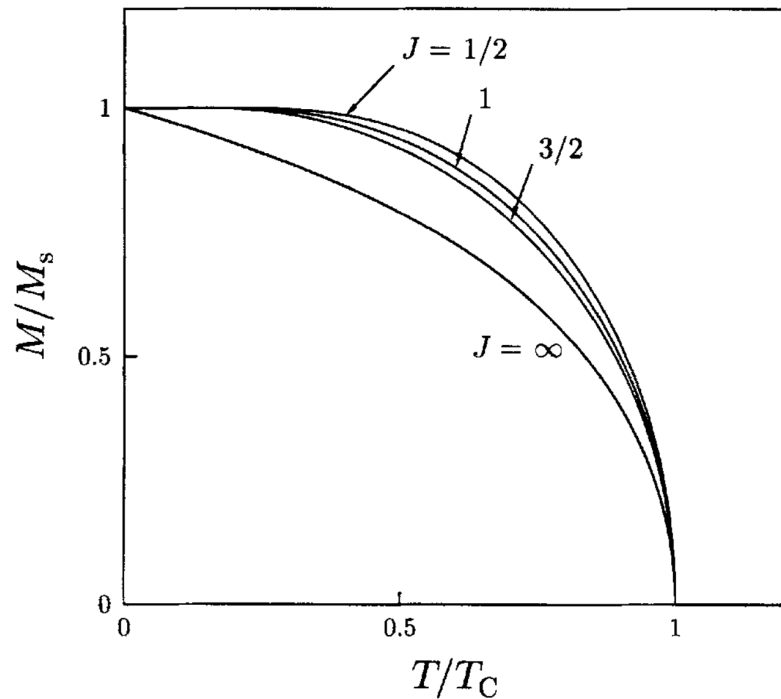


What if $d \leq 2$?

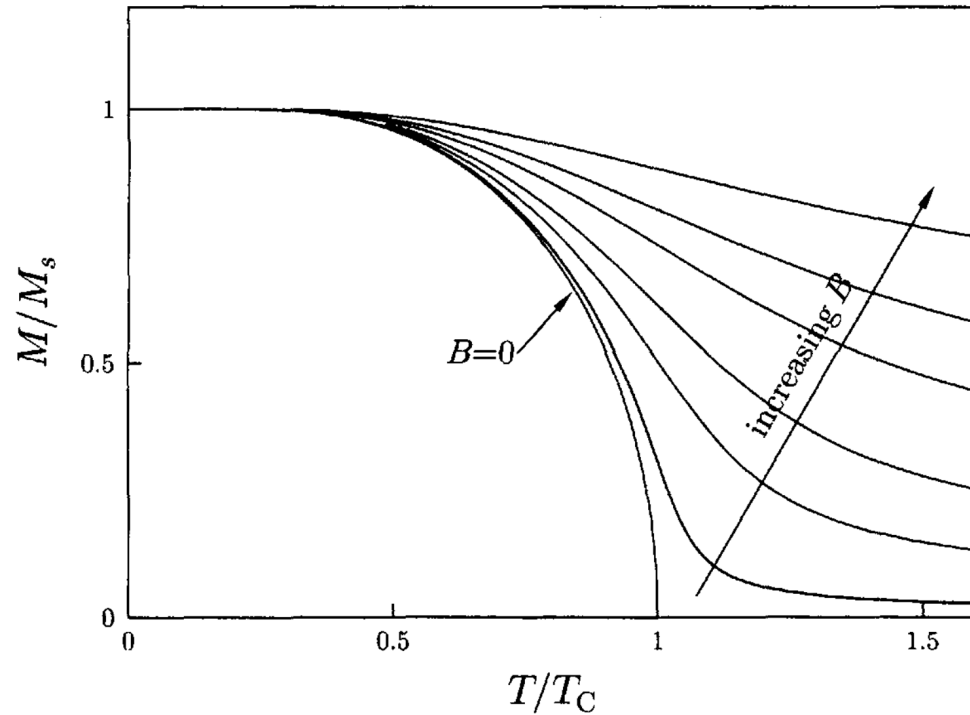
1.4 Magnetic ordering-ferromagnet

- Magnetization curve

At $B = 0$



$B \neq 0$



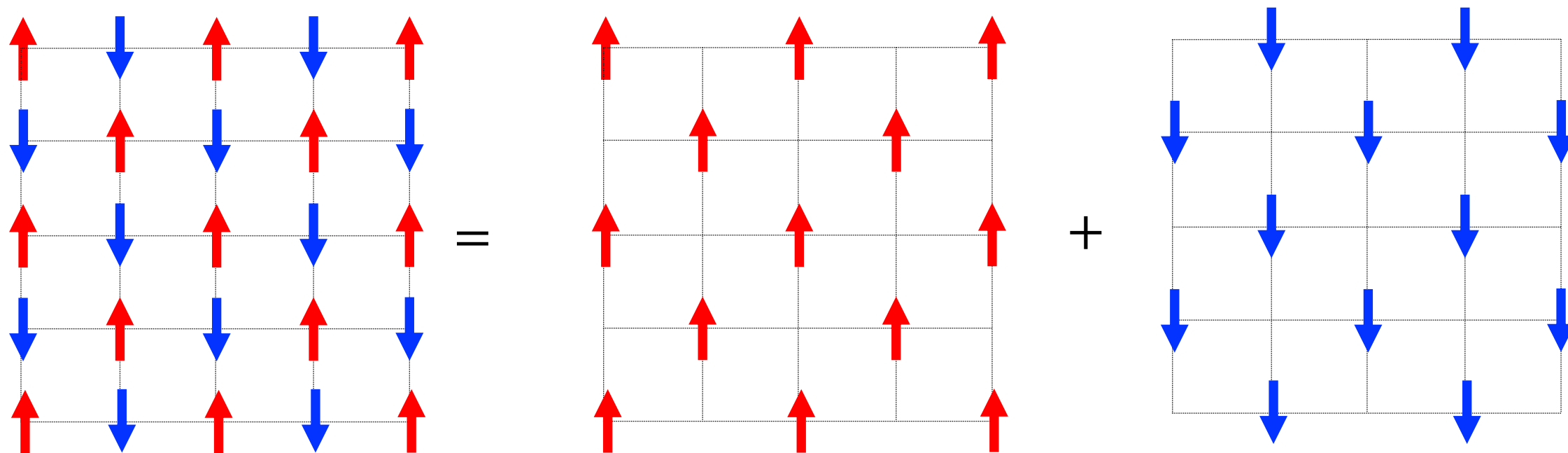
1.4 Magnetic ordering-antiferromagnet

- Hamiltonian

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j - K \sum_i S_{iz}^2 - g\mu_B \sum_i \vec{B} \cdot \vec{S}_i$$

The solution to $d \leq 2$

$J > 0$

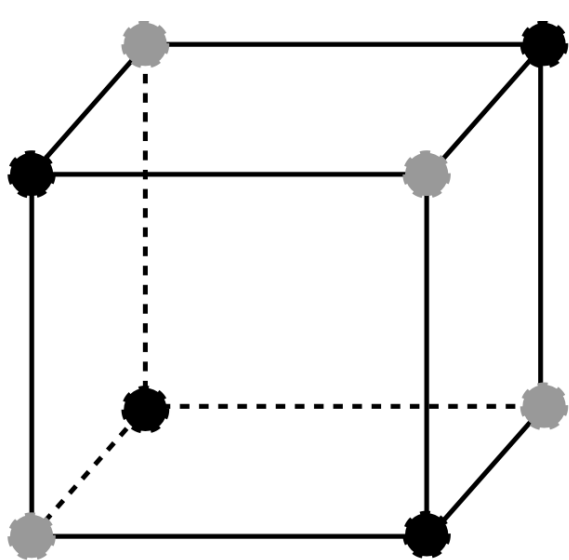


1.4 Magnetic ordering-antiferromagnet

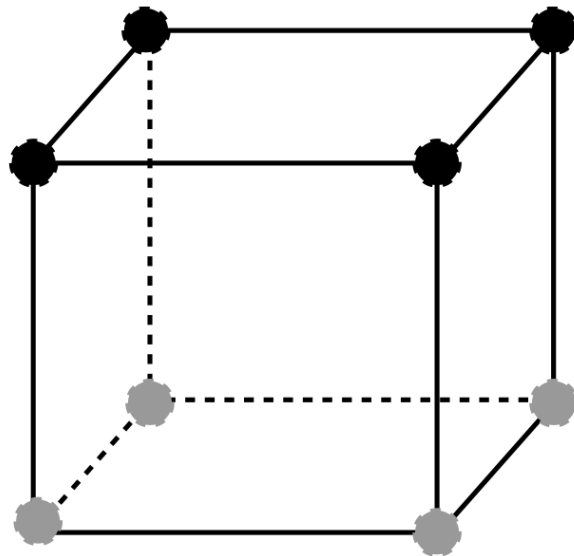


南開大學
Nankai University

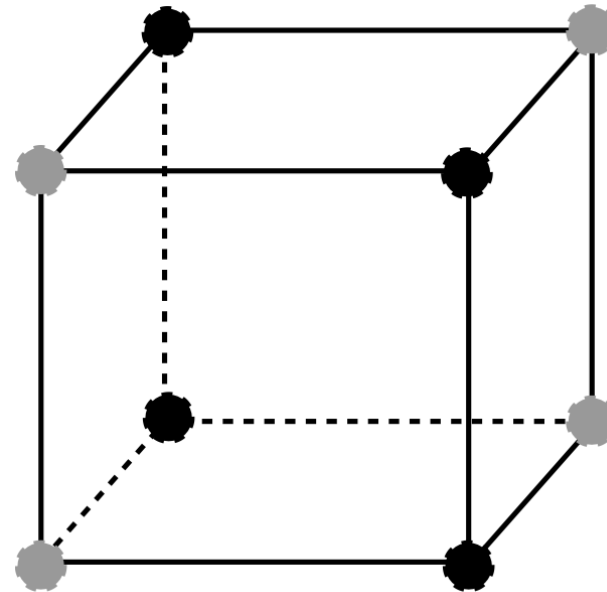
G-type



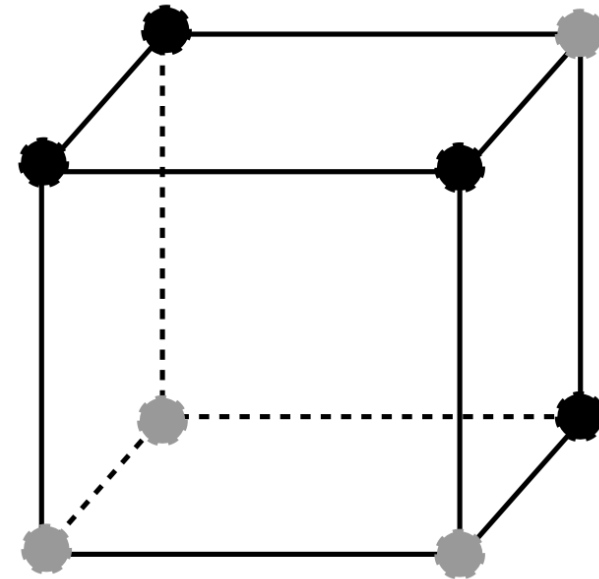
A-type



C-type



F-type

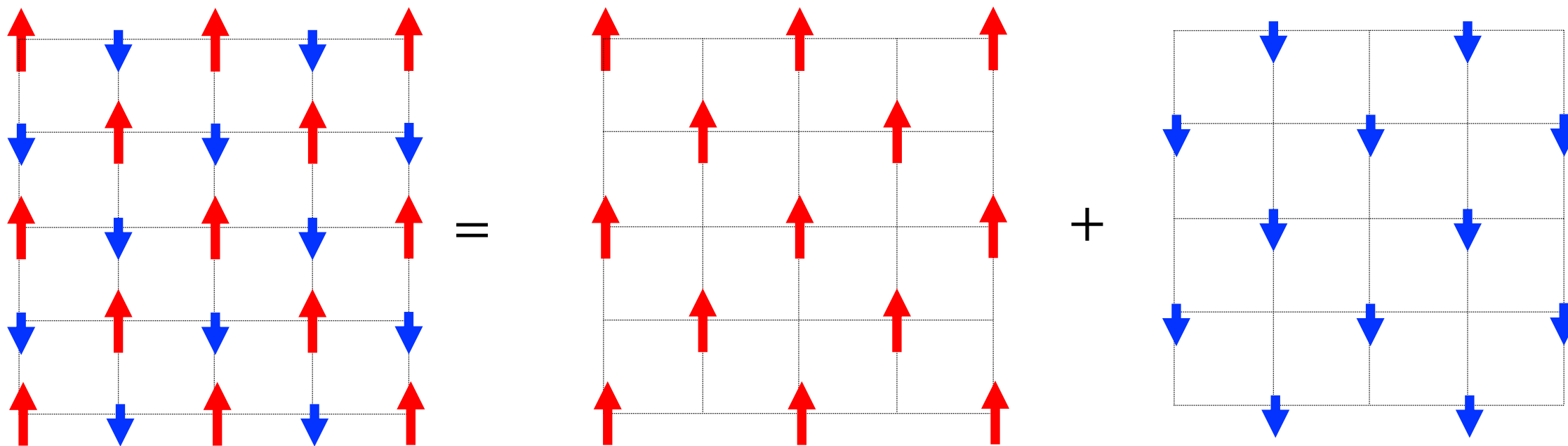


1.4 Magnetic ordering-ferrimagnet

- Hamiltonian

$$J > 0, S_1 \neq S_2$$

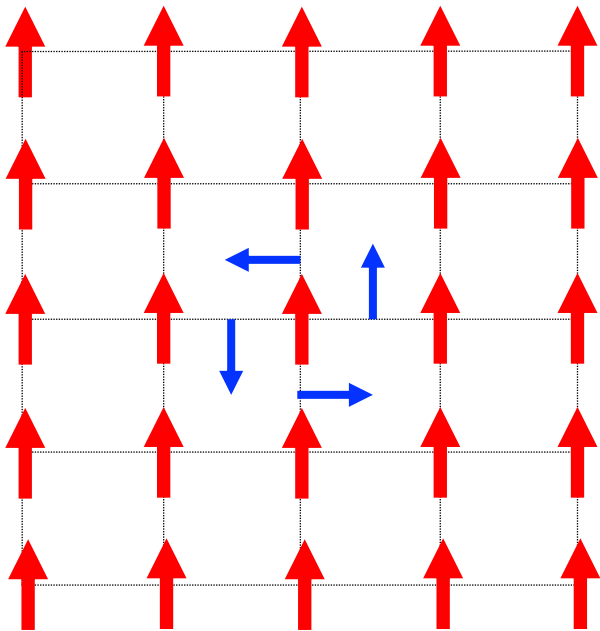
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j - K \sum_i S_{iz}^2 - g\mu_B \sum_i \vec{B} \cdot \vec{S}_i$$



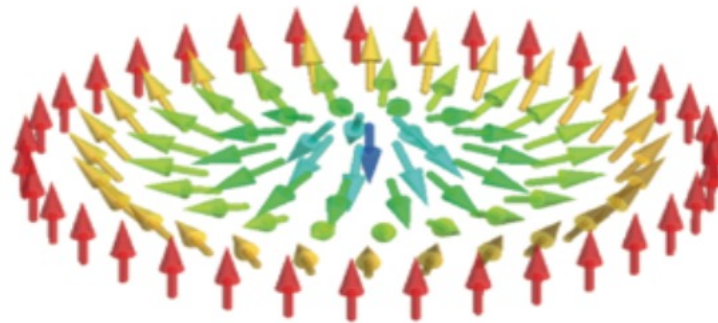
1.4 Magnetic ordering-noncolinear

- Hamiltonian

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + \sum_{\langle ij \rangle} (\vec{D} \times \vec{e}_{ij}) \cdot (\vec{S}_i \times \vec{S}_j) - K \sum_i S_{iz}^2 - g\mu_B \sum_i \vec{B} \cdot \vec{S}_i$$



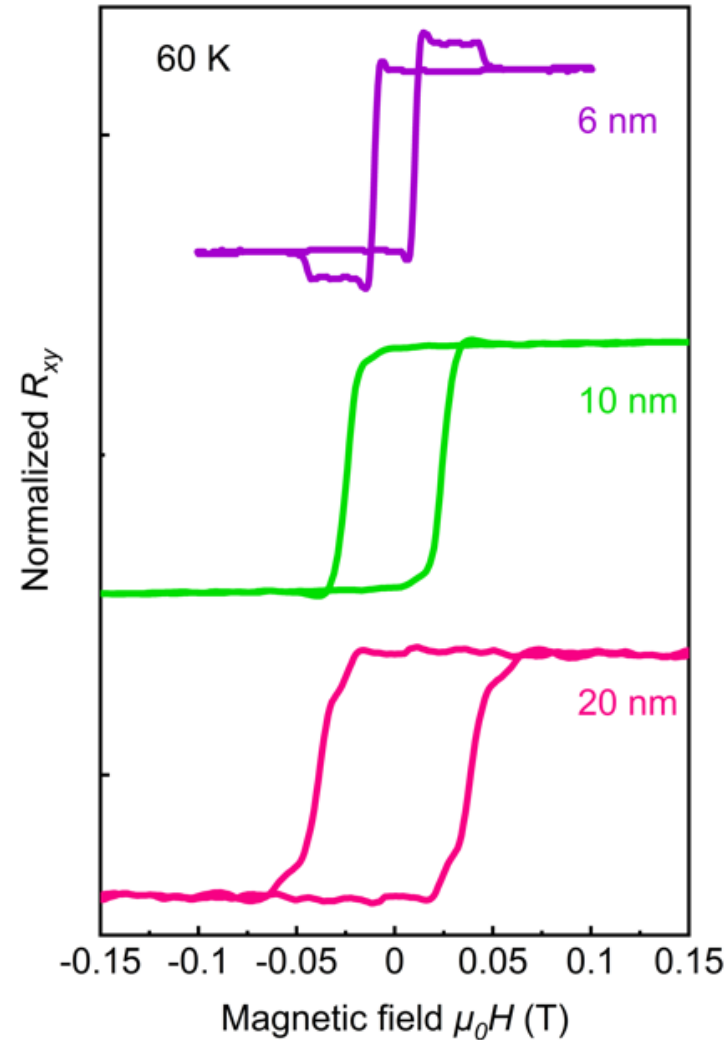
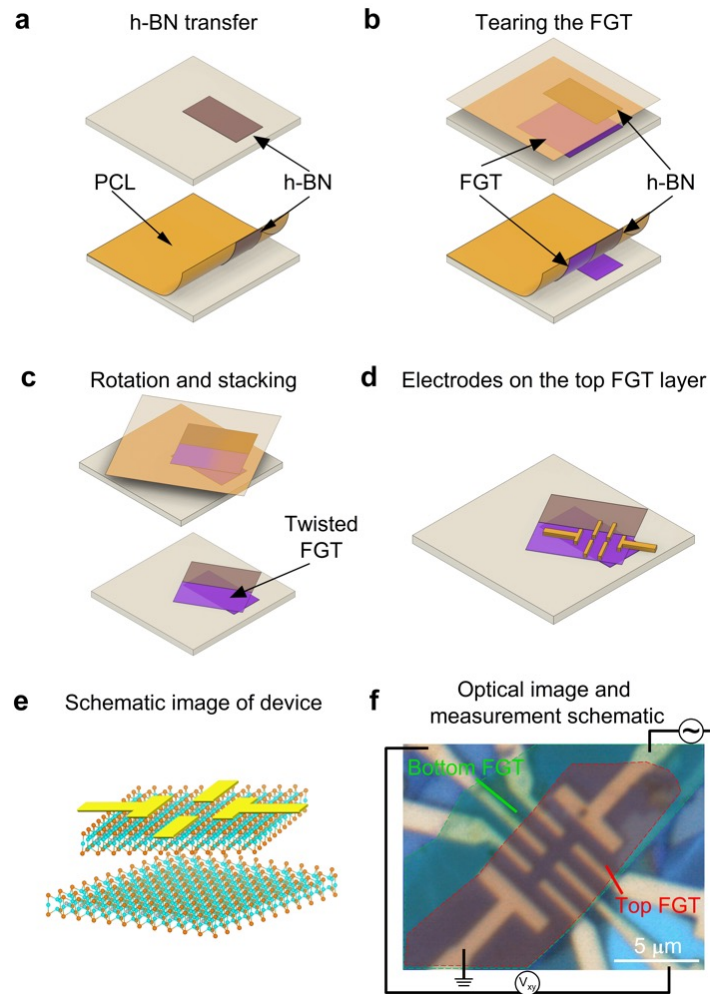
Néel-type skyrmion



Bloch-type skyrmion

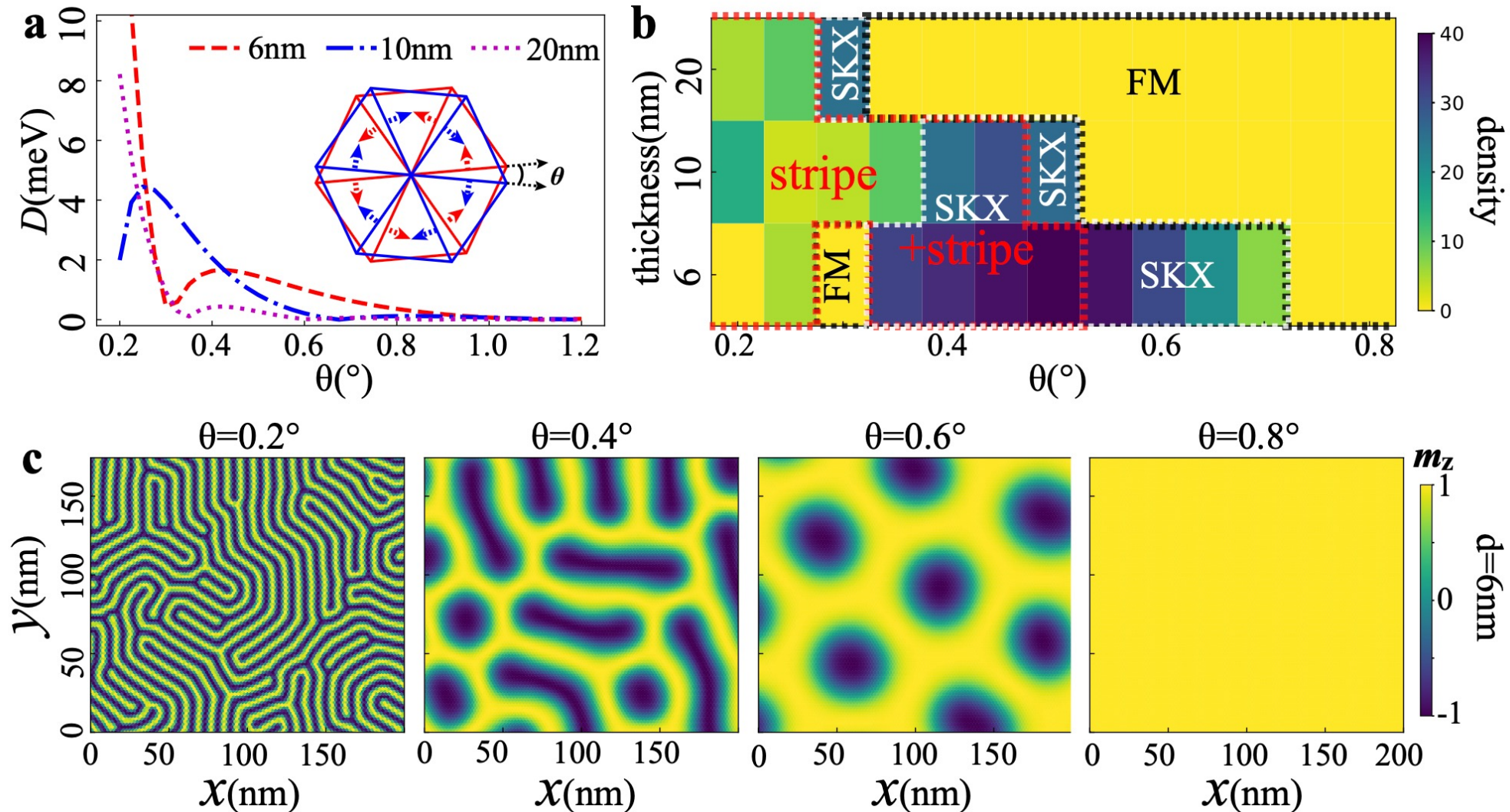


1.4 Magnetic ordering-noncolinear



Kim, Zhang, YHL, *et. al.*, Topological Hall effect in twisted Fe₃GeTe₂ metallic system, *Nat. Commun.* **17**, 2931(2026)

1.4 Magnetic ordering-noncolinear



Kim, Zhang, YHL, *et. al.*, Topological Hall effect in twisted Fe_3GeTe_2 metallic system, *Nat. Commun.* **17**, 2931(2026)

1.4 Magnetic ordering-disordered

- Hamiltonian

$$J > 0$$

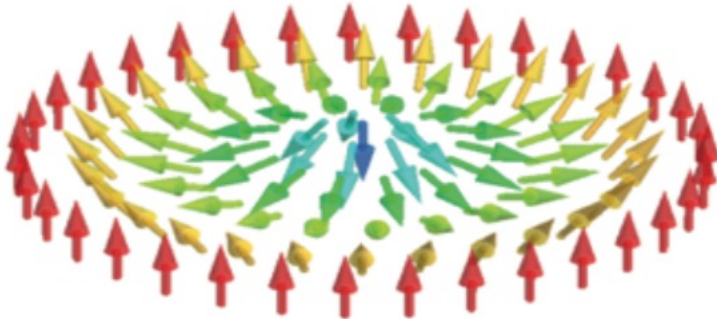
$$H = J \sum_{\langle ij \rangle} S_i \cdot S_j - K \sum_i S_{iz}^2 - g\mu_B \sum_i B \cdot S_i$$

frustration

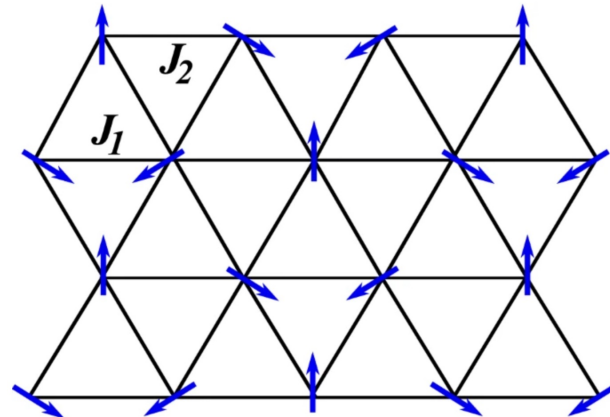
?



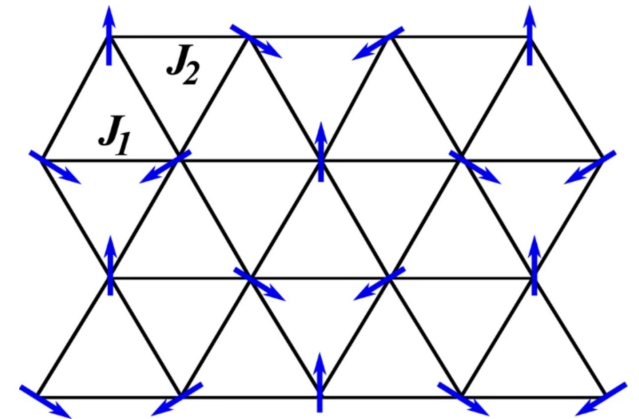
Skyrmion?



Spin liquid?



Spin glass?



How to determine the magnetic ground state???

1. Static magnetic properties

- Magnetic moment
- Magnetic anisotropy and Exchange interaction
- Magnetic ordering

2. Dynamic magnetic properties

- Landau-Lifshitz-Gilbert (LLG) equation
- Magnetic resonance
- Magnetic excitation

3. Magnetic effect

2.1 LLG equation

$$\frac{d\vec{M}}{dt} = -\gamma \left(\vec{M} \times \vec{H}_{eff} - \eta \vec{M} \times \frac{d\vec{M}}{dt} \right) + \dots \quad (\text{SOT, STT, T...})$$

γ : gyromagnetic ratio

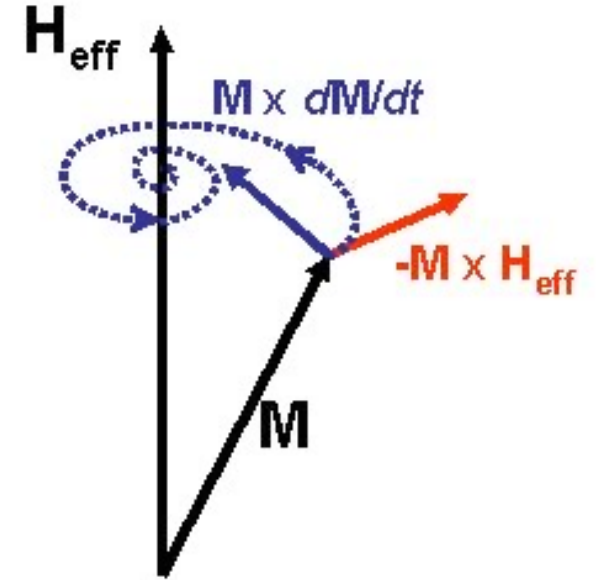
η : damping factor

Effective field:
$$\vec{H}_{eff} = \frac{\partial H}{\partial \vec{M}}$$

Hamiltonian

$$H = -J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j - K \sum_i S_{iz}^2 - g\mu_B \sum_i \vec{B} \cdot \vec{S}_i + \dots$$

$$= \int dr [Ja^2(\nabla m)^2 - K(m_z)^2 - g\mu_B(B \cdot m) + \dots] \quad \text{For FM}$$



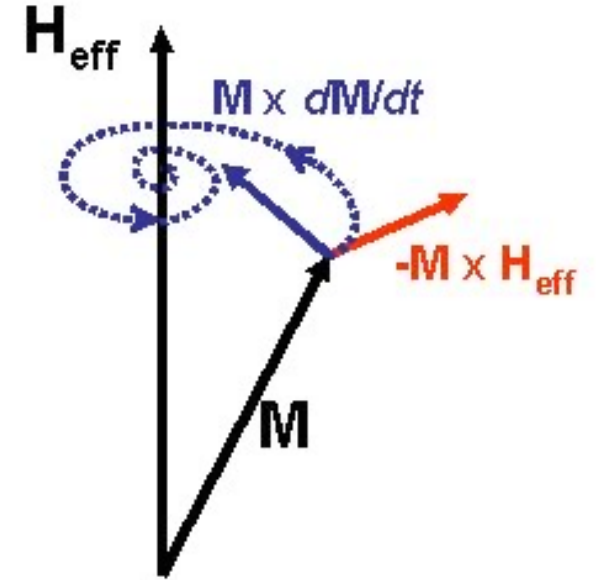
2.2 Magnetic resonance-FMR

$$\frac{d\vec{M}}{dt} = -\gamma \left(\vec{M} \times \vec{H}_{eff} - \eta \vec{M} \times \frac{d\vec{M}}{dt} \right) + \dots \quad (\text{SOT, STT, Tem...})$$

$$\frac{d\vec{M}}{dt} = -\gamma \vec{M} \times \vec{H}_{eff} \quad \Rightarrow \quad \begin{aligned} \frac{dM_x}{dt} &= \gamma M_y (\mu_0 H + K M_z) \\ \frac{dM_y}{dt} &= -\gamma M_x (\mu_0 H + K M_z) \\ \frac{dM_z}{dt} &\approx 0 \end{aligned}$$

$$M_x = m \cos \omega t, M_y = m \sin \omega t, M_z = 1 \Rightarrow \omega = \gamma H_{eff} \quad \text{Kittle formula}$$

$$H_{eff} \sim 2KM \quad \Rightarrow \quad f = \frac{\omega}{2\pi} \sim \text{GHz}$$



2.2 Magnetic resonance-AFMR

Hamiltonian

$$\begin{aligned} H &= J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j - K \sum_i S_{iz}^2 - g\mu_B \sum_i \vec{B} \cdot \vec{S}_i \\ &= \int dr \{ J(2m_1 \cdot m_2) - K[(m_1^z)^2 + (m_2^z)^2] - g\mu_B B \cdot (m_1 + m_2) \} \\ &= \int dr \{ 2Jm_1 \cdot m_2 - K[(m_1^z)^2 + (m_2^z)^2] - g\mu_B B \cdot (m_1 + m_2) \} \end{aligned}$$

Effective field

$$\vec{H}_{eff}^1 = \frac{\partial H}{\partial \vec{M}_1} = 2J\vec{M}_2 - 2K\vec{M}_1 - g\mu_B \vec{B} \qquad \vec{H}_{eff}^2 = \frac{\partial H}{\partial \vec{M}_2} = 2J\vec{M}_1 - 2K\vec{M}_2 - g\mu_B \vec{B}$$

2.2 Magnetic resonance-AFMR



$$\begin{aligned} \frac{d\vec{M}_1}{dt} &= -\gamma \vec{M}_1 \times \vec{H}_{eff}^1 \\ \frac{d\vec{M}_2}{dt} &= -\gamma \vec{M}_2 \times \vec{H}_{eff}^2 \end{aligned} \quad \Rightarrow \quad \frac{d}{dt} \begin{bmatrix} M_1^x \\ M_1^y \\ M_2^x \\ M_2^y \end{bmatrix} = -\gamma \begin{bmatrix} M_1^y H_{eff}^1 \\ -M_1^x H_{eff}^1 \\ M_2^y H_{eff}^2 \\ -M_2^x H_{eff}^2 \end{bmatrix}$$

Solve this eigenequation, we obtain:

$$f = \frac{\omega}{2\pi} = \gamma \sqrt{H_E(H_A \pm H_B)} \sim \text{THz}$$

AFMR is on the order of THz, which is about three orders of magnitude larger than FMR!!!

Fast dynamics !!!

2.3 Magnetic excitation-spin wave

- Goldstone theorem

continuous symmetry is spontaneous breaking

$$T > 0$$

Hamiltonian

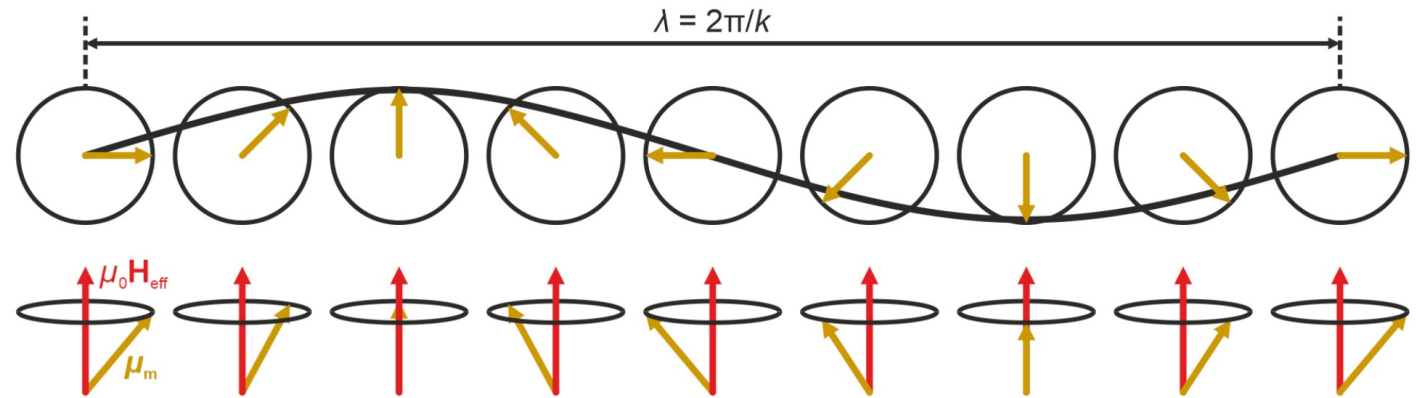
$$H = -J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

$$\frac{d\vec{S}_i}{dt} = -\frac{i}{\hbar} [\vec{S}_i, H] = \frac{2J}{\hbar} [\vec{S}_i, \vec{S}_j \times (\vec{S}_{j-1} + \vec{S}_{j+1})]$$

$$\hbar \frac{dS_j^x}{dt} = 2JS(2S_j^y - S_{j-1}^y - S_{j+1}^y)$$

$$\hbar \frac{dS_j^y}{dt} = -2JS(2S_j^x - S_{j-1}^x - S_{j+1}^x)$$

gapless boson excitation!!!



$$S_j^x = u \exp[i(jka - \omega_k t)]$$

$$S_j^y = v \exp[i(jka - \omega_k t)]$$

$$\epsilon_k = \hbar \omega_k = 4JS(1 - \cos ka)$$

2.3 Magnetic excitation-HP transformation



$$[S_\alpha, S_\beta] = i\epsilon_{\alpha\beta\gamma}\hbar S_\gamma$$

$$|s, s - n\rangle \mapsto \frac{1}{\sqrt{n!}} (a^\dagger)^n |0\rangle_B$$

$$S^2 |s, m_s\rangle = \hbar^2 s(s+1) |s, m_s\rangle,$$

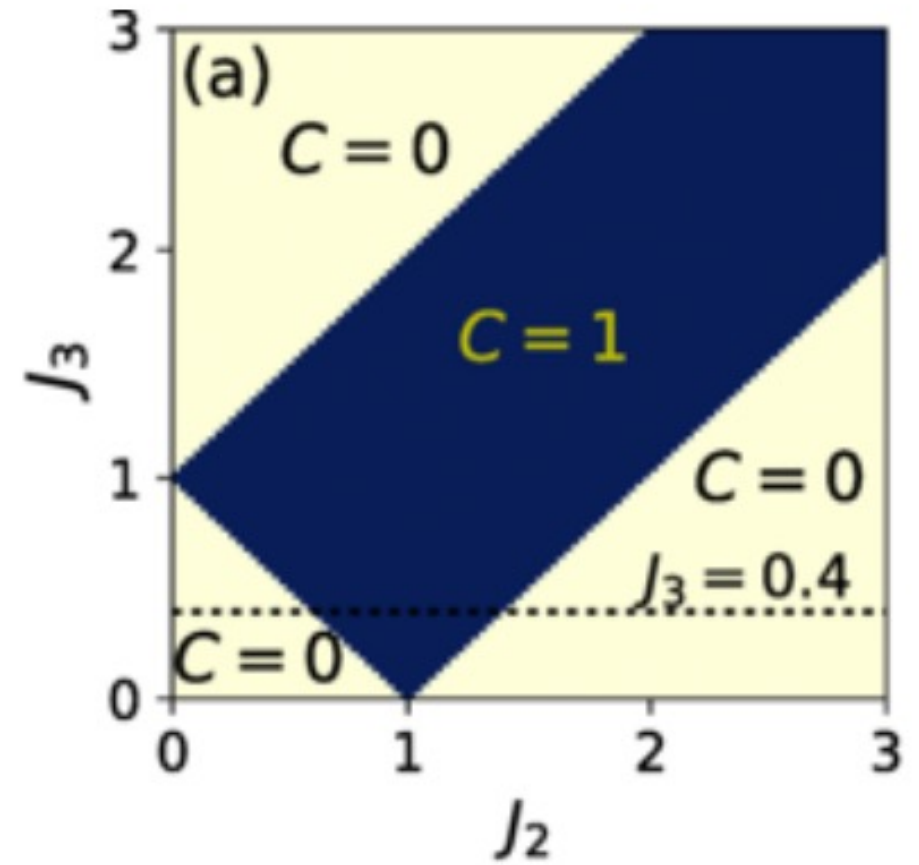
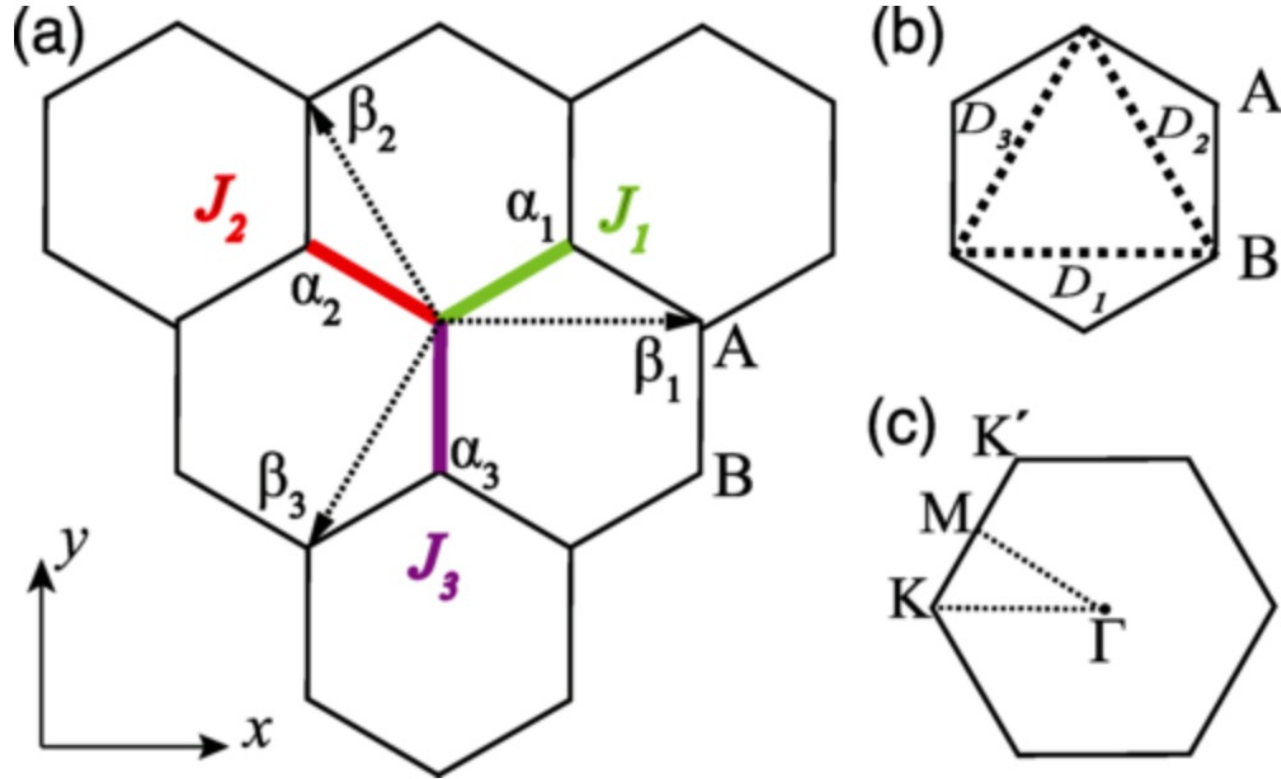
$$S_z |s, m_s\rangle = \hbar m_s |s, m_s\rangle.$$

$$S_+ = S_x + iS_y, S_- = S_x - iS_y$$

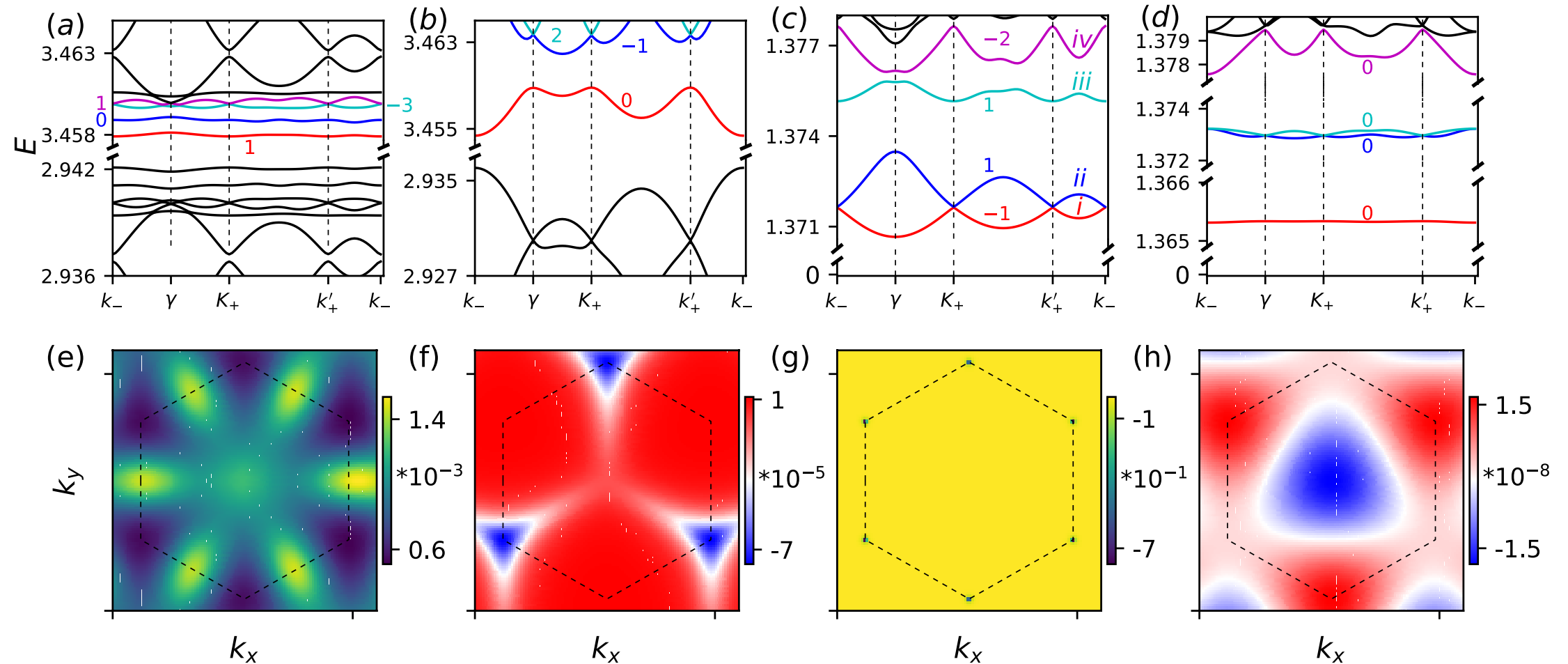
Holstein-Primakoff Transformation

$$S_+ = \hbar\sqrt{2s}\sqrt{1 - \frac{a^\dagger a}{2s}} a, \quad S_- = \hbar\sqrt{2s}a^\dagger \sqrt{1 - \frac{a^\dagger a}{2s}}, \quad S_z = \hbar(s - a^\dagger a).$$

2.3 Magnetic excitation



2.3 Magnetic excitation



1. Static magnetic properties

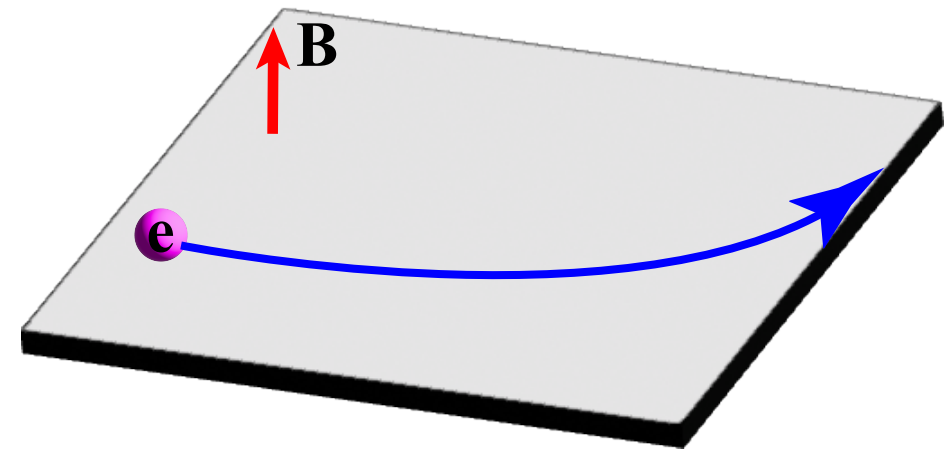
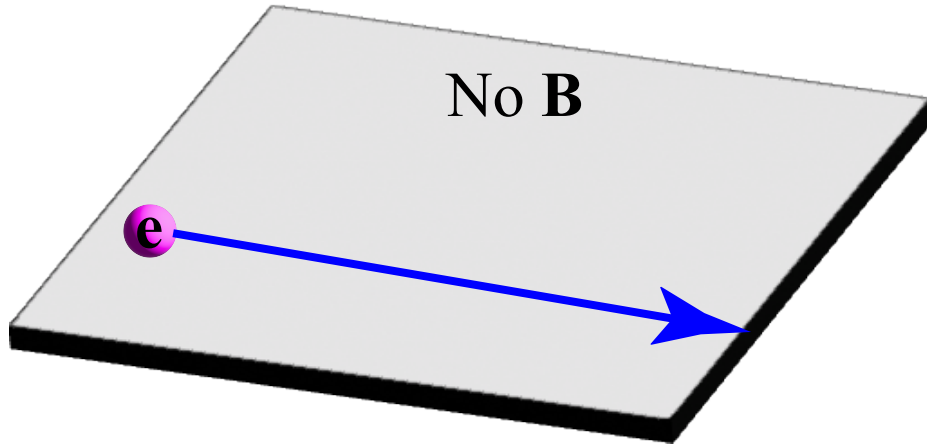
- Magnetic moment
- Magnetic anisotropy and Exchange interaction
- Magnetic ordering

2. Dynamic magnetic properties

- Landau-Lifshitz-Gilbert (LLG) equation
- Magnetic resonance
- Magnetic excitation

3. Magnetic effect

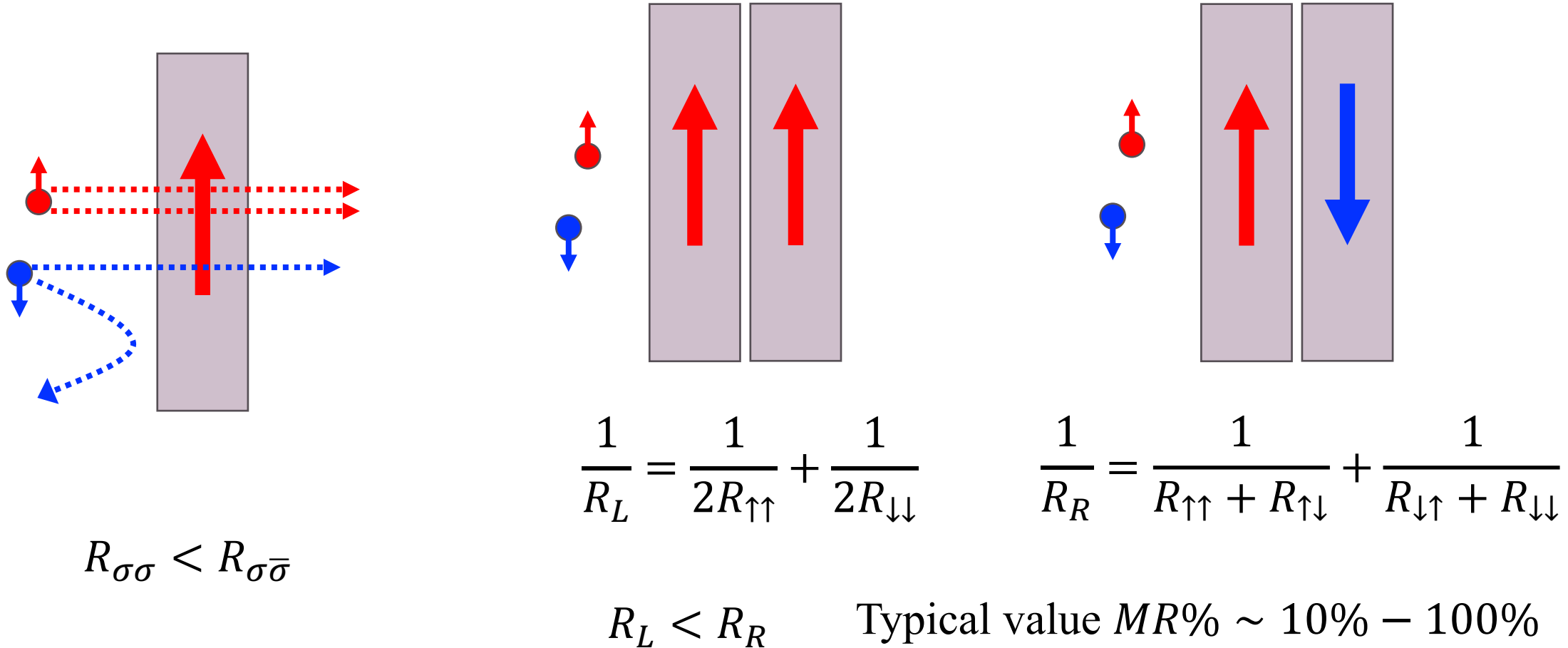
3.1 Magnetoresistance



$$MR\% = \frac{R(\mu_0 \vec{B}) - R(0)}{R(0)} \times 100\%$$

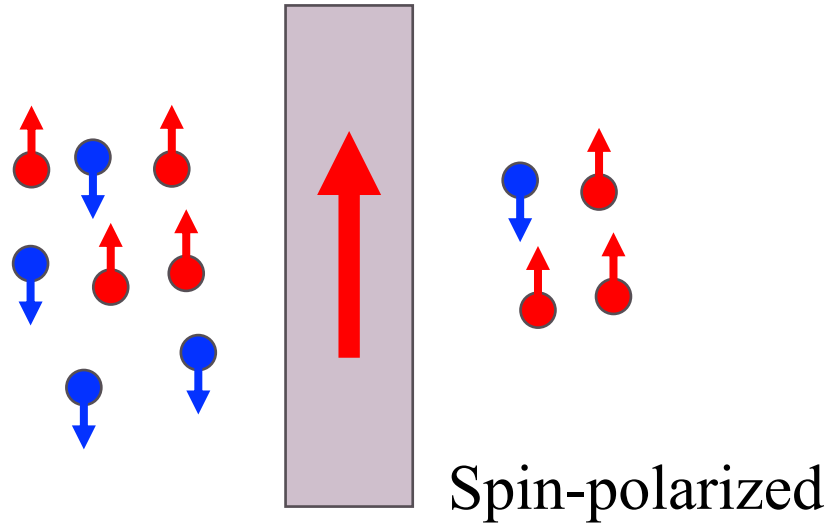
Typically < 2%

3.2 Giant Magnetoresistance



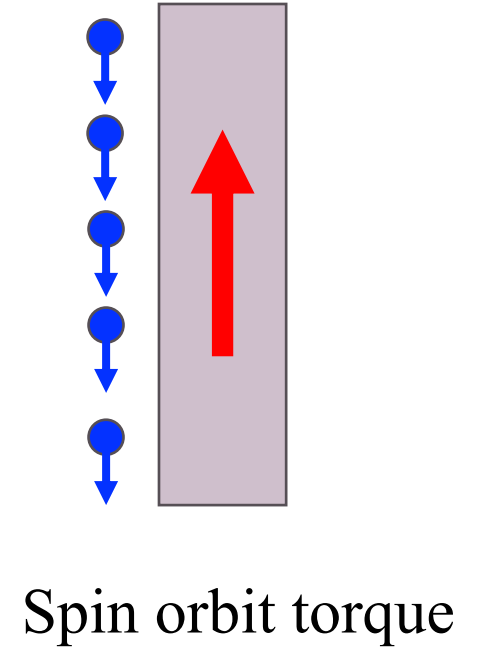
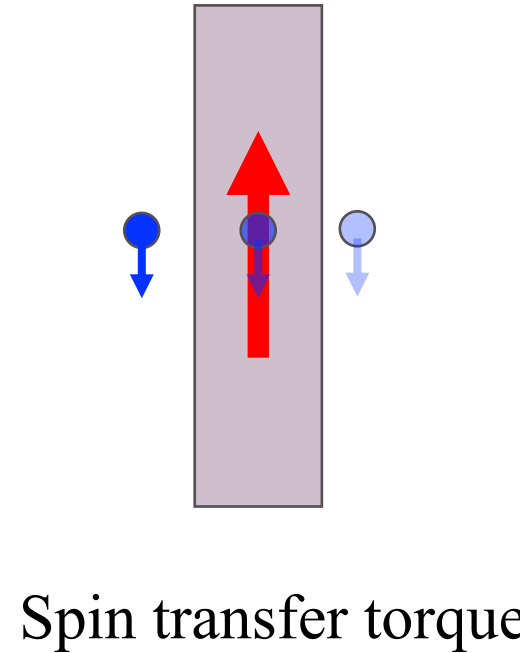
The Nobel Prize in Physics 2007 was awarded jointly to Albert Fert and Peter Grünberg "for the discovery of ***Giant Magnetoresistance***"

3.3 Spin pumping and spin torque



Unpolarized
electron
current

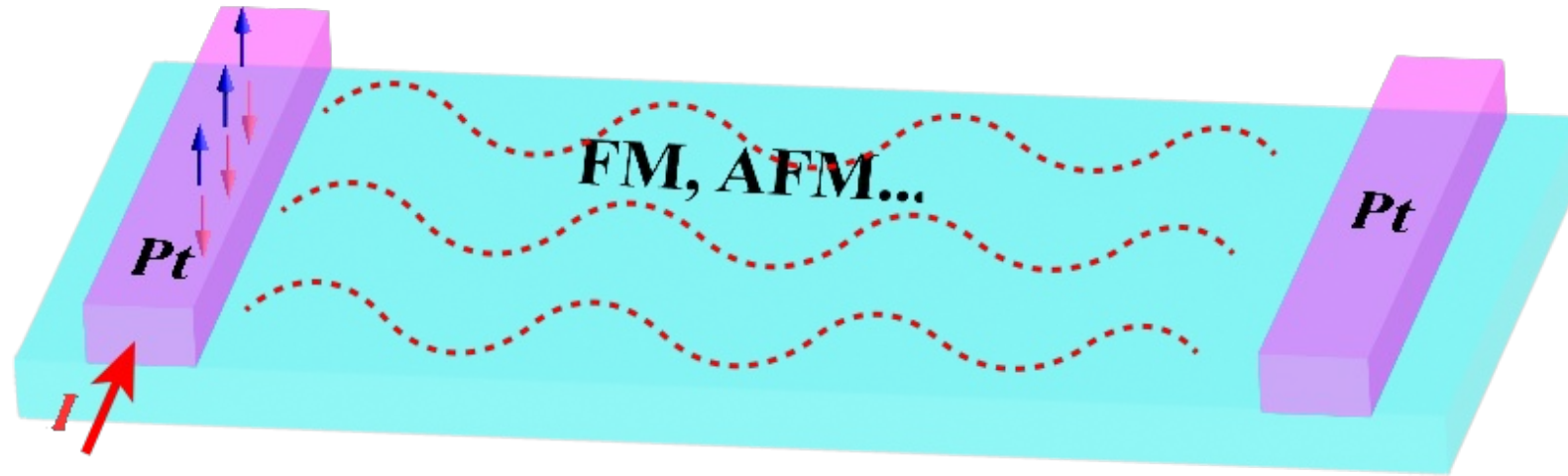
Spin pumping



$$H_{eff} = J\vec{S} \cdot \vec{S}$$

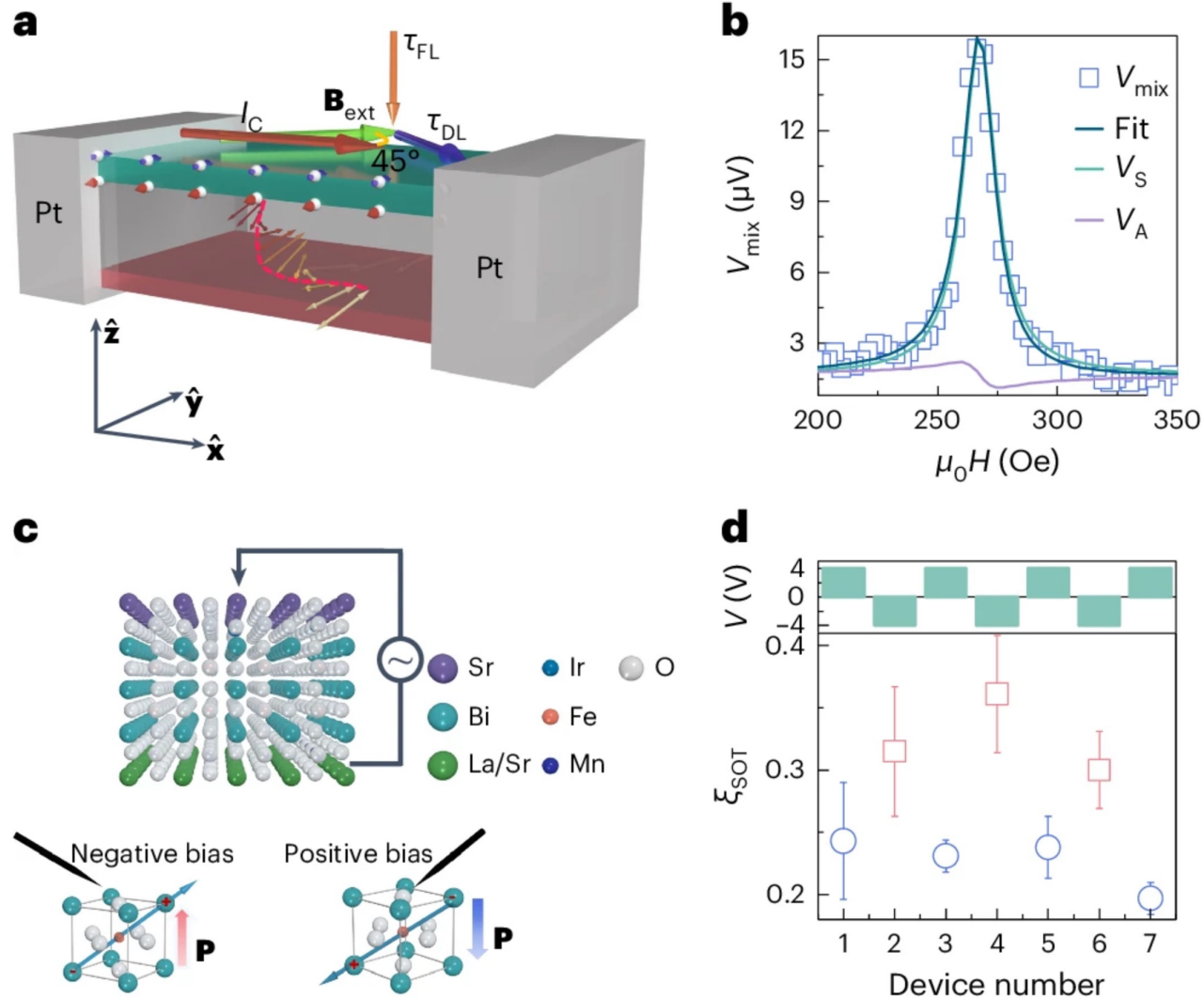
$$\tau_{field} = \frac{\partial \vec{S}}{\partial t} = c_1 \vec{S} \times \vec{S}$$
$$\tau_{damping} = c_2 (\vec{S} \times \vec{S}) \times \vec{S}$$

3.4 Spintronics



- Spin Hall effect and inverse spin Hall effect

3.4 Spintronics



$$J_s = \frac{G_L}{S} \int d\mathbf{r}^2 [\mathbf{L}(\mathbf{r}) \cdot \mathbf{s}]^2 + \frac{G_m}{S} \int d\mathbf{r}^2 [\mathbf{m}(\mathbf{r}) \cdot \mathbf{s}]^2$$

$$\xi = \frac{J_s(-\mathbf{P}) - J_s(\mathbf{P})}{J_s(-\mathbf{P}) + J_s(\mathbf{P})}$$

Thanks for your attention