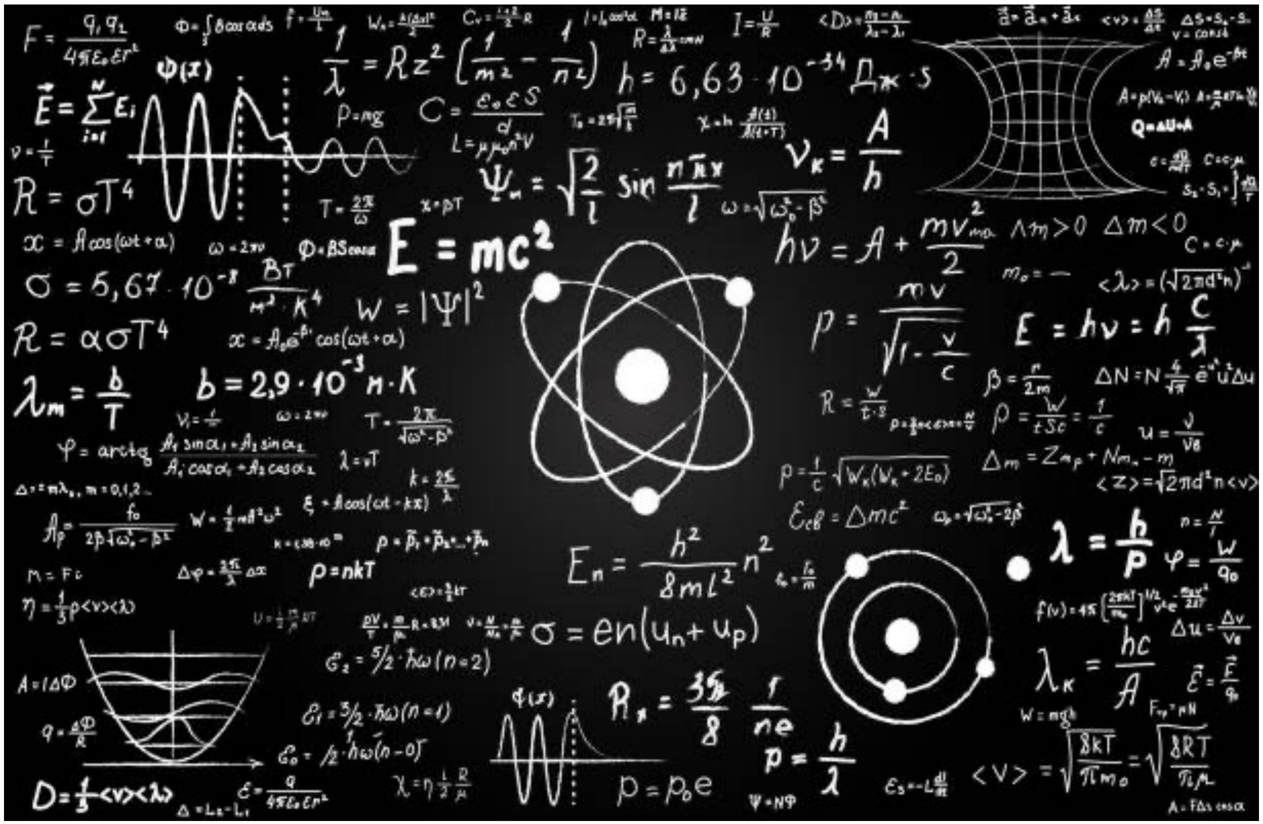
Atomic Physics

Chapter 1

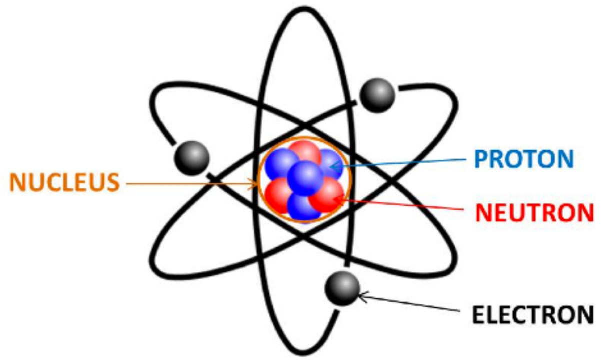
Basic Properties of Atom



An atom is the smallest unchangeable component of a chemical element.

1. Unchangeable means in this case by chemical means

2. Moderate temperatures: kT < eV



Mass range: 1.67×10^{-27} to 4.52×10^{-25} kg

Electric charge: zero (neutral), or ion charge

Diameter range: 62 pm (He) to 520 pm (Cs)

Components: Electrons and compact nucleus of protons and neutrons

The mass of an atom

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Atomic mass unit (AMU):

1u: 1/12 of the mass of a neutral carbon atom with nuclear charge 6 and mass number 12

Mass number (A):

The total number of protons and neutrons in nucleus

Mole (mol):

1 mol is the quantity of a substance that contains the same number of particles (atoms or molecules) as 0.012 kg of carbon ^{12}C.

1 mol of atoms or molecules with atomic mass number A AMU has a mass of A grams.

The relation between 1u and N_A

$$1u = \frac{1}{N_A} = 1.660539040(20) \times 10^{-27} \text{ kg}$$

Electronvolt

$$1 \text{ eV} = 1.602176565(35) \times 10^{-19} \text{ C} \times 1 \text{ V}$$
$$= 1.602176565(35) \times 10^{-19} \text{ J}$$

Mass-energy equivalence

$$E = mc^2$$

1u transfer to eV

$$1 \text{ u} = 931.478 \times 10^6 \text{ eV}/c^2$$
$$= 931.478 \text{ MeV}/c^2$$

The mass of an atom

The mass of electron:

$$
\begin{aligned}
m_e &= 9.10938356(11) \times 10^{-31}\ \mathrm{kg} \\
&= 5.48579909070(16) \times 10^{-4}\ \mathrm{u} \\
&= 0.5109989461(31)\ \mathrm{MeV}
\end{aligned}
$$

The mass of proton:

$$
\begin{aligned}
m_p &= 1.672621898(21) \times 10^{-27}\ \mathrm{kg} \\
&= 1.007276466879(91)\ \mathrm{u} \\
&= 938.2720813(58)\ \mathrm{MeV}
\end{aligned}
$$

The mass of neutron:

$$
\begin{aligned}
m_n &= 1.674927471(21) \times 10^{-27}\ \mathrm{kg} \\
&= 1.00866491588(49)\ \mathrm{u} \\
&= 939.5654133(58)\ \mathrm{MeV}
\end{aligned}
$$

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Avogadro's number is a Bridge from macroscopic to microscopic physics.

1 mole of any substance contains the same number (N_A) of atoms (molecules)

$$N_{\mathrm{A}} = \frac{\text{Mass of 1 mole of the substance}}{\text{Mass of an atom}}$$
$$= 6.02214078(18) \times 10^{23}\ \mathrm{mol}^{-1}$$

1. The Faraday constant and elementary charge

$$F = N_{\mathrm{A}} e$$

2. Gas constant and Boltzmann constant

$$R = k_{\mathrm{B}} N_{\mathrm{A}}$$

3. Molar volume and atomic volume

$$V_{\mathrm{m}} = V_{\mathrm{atom}} N_{\mathrm{A}}$$

The Faraday's constant

$$F = N_{\mathrm{A}} \cdot e = 96,485.3383(83)\,\mathrm{C/mol}$$

is the electric charge transported to the electrode in an electrolytic cell, when 1 mol of singly charged ions with mass m_X and elementary charge e has been deposited at the electrode.

Therefore, weighing the mass increase Δm of the electrode after a charge Q has been transferred, yields:

$$\Delta m = \frac{Q}{e} m_{\mathrm{X}} = \frac{Q}{e} \frac{M_{\mathrm{X}}}{N_{\mathrm{A}}}$$

$$\Rightarrow N_{\mathrm{A}} = \frac{Q}{e} \frac{M_{\mathrm{X}}}{\Delta m}$$

From measurements of the absolute mass m of atoms X and the molar mass M_X, the Avogadro constant

$$N_A = M_X / m_X$$

can be directly determined.

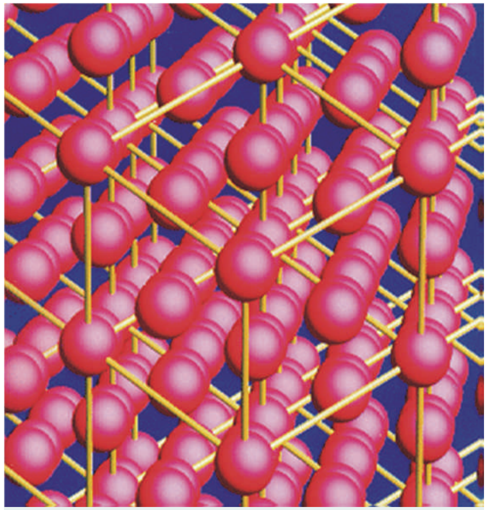
The molar mass for gas is defined as the mass of a gas of atoms X within the molar volume V = 22.4 dm^3 under normal conditions p and T .

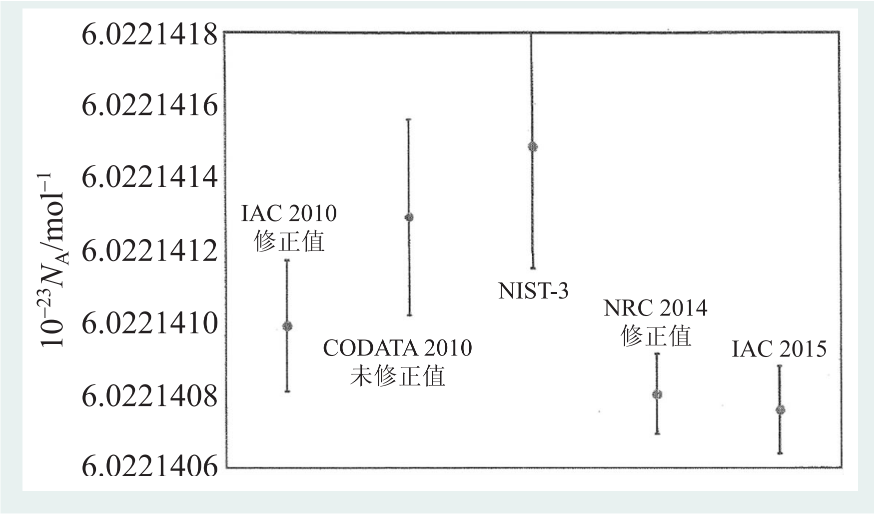
The molar mass can be also obtained for nongaseous substances from the definition

$$M_X = 0.012 m_X / m(^{12}C)\ \mathrm{kg}$$

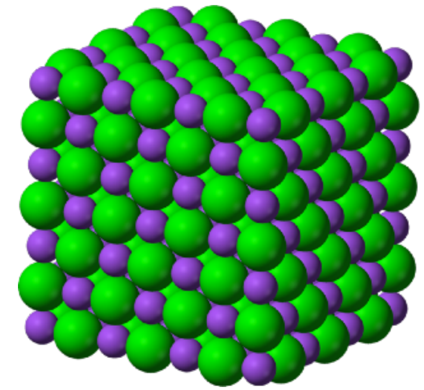
Avogadro's Number measurements

Method	Fundamental constant	Avogadro's number
General gas equation	Universal gas constant R	
Barometric pressure formula (*Perrin*)	Boltzmann's constant k	$N_A = R/k$
Diffusion (*Einstein*)		
Torsionsal oscillations (*Kappler*)		
Electrolysis	Faraday's constant F	$N_A = F/e$
Millikan's oil-drop experiment	Elementary charge e	
X-ray diffraction and interferometry	Distance d between crystal planes in a cubic crystal	$N_A = (V/a^3)\frac{M_m}{M_c}$ for cubic primitive crystal
Measurement of atom number N in a single crystal with mass M_c and molar mass M_m	$N_A = N \cdot \frac{M_m}{M_c}$	$N_A = 4\,M_m/\varrho a^3$ for cubic face centered crystal





The size of atom



Assume that the masses of 1 mole atoms is A, and the atom is spherical

$$\frac{4}{3}\pi r^3 N_{\rm A} = \frac{A}{\rho}$$

The radius of atom → r

The density of substance → ρ

The radius of atom

$$r = \left(\frac{3A}{4\pi\rho N_{\rm A}}\right)^{\frac{1}{3}}$$

The units for the radius of atom

$$1\ \text{nm} = 10^{-9}\ \text{m},\quad 1\ \text{Å} = 10^{-10}\ \text{m},$$

$$1\ \text{pm} = 10^{-12}\ \text{m},\quad 1\ \text{fm} = 10^{-15}\ \text{m}$$

The size of atom

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Elements	A	Density ρ (g/cm³)	Radius *r* (nm)
Li	7	0.7	
Al	27	2.7	
Cu	63	8.9	
S	32	2.07	
Pb	207	11.34	

The size of atom

Elements	A	Density ρ (g/cm³)	Radius *r* (nm)
Li	7	0.7	0.16
Al	27	2.7	0.16
Cu	63	8.9	0.14
S	32	2.07	0.18
Pb	207	11.34	0.19

The size of atom

The unit is pm

1	H 25																	He
2	Li 145	Be 105											B 85	C 70	N 65	O 60	F 50	Ne
3	Na 180	Mg 150											Al 125	Si 110	P 100	S 100	Cl 100	Ar
4	K 220	Ca 180	Sc 160	Ti 140	V 135	Cr 140	Mn 140	Fe 140	Co 135	Ni 135	Cu 135	Zn 135	Ga 130	Ge 125	As 115	Se 115	Br 115	Kr
5	Rb 235	Sr 200	Y 180	Zr 155	Nb 145	Mo 145	Tc 135	Ru 130	Rh 135	Pd 140	Ag 160	Cd 155	In 155	Sn 145	Sb 145	Te 140	I 140	Xe
6	Cs 260	Ba 215	*	Hf 155	Ta 145	W 135	Re 135	Os 130	Ir 135	Pt 135	Au 135	Hg 150	Tl 190	Pb 180	Bi 160	Po 190	At	Rn
7	Fr	Ra 215	**	Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn	Nh	Fl	Mc	Lv	Ts	Og
Lanthanides	*	La 195	Ce 185	Pr 185	Nd 185	Pm 185	Sm 185	Eu 185	Gd 180	Tb 175	Dy 175	Ho 175	Er 175	Tm 175	Yb 175	Lu 175		
Actinides	**	Ac 195	Th 180	Pa 180	U 175	Np 175	Pu 175	Am 175	Cm	Bk	Cf	Es	Fm	Md	No	Lr		

Determination of the size of atom

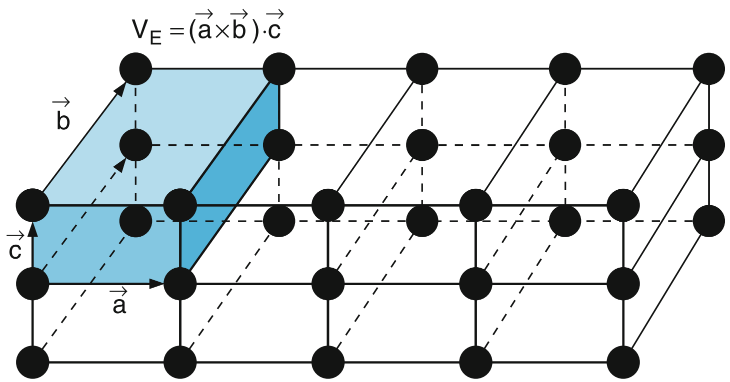
1. From the Covolume(协体积) in Van der Waals equation

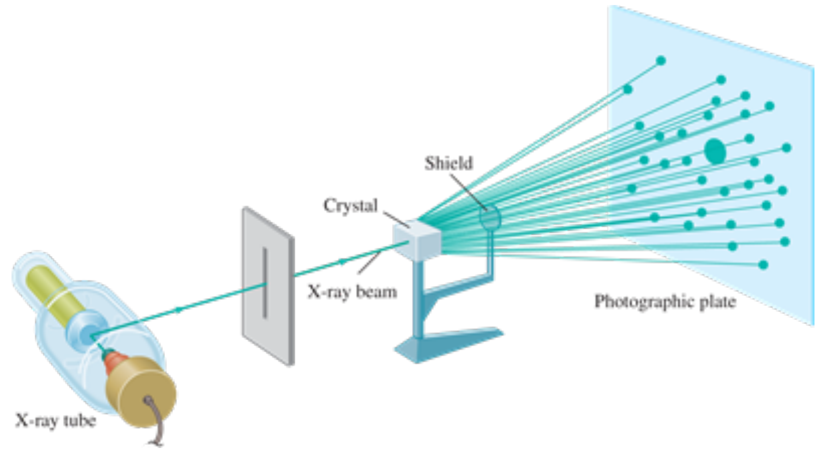
$$(P + a/V^2)(V - b) = RT$$

where, the quantity b, is equal to the fourfold volume of the particles

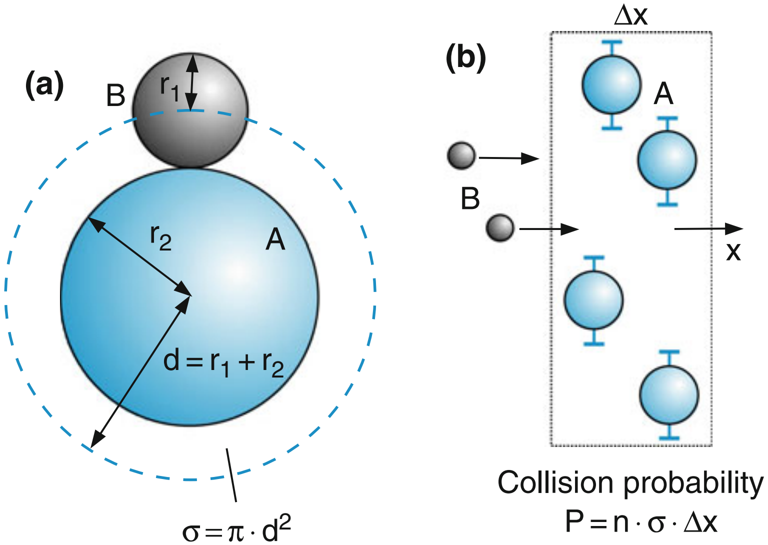
$$b = 4\frac{4\pi}{3}r^3 N_\mathrm{A}$$

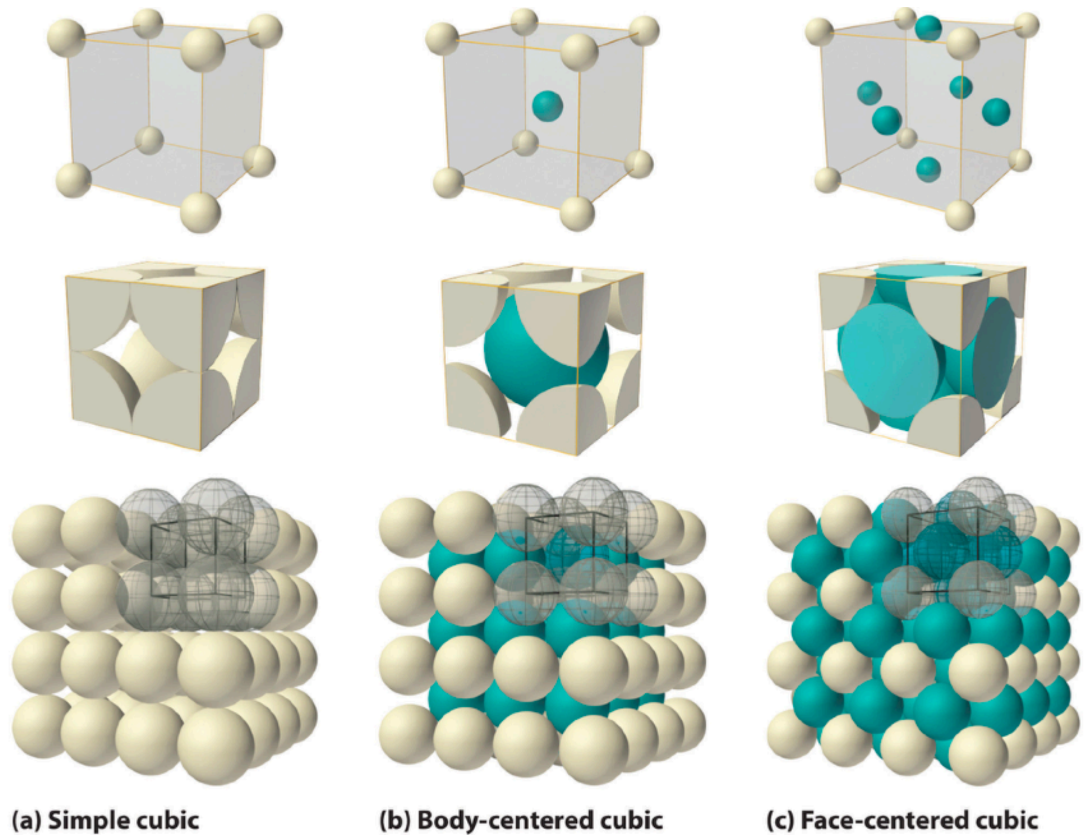
2. From X-ray diffraction measurements on crystals





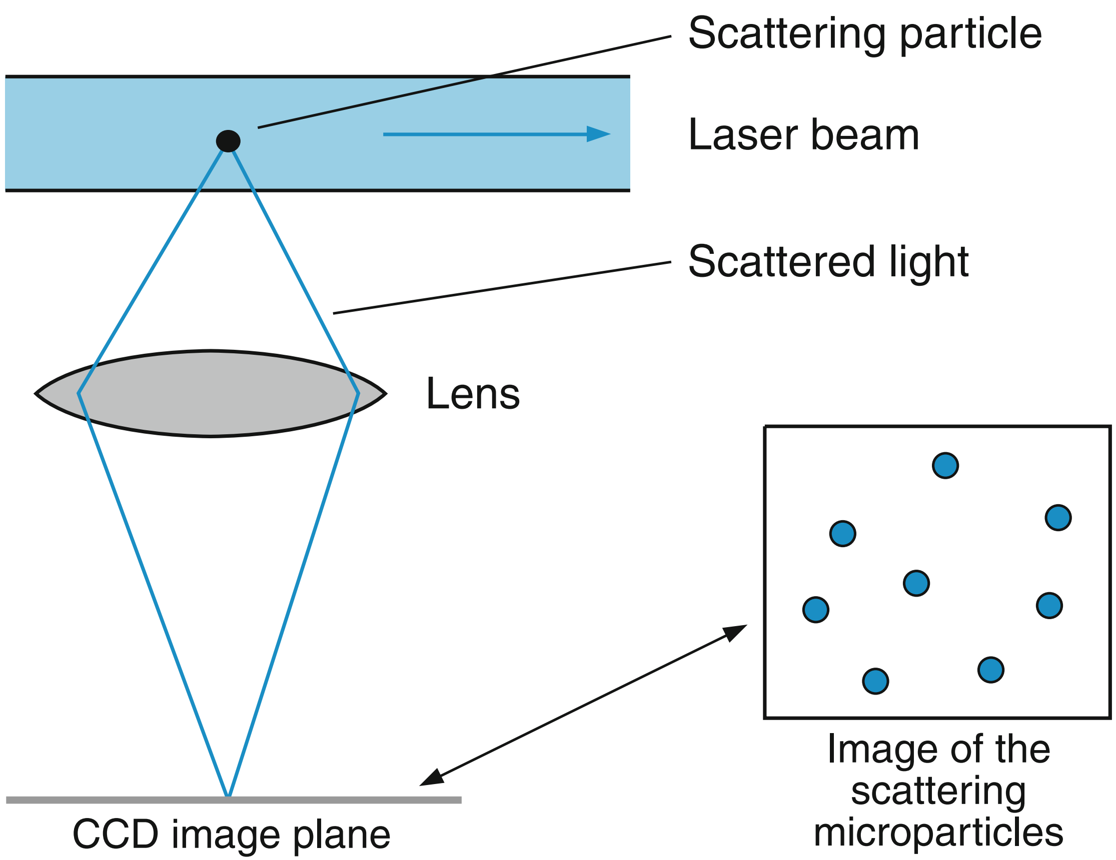
3. From the interaction cross section



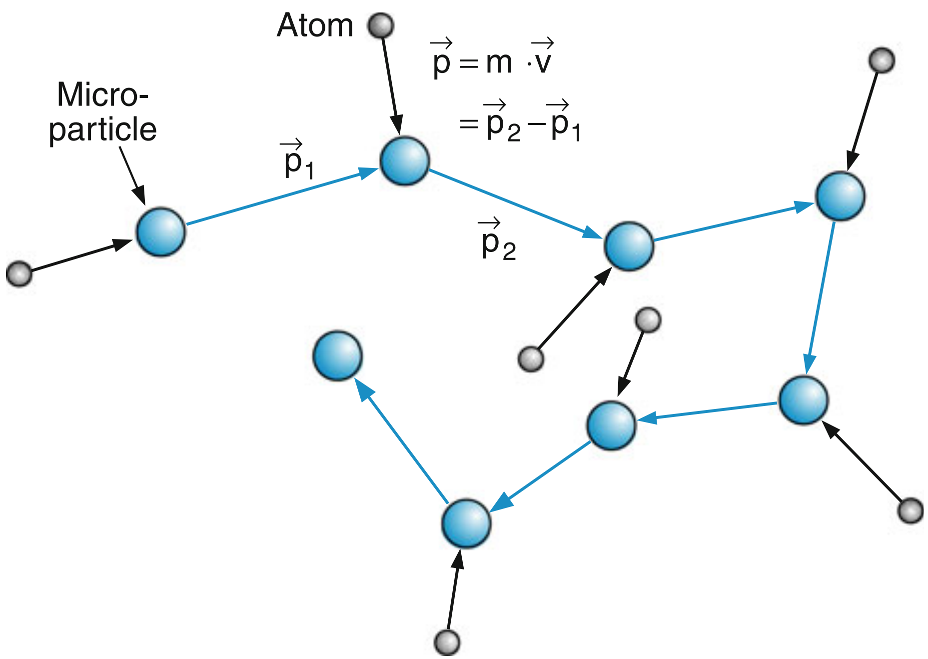


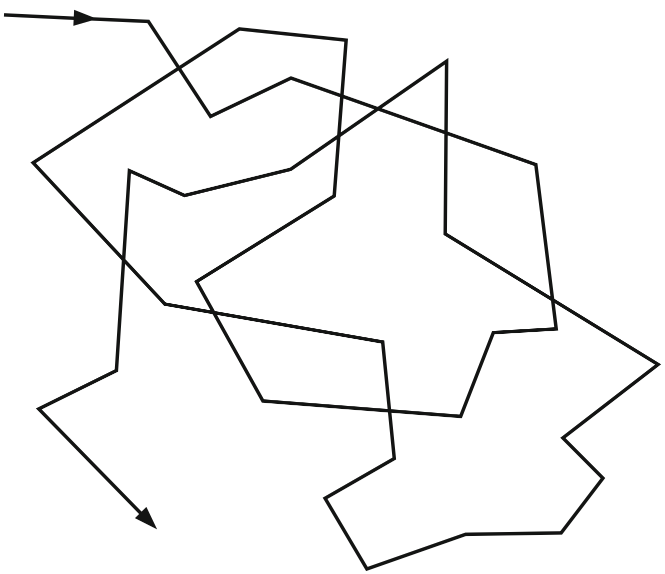
Can One See Atoms?

Scattering of visible light by single atoms. Each image point corresponds to one atom



Brownian Motion: small particles suspended in liquids performed small irregular movements, which can be viewed under a microscope





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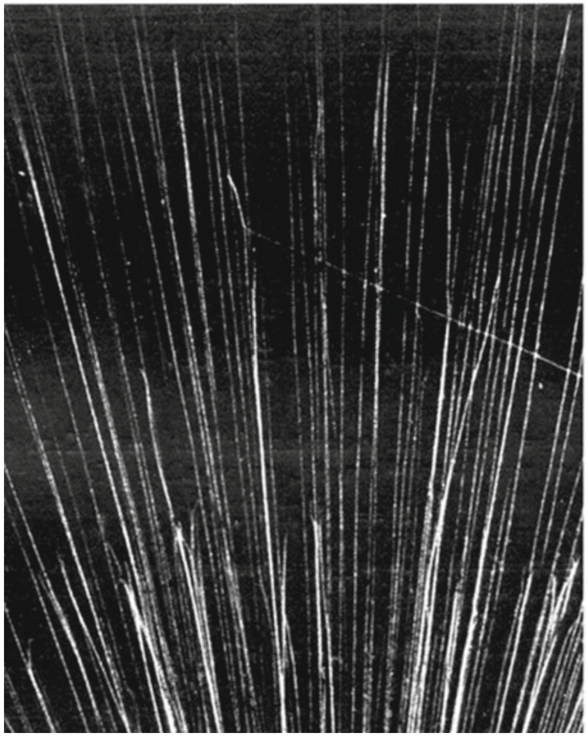
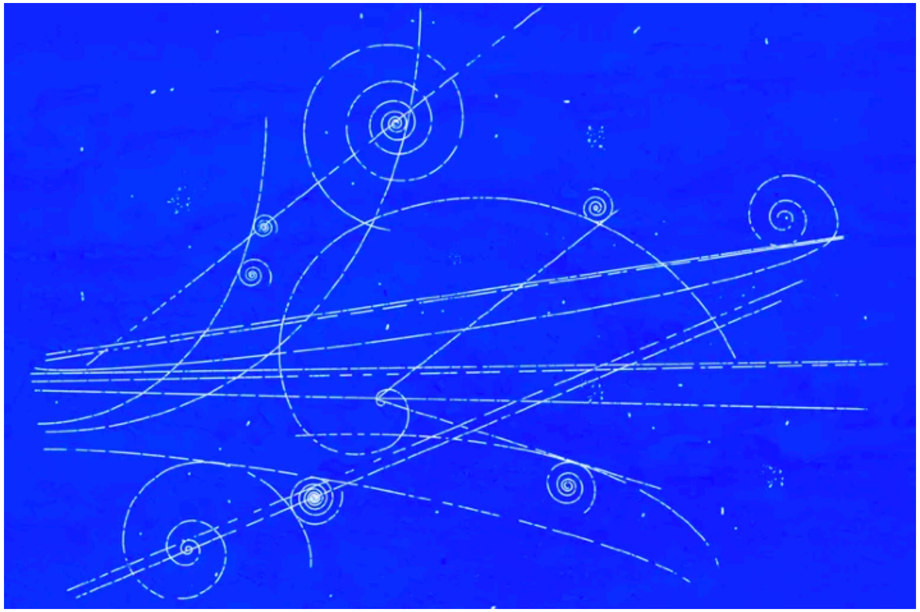
Cloud Chamber: Incident particles with sufficient kinetic energy can ionize the atoms or molecules in the cloud chamber, which is filled with supersaturated water vapor.





How to Make a Cloud Chamber

https://www.thoughtco.com/how-to-make-a-cloud-chamber-415380

Microscopes with Atomic Resolution:

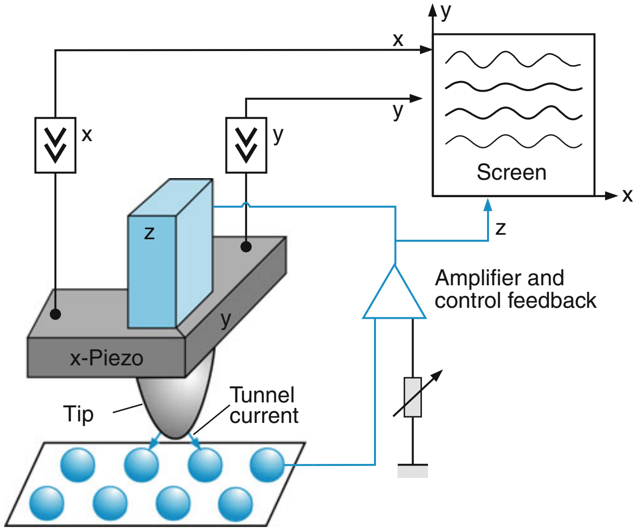
Field Emission Microscope

Transmission Electron Microscope

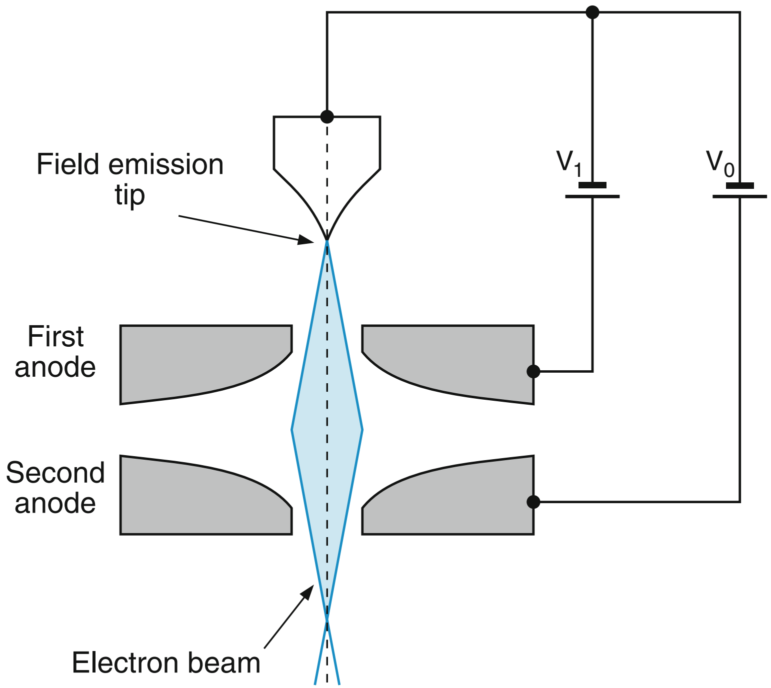
Scanning Electron Microscope

Scanning Tunneling Microscope

Atomic Force Microscope

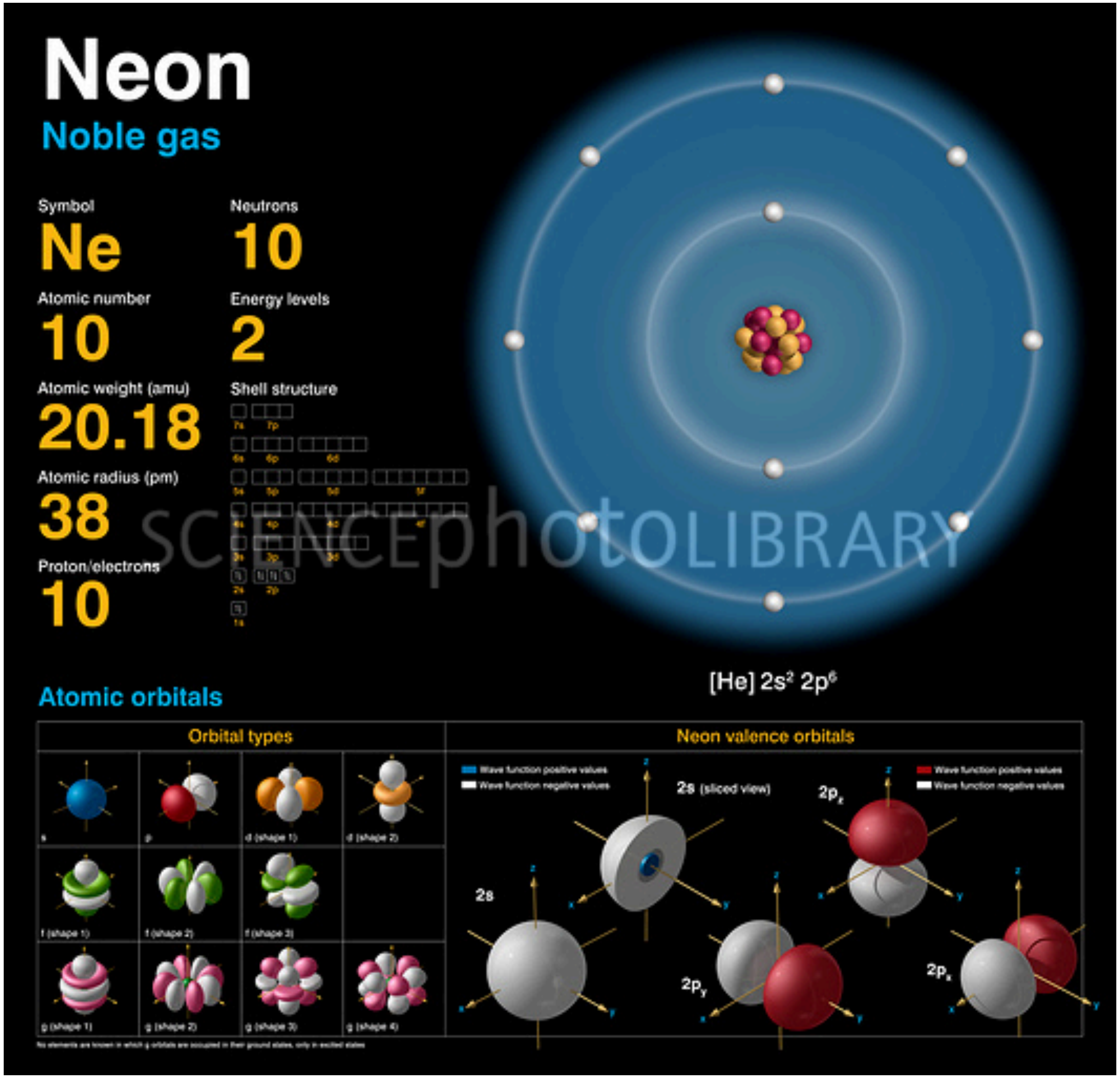


Scanning Tunneling Microscope



Field Emission Microscope

Element symbol



Various experimental investigations had shown already at the end of the 19th century that matter consists of electrically charged particles. The essential evidence came from:

1. Investigations of electrolytic conductivity in polar liquids
2. Experiments on gas discharges
3. Observations of the influence of magnetic fields on the electric current in metals and semiconductors
4. The discovery that particles emitted from radioactive substances show different deflections in magnetic fields.

The Electric Structure of Atoms

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Atoms are built up of charged particles. They can therefore not be "indivisible," but have a substructure, which, however, was unknown at this time. The electrically charged positive and negative constituents have different masses.

This raises the questions:

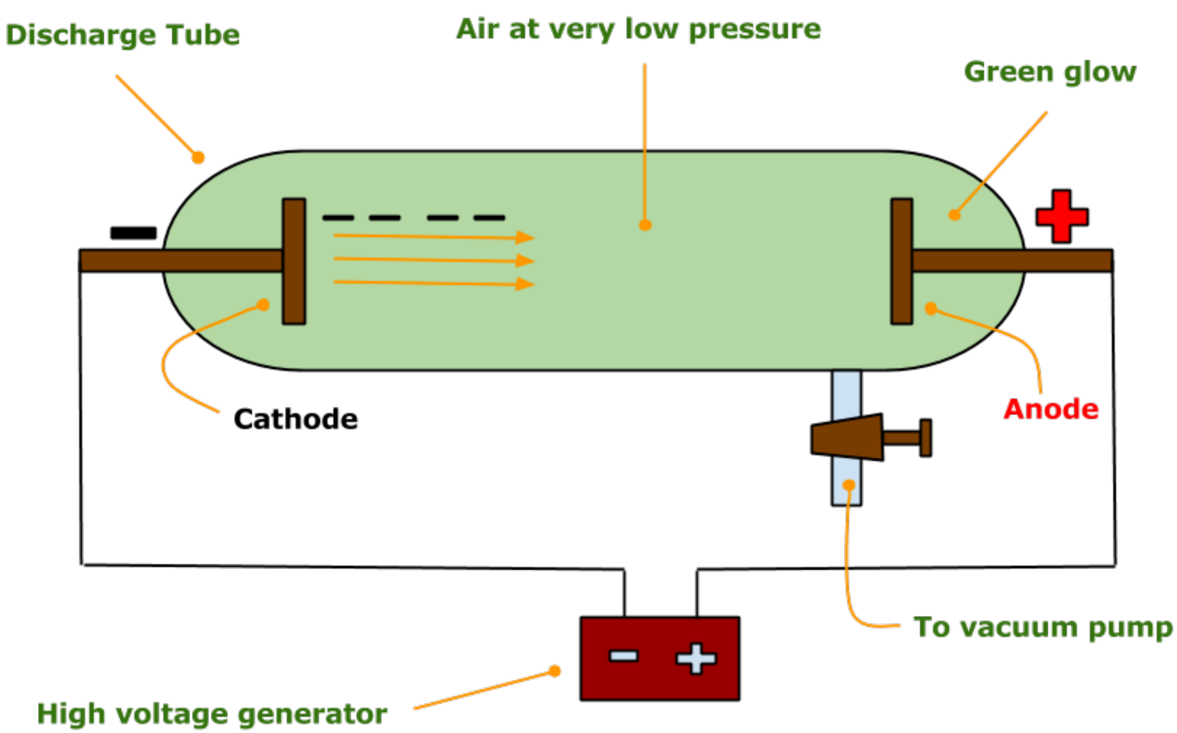
- What properties do these constituents have?
- What force keeps them together to form stable atoms?
- What is the charge distribution inside the atom?
- How can the microscopic properties of matter be explained

by this model?

Investigations of gas discharges have all contributed much to our understanding of the electric structure of atoms. It is worthwhile to note that the essential experimental progress was only possible after the improvement of vacuum technology.

In a gas discharge tube at low pressures, Hittorf observed particle rays emitted from the cathode that followed (without external fields) straight lines. He could prove this by the shadow that was produced on a fluorescent screen when obstacles were put in the path of the cathode rays.

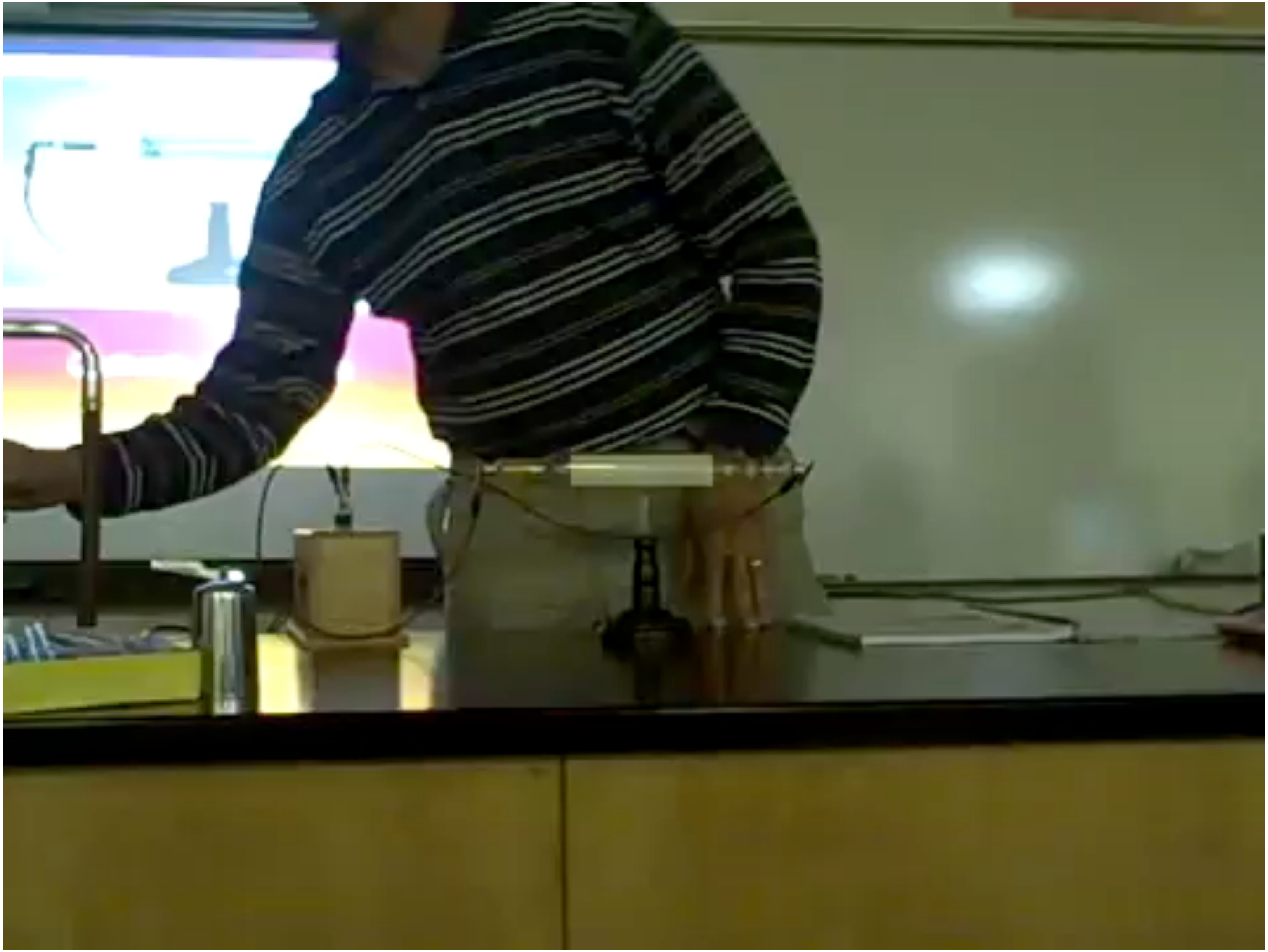
Cathode ray tube



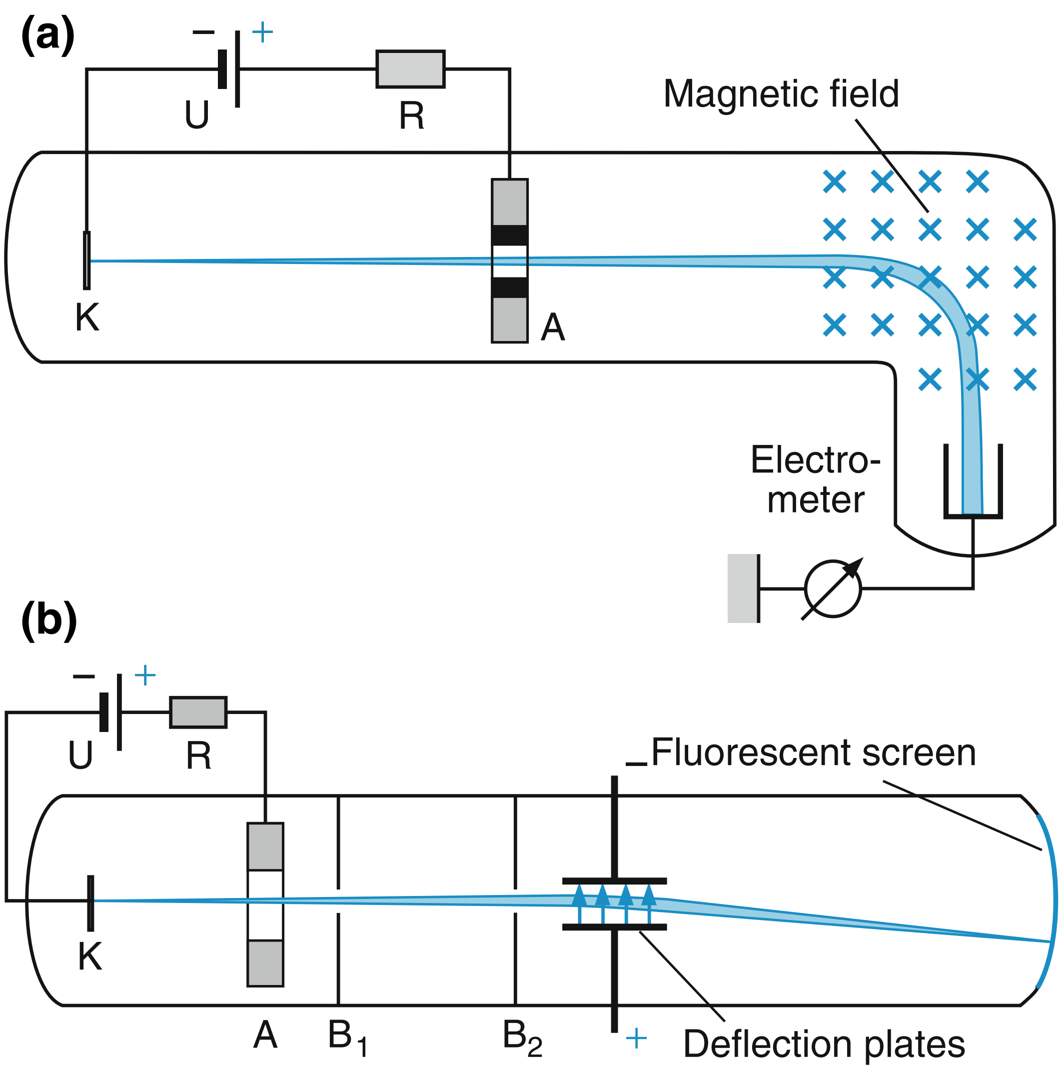
Production of cathode rays

Obtained in 1895 by J.B. Perrin and with an improved apparatus in 1897 by Thomson

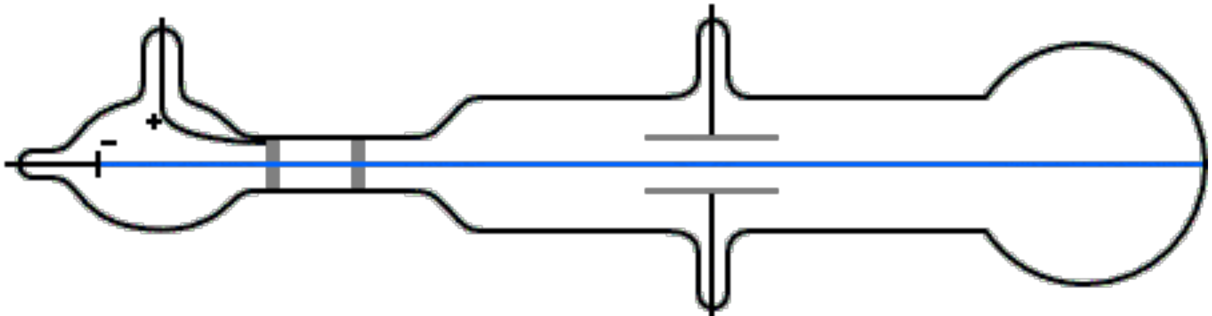
Cathode ray tube



Cathode ray in external field



1897, J. J. Thomson found electron (corpuscles)



http://web.lemoyne.edu/~giunta/thomson1897.html

1. They travel in straight lines.
2. They are independent of the material composition of the cathode.
3. Applying electric field in the path of cathode ray deflects the ray towards positively charged plate. Hence cathode ray consists of negatively charged particles.

The discovery of electron

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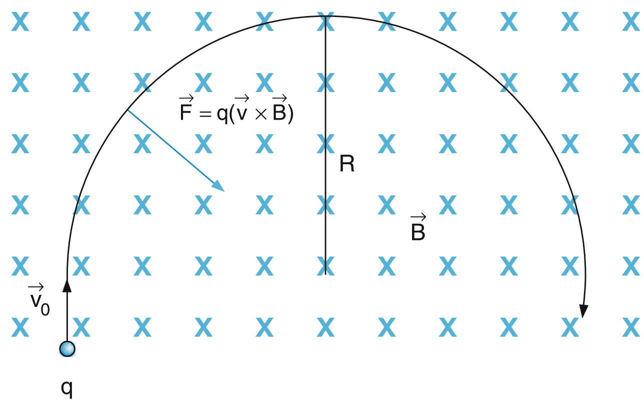
The negative light particles of the cathode rays were named electrons after a proposal by J. Stoney and G. Fitzgerald in 1897. The positively charged heavy particle were named ions according to the existing name for charged atoms or molecules in the electrolysis.

These discoveries gave the following picture of the charged constituents of atoms: atoms consist of negatively charged electrons and positively charged particles that just compensate the negative charge to make the whole atom neutral.

Charge-to-Mass Ratio for the Electron



Lorentz Force

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

Circle motion

$$\frac{mv^2}{R} = evB_z$$

Acceleration voltage V

$$(m/2)v^2 = eV$$

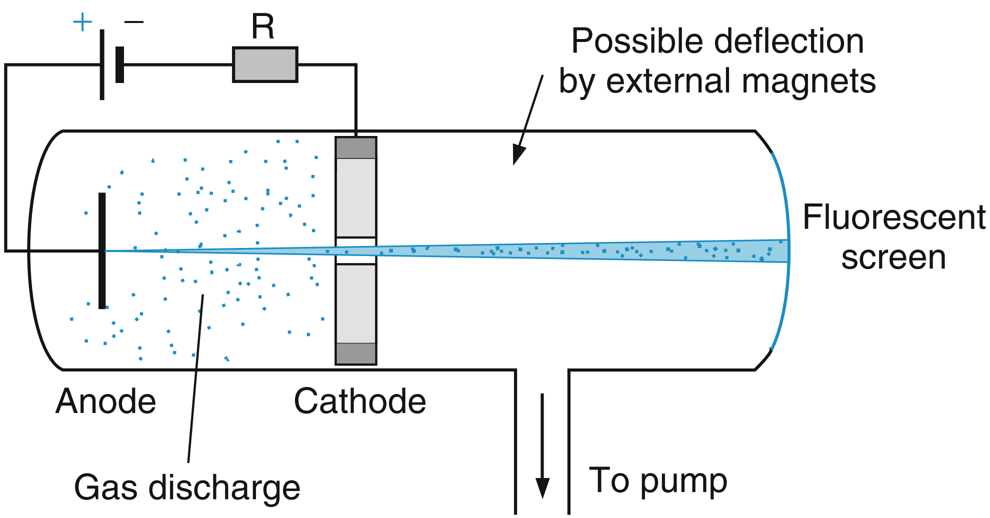
Charge-to-Mass ration

$$\frac{e}{m} = \frac{2V}{R^2B^2}.$$

Gas.	Value of W/	I.	m/e	v.
		Tube		
Air	4.6×10^{11}	230	$.57 \times 10^{-7}$	4×10^{9}
Air	1.8×10^{12}	350	$.34 \times 10^{-7}$	1×10^{10}
Air	6.1×10^{11}	230	$.43 \times 10^{-7}$	5.4×10^{9}
Air	2.5×10^{12}	400	$.32 \times 10^{-7}$	1.2×10^{10}
Air	5.5×10^{11}	230	$.48 \times 10^{-7}$	4.8×10^{9}
Air	1×10^{12}	285	$.4 \times 10^{-7}$	7×10^{9}
Air	1×10^{12}	285	$.4 \times 10^{-7}$	7×10^{9}
Hydrogen	6×10^{12}	205	$.35 \times 10^{-7}$	6×10^{9}
Hydrogen	2.1×10^{12}	460	$.5 \times 10^{-7}$	9.2×10^{9}
Carbonic	8.4×10^{11}	260	$.4 \times 10^{-7}$	7.5×10^{9}
Carbonic	1.47×10^{12}	340	$.4 \times 10^{-7}$	8.5×10^{9}
Carbonic	3.0×10^{12}	480	$.39 \times 10^{-7}$	1.3×10^{10}

Kanalstrahlen

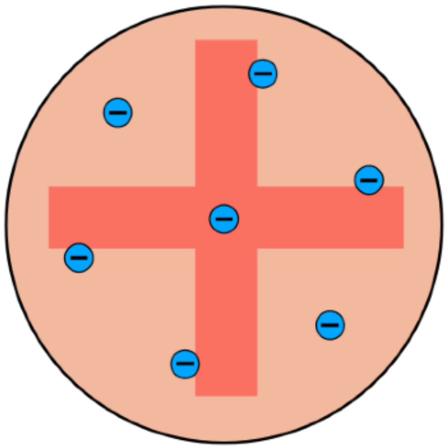
Thomson could also show that the ratio e/m was independent of the cathode material, but was about 10^4 times larger than that for the "Kanalstrahlen" discovered in 1886 by Eugen Goldstein (1850–1930) in a discharge tube, which fly through a hole in the cathode in the opposite direction of the cathode rays.

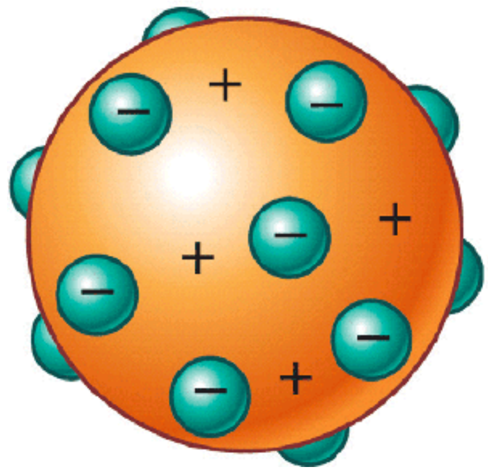


Wilhelm Wien (1864–1928) measured in 1897 the value of e/m for the particles in the Kanalstrahlen and he proved that they are positively charged atoms of the gas inside the discharge tube.

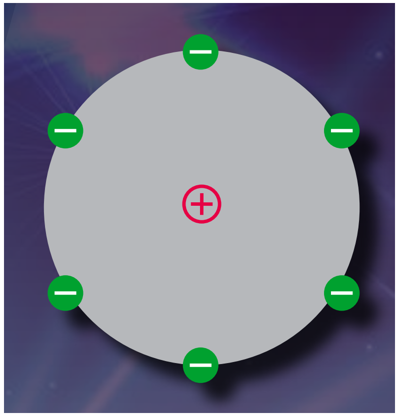
Contemporary theories of atom

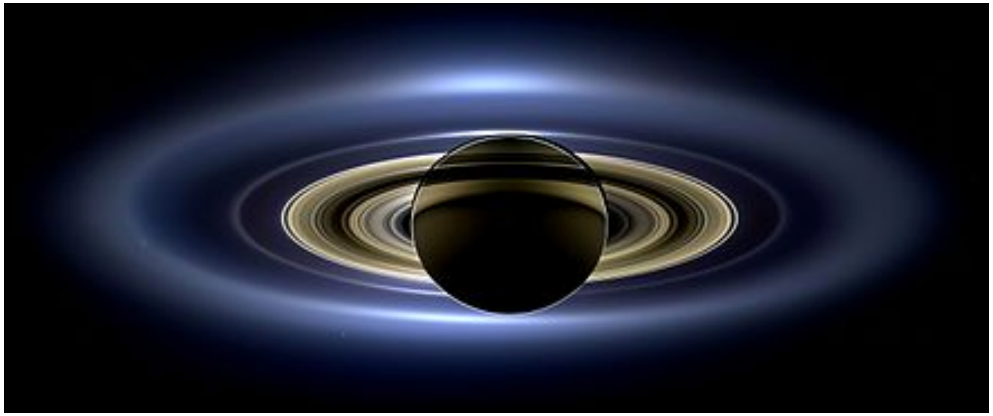
1. Plum pudding model (Lord Kelvin and J. J. Thomson)



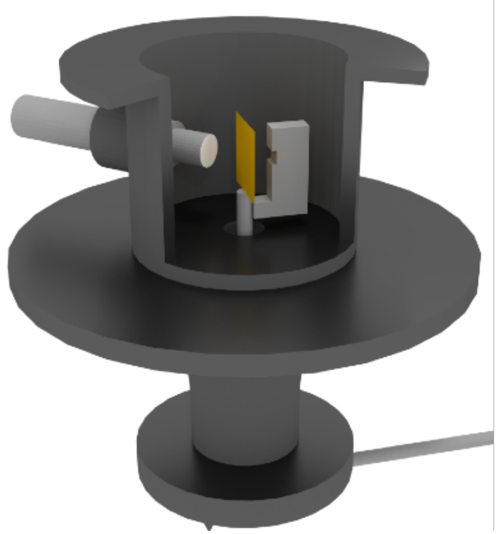


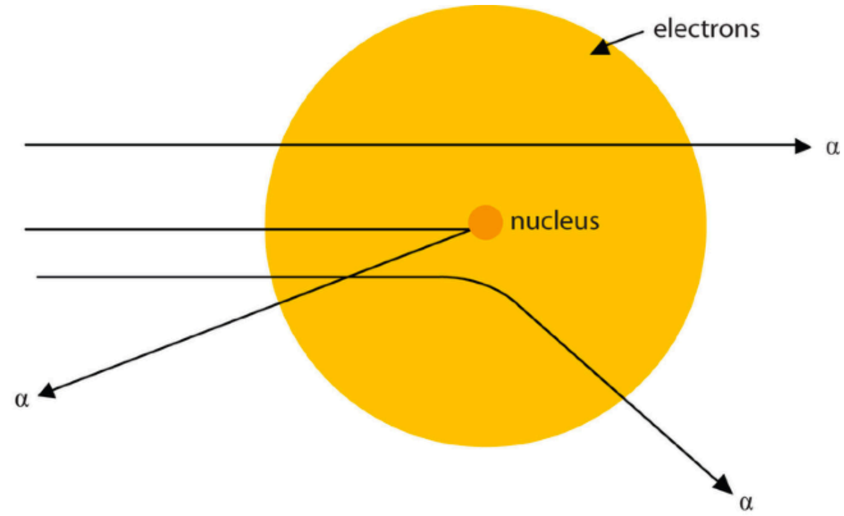
2. Saturnian model of the atom (H. Nagaoka)

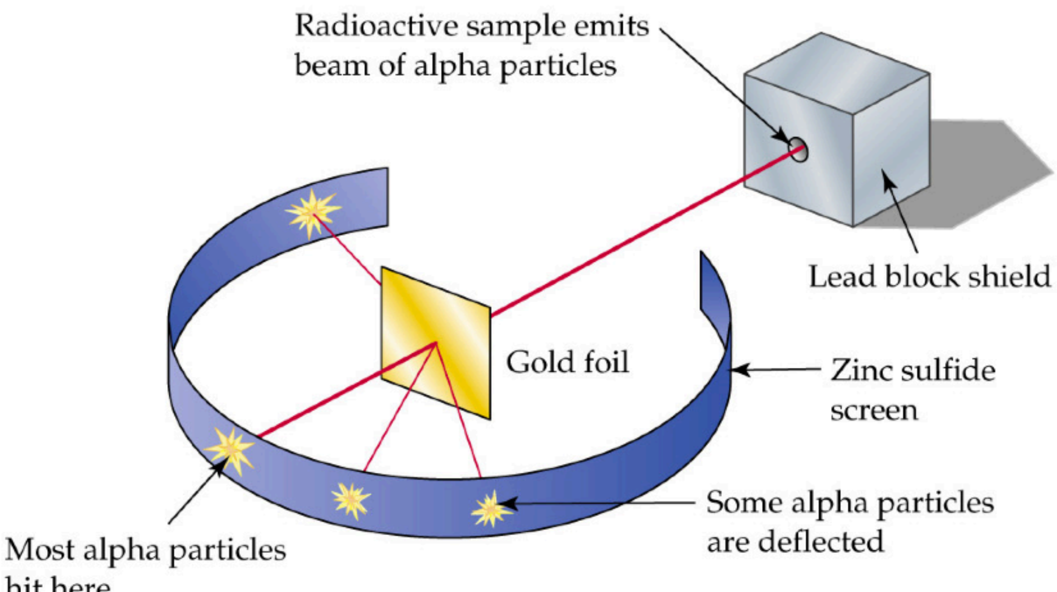




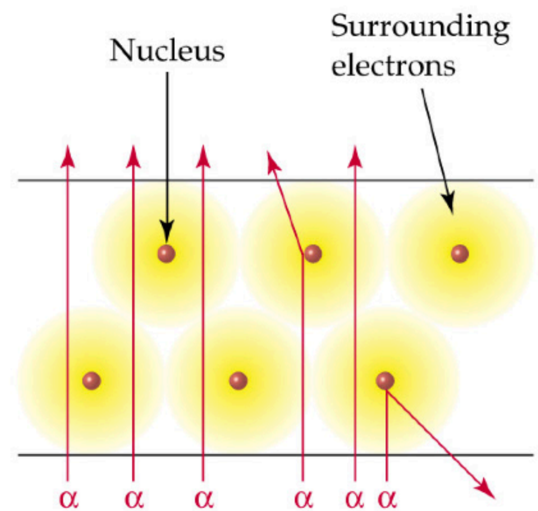
Geiger-Marsden experiment







(a)



(b)

Implications of plum pudding model

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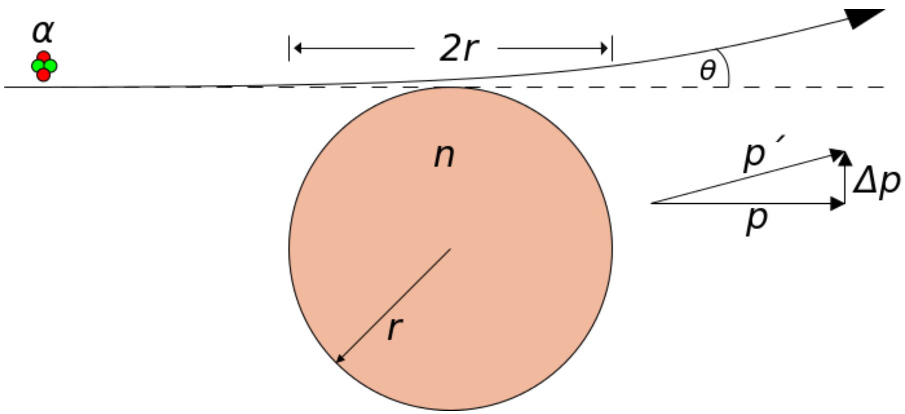
Using classical physics, the alpha particle's lateral change in momentum Δp can be approximated using the impulse of force relationship and the Coulomb force expression:



$$\Delta p = F\Delta t = k\frac{Q_\alpha Q_n}{r^2}\frac{2r}{v_\alpha}$$

The maximum deflection angle:

$$\theta \approx \frac{\Delta p}{p} < k\frac{2Q_\alpha Q_n}{m_\alpha r v_\alpha^2} = 0.000326 \text{ rad}$$

where, r: radius of a gold atom

Q_n: positive charge of gold atom

Q_α: charge of alpha particle.

k: Coulomb's constant

m_α: mass of alpha particle

v_α: velocity of alpha particle

Implications of plum pudding model

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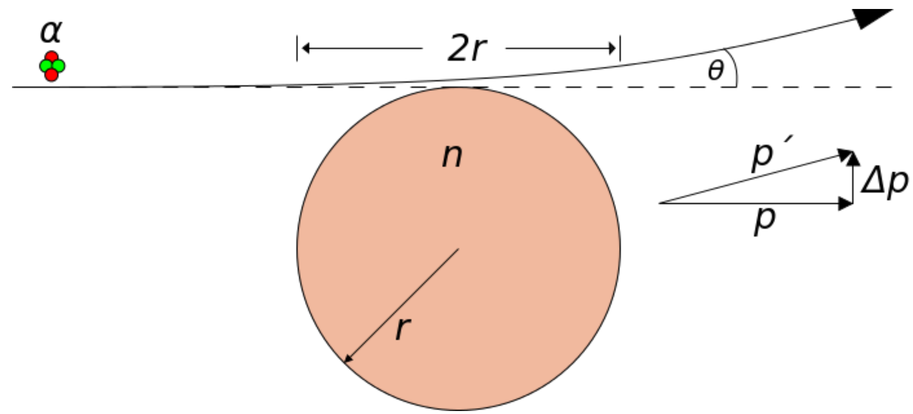

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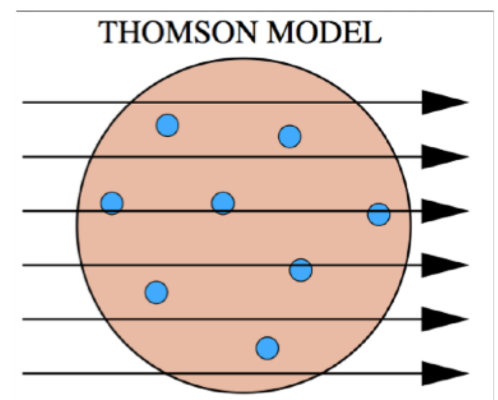


Fig – Thomson Lpum Pudding Model   Source - Wikipedia

where, r: radius of a gold atom

Q_n: positive charge of gold atom

Q_α: charge of alpha particle.

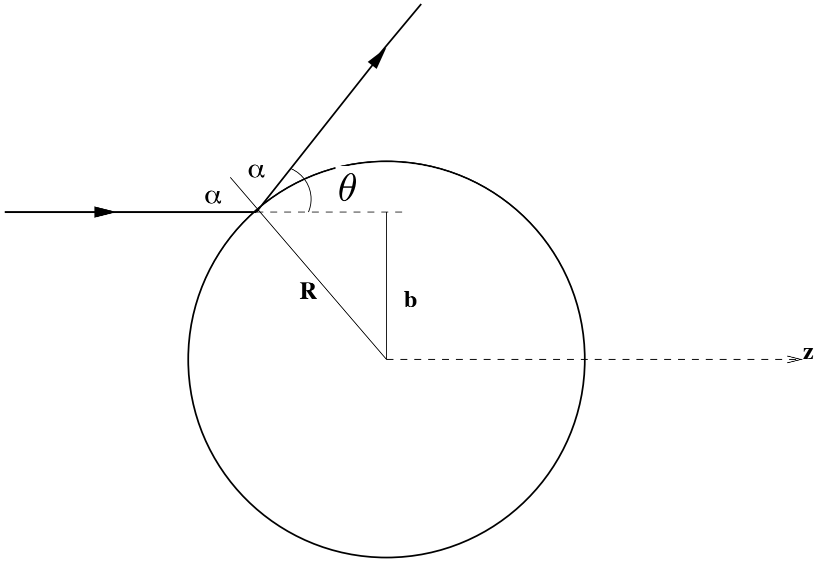
k: Coulomb's constant

m_α: mass of alpha particle

v_α: velocity of alpha particle

Scattering in a hard sphere

Consider one of these particles incident with velocity v and impact parameter b.



R: radius of the sphere

α: incident angle

b: impact parameter $b = R \sin \alpha$

The particle is deflected through an angle $\theta = \pi - 2\alpha$, related to the impact parameter by

$$b = R \cos \frac{\theta}{2}.$$

Trajectories in Scattering

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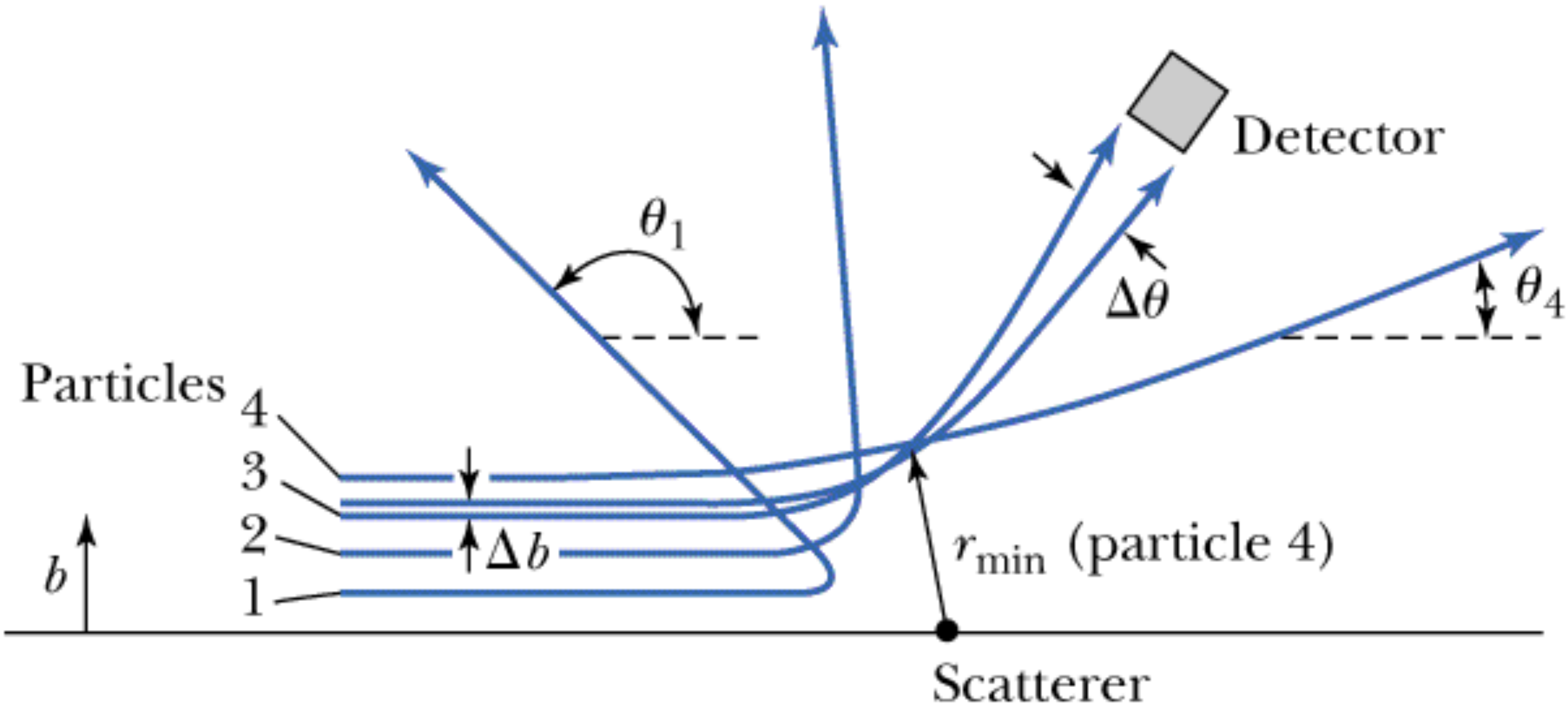
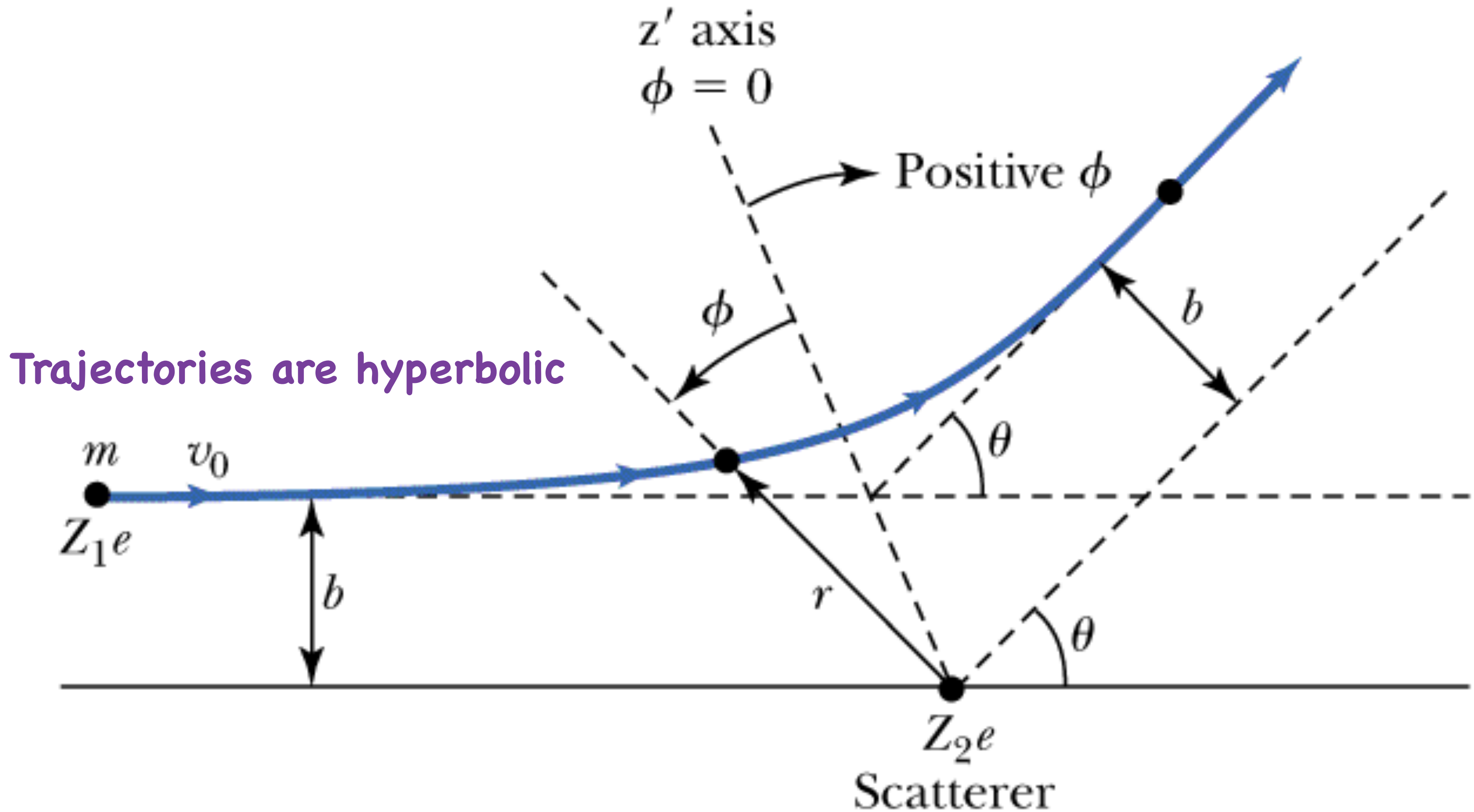
Trajectories are strongly dependent on the impact parameter



The key concept in Rutherford scattering is the relationship between the impact parameter b and the scattering angle θ.



Rutherford Scattering Formula

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1. Basic knowledge

The Coulomb force;

The Newton's laws;

The conservation of linear momentum;

The conservation of angular momentum.

2. Assumptions

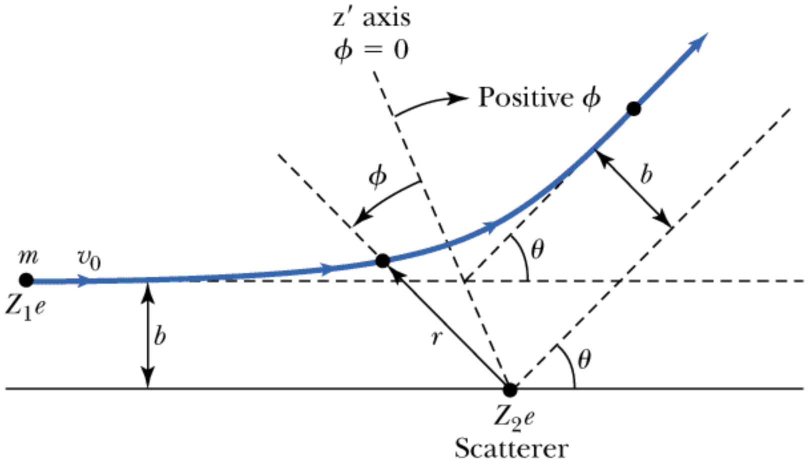
Single scattering

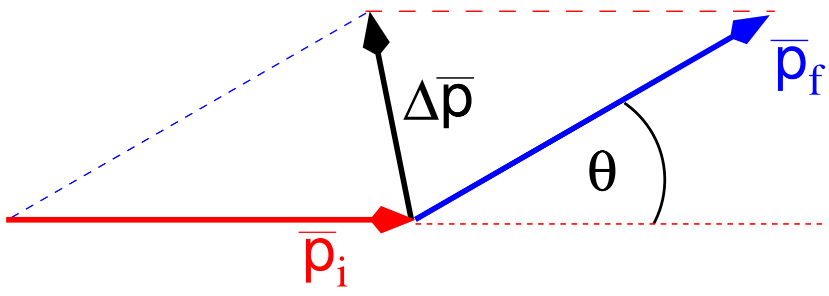
Only consideration Coulomb force

The effect of electrons in nuclei is neglected

The target is static

Momentum change in Rutherford scattering



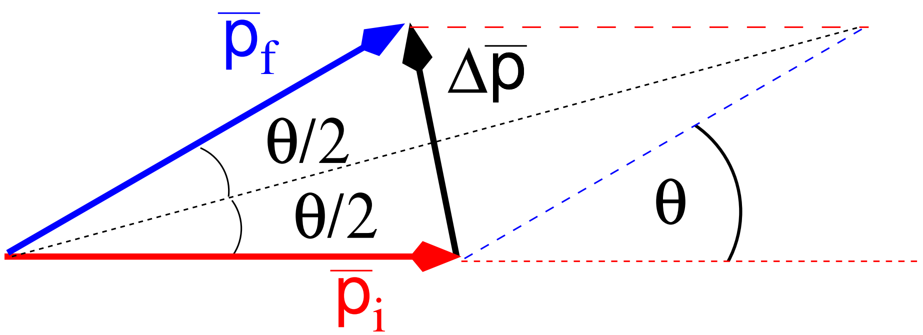


Elastic scattering

$$|\vec{p}_i| = |\vec{p}_f| = p$$

Momentum change

$$\Delta p = 2p\sin(\theta/2)$$



From the Newton's second law

$$\vec{F} = \frac{d\vec{p}}{dt} \Longrightarrow \Delta\vec{p} = \int \vec{F} dt$$

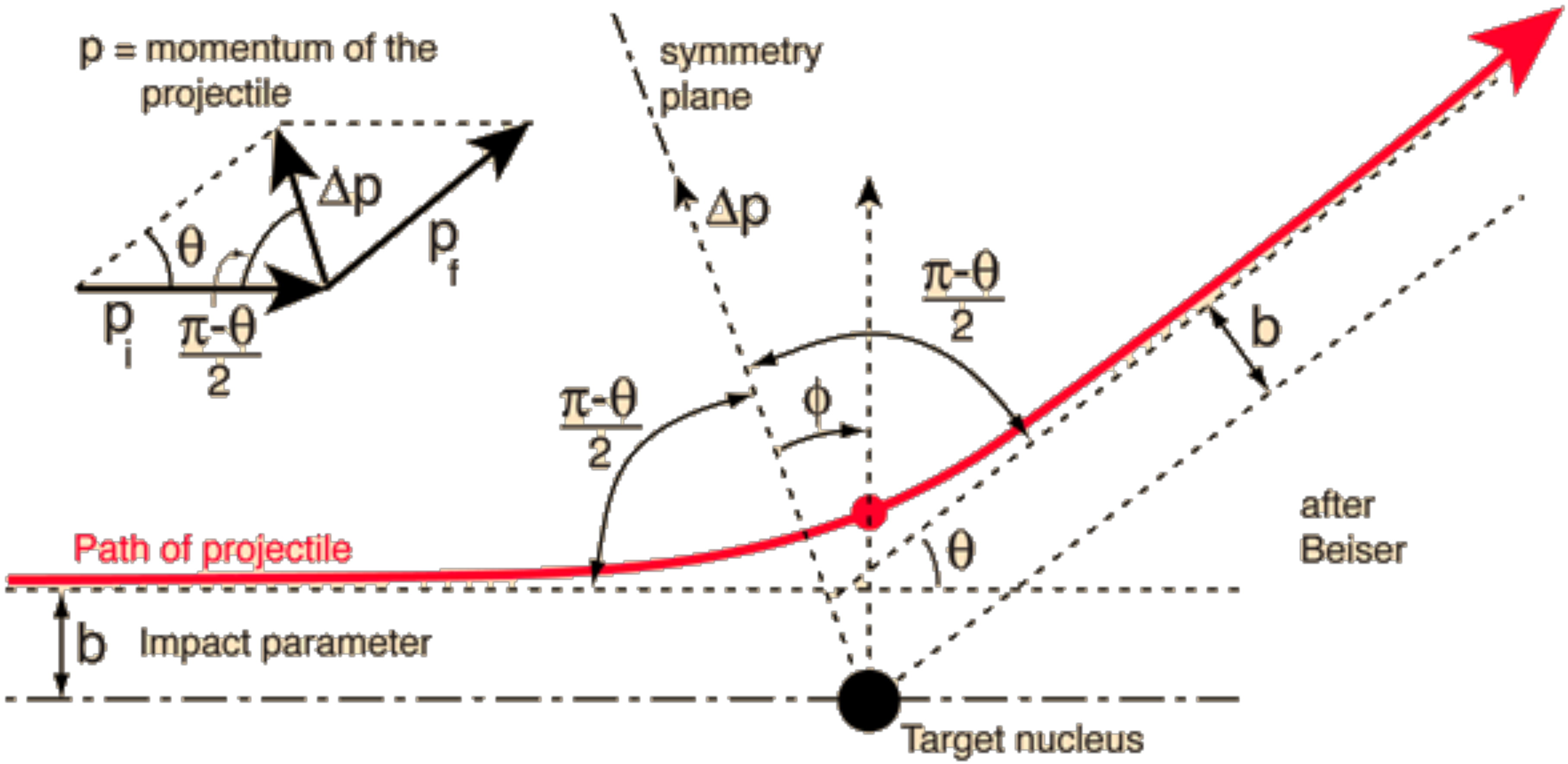
The force is the Coulomb force

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{Z_1 Z_2 e^2}{r^2} \frac{\vec{r}}{r}$$

Before we start integrating let us note that the trajectories are symmetric with respect to the line defined by the distance of the closest approach

Rutherford Scattering Formula

Trajectories are symmetric with respect to angle ϕ



The symmetry with respect to the line at ϕ = 0 implies

$$\Delta\vec{p} = \int \vec{F}dt \Longrightarrow \Delta p = \int F\cos\phi dt$$

So,

$$\Delta p = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\int \frac{1}{r^2}\cos\phi dt$$

This integral can be carried over with a help of conservation of angular momentum.

The angular momentum is

$$\vec{L} = \vec{r}\times\vec{p} = m\vec{r}\times\vec{v} = m\vec{r}\times\left(\frac{d\vec{r}}{dt} + r\frac{d\vec{\phi}}{dt}\right) = mr\vec{r}\times\frac{d\vec{\phi}}{dt}$$

The magnitude of angular momentum

$$L = |\vec{L}| = mr^2 \frac{d\vec{\phi}}{dt}$$

From the initial condition

$$L = mv_0 b$$

Since the angular momentum is conserved

$$mr^2 \frac{d\phi}{dt} = mv_0 b$$

$$\frac{dt}{r^2} = \frac{d\phi}{v_0 b}$$

Thus the change of momentum

$$\Delta p = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0} \int \frac{dt}{r^2} \cos\phi = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0} \int \frac{d\phi}{v_0 b} \cos\phi$$

$$= \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0} \frac{1}{v_0 b} \int_{\phi_<}^{\phi_>} d\phi \cos\phi$$

The limits for integration are

$$\phi_> = \frac{1}{2}(\pi - \theta)$$

$$\phi_< = -\frac{1}{2}(\pi - \theta)$$

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The integral is

$$\Delta p = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{1}{v_0 b}\int_{\phi_<}^{\phi_>} d\phi \cos\phi = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{1}{v_0 b}(\sin\phi_> - \sin\phi_<)$$

$$= \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{2}{v_0 b}\cos\frac{\theta}{2}$$

Since

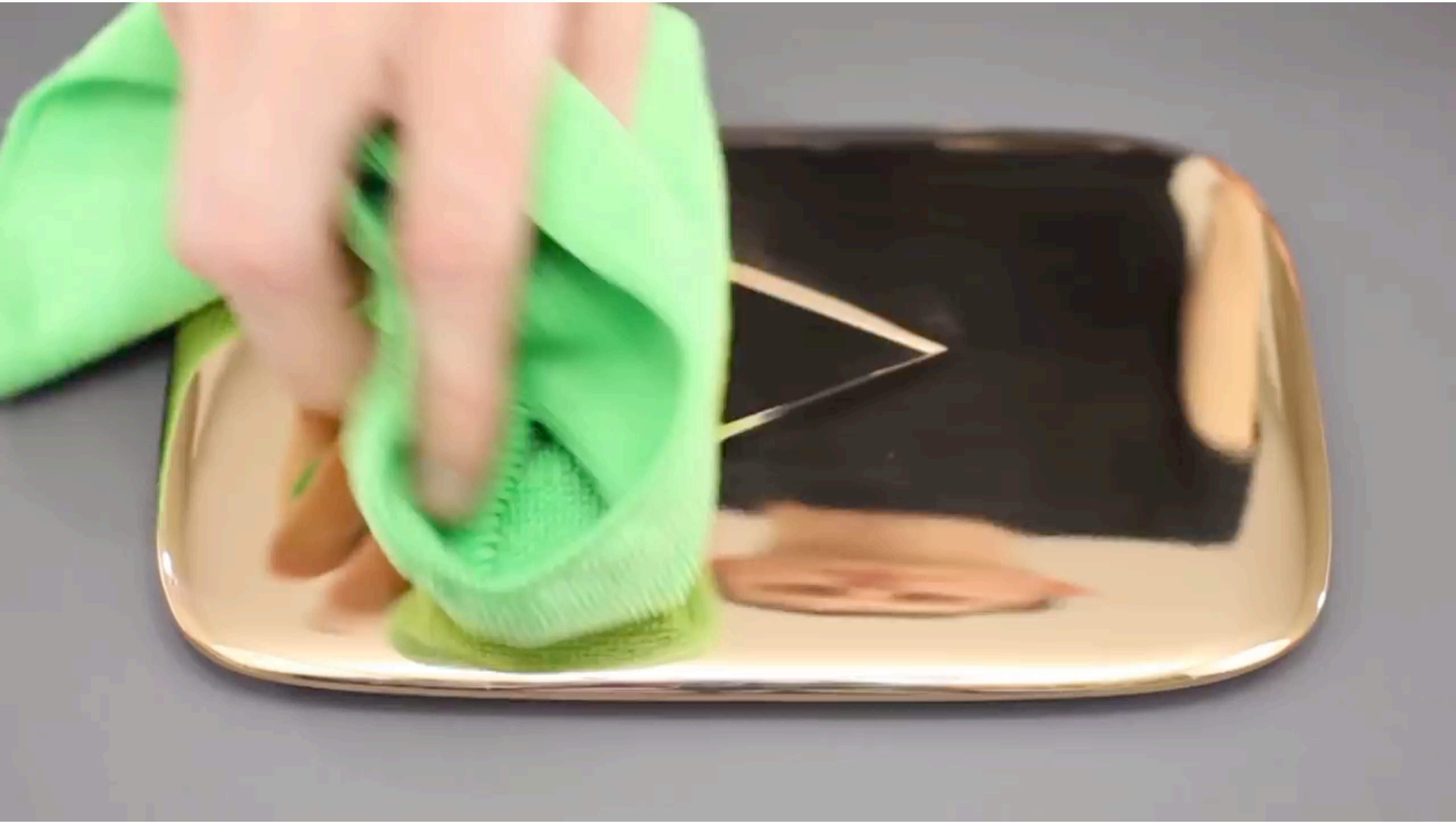
$$\Delta p = 2p\sin(\theta/2)$$

The impact parameter is expressed as

$$b = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{1}{p v_0}\frac{1}{\tan(\theta/2)}$$

$$= \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{1}{2E}\frac{1}{\tan(\theta/2)}$$

with E being the initial kinetic energy for the projectile



The reduced mass

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

Energy conservation demands

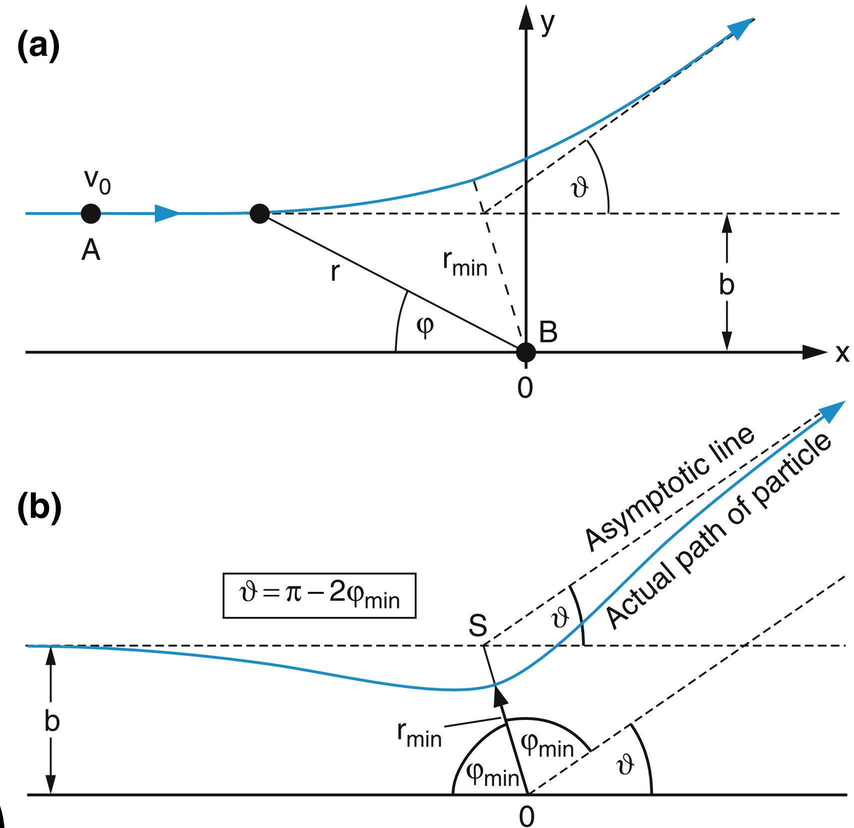
$$\frac{1}{2}\mu v^2 + E_{\text{pot}}(r) = \frac{1}{2}\mu v_0^2 = \text{const},$$

v_0 is the initial velocity

The angular momentum L

$$\boldsymbol{L} = \mu(\boldsymbol{r} \times \boldsymbol{v}) = \mu\left(\boldsymbol{r} \times \left[\frac{\mathrm{d}r}{\mathrm{d}t}\hat{e}_r + r\frac{\mathrm{d}\varphi}{\mathrm{d}t}\hat{e}_t\right]\right)$$

$$= \mu r\dot{\varphi}\left(\boldsymbol{r} \times \hat{e}_t\right),$$



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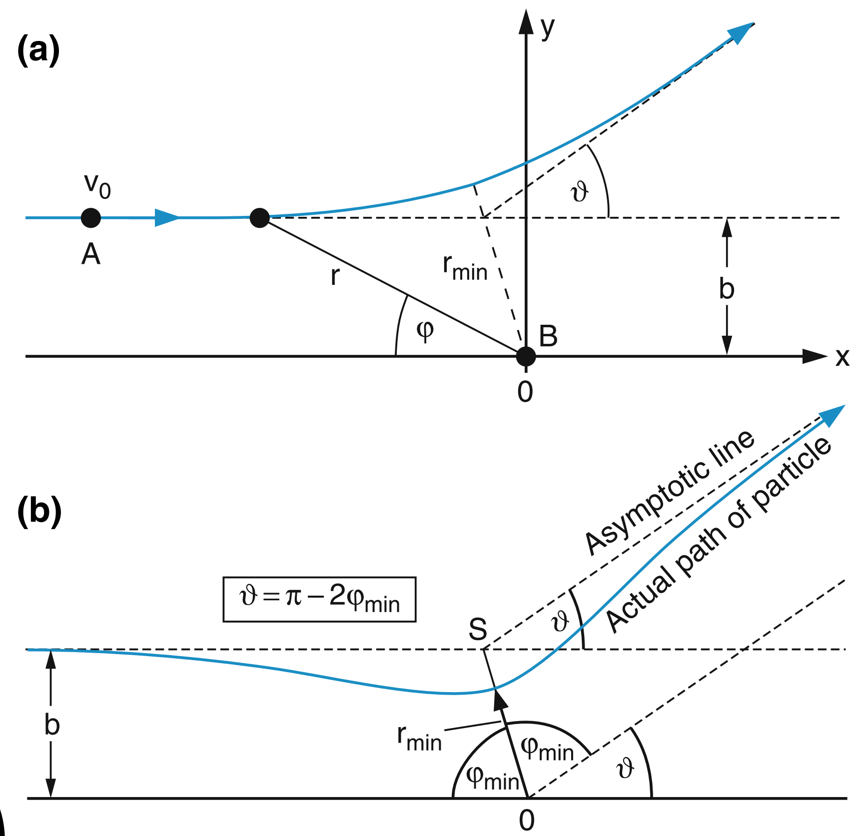
$$= \mu r\dot{\varphi}\left(\boldsymbol{r}\times\hat{e}_t\right),$$

$$= \mu r^2\dot{\varphi} = \mu v_0 b,$$



The kinetic energy in center of mass frame

$$E_{\text{kin}} = \frac{1}{2}\mu v^2 = \frac{1}{2}\mu\left(\dot{r}^2 + r^2\dot{\varphi}^2\right)$$
$$= \frac{1}{2}\mu\dot{r}^2 + \frac{L^2}{2\mu r^2}.$$

The total energy

$$E_{\text{total}} = E_0 = \frac{1}{2}\mu\dot{r}^2 + \frac{L^2}{2\mu r^2} + E_{\text{pot}}(r) = \text{const.}$$

The derivatives of radii and angle are

$$\dot{r} = \left[\frac{2}{\mu}\left(E_0 - E_{\text{pot}}(r) - \frac{L^2}{2\mu r^2}\right)\right]^{1/2}$$
$$\dot{\varphi} = \frac{L}{\mu r^2}.$$

Since for a spherically symmetric potential this path must be mirror-symmetric to the line OS

The relation the asymptotic scattering angle θ to the polar angle by

$$\vartheta = \pi - 2\varphi_{\min}.$$

This yields the relation

$$\varphi_{\min} = \int_{\varphi=0}^{\varphi_{\min}} \mathrm{d}\varphi = \int_{r=-\infty}^{r_{\min}} \frac{\mathrm{d}\varphi}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}r}\mathrm{d}r$$

$$= \int_{r=-\infty}^{r_{\min}} (\dot{\varphi}/\dot{r})\,\mathrm{d}r = \int_{r_{\min}}^{+\infty} \frac{\dot{\varphi}}{\dot{r}}\mathrm{d}r.$$

The scattering angle in the CM-frame becomes

$$\vartheta(E_0, L) = \pi - 2\int_{r_{\min}}^{+\infty} \frac{(L/(\mu r^2))\,\mathrm{d}r}{\left[\frac{2}{\mu}\left(E_0 - E_{\mathrm{pot}}(r) - \frac{L^2}{2\mu r^2}\right)\right]^{1/2}}.$$

The amount of the angular momentum

$$L = \mu r v \sin\varphi = \mu b v_0 \Rightarrow L^2 = \mu^2 b^2 v_0^2 = 2\mu b^2 E_0$$

Therefore

$$\vartheta(E_0, b) = \pi - 2b \int_{r_{\min}}^{+\infty} \frac{\mathrm{d}r}{r^2\left[1 - \frac{b^2}{r^2} - \frac{E_{\mathrm{pot}}(r)}{E_0}\right]^{1/2}}$$

The lower integration limit r_{min} is fixed by the condition

$$\dot{r}(r_{\min}) = 0$$

which gives

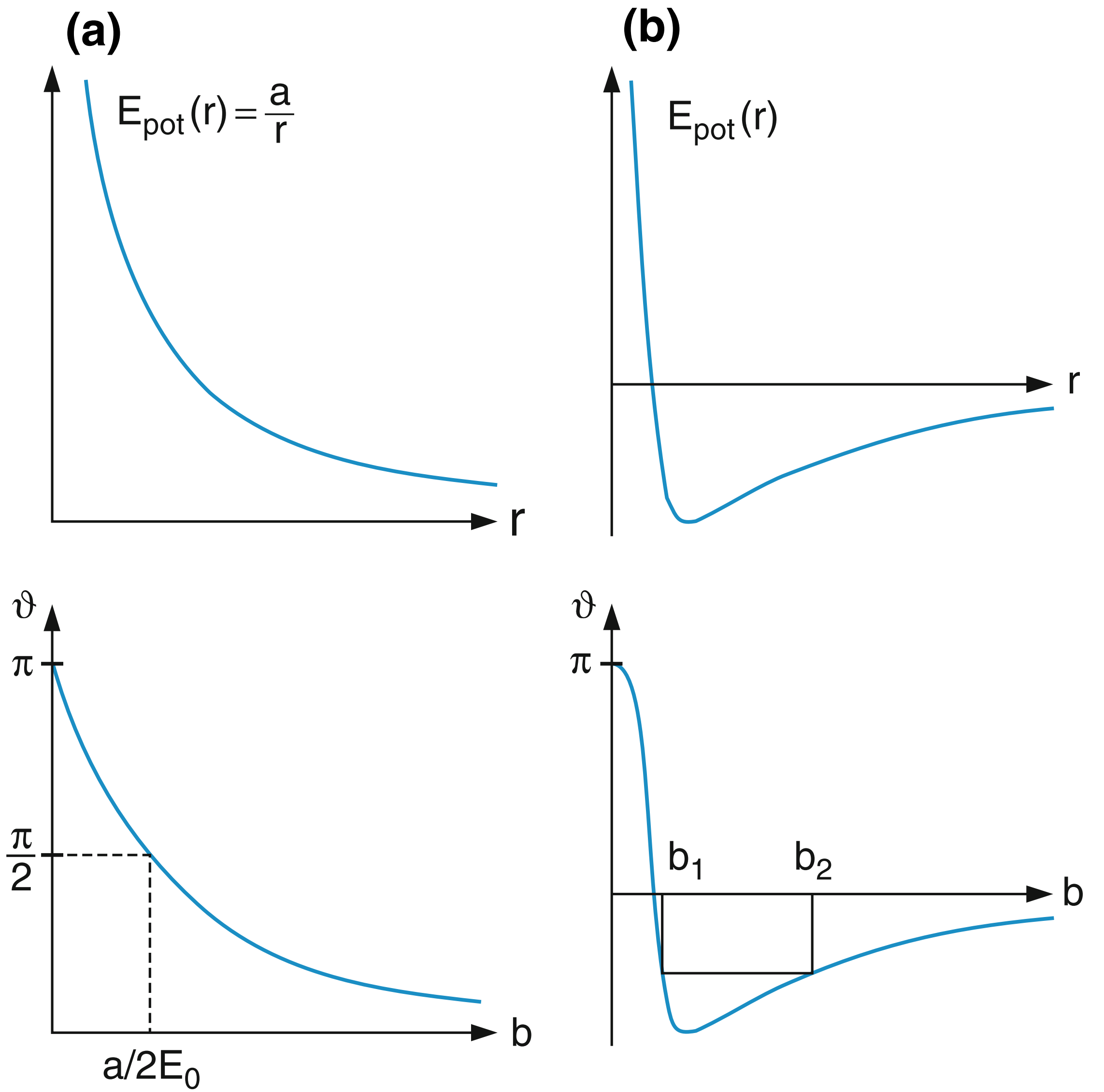
$$r_{\min} = \frac{b}{\left[1 - \frac{E_{\mathrm{pot}}(r_{\min})}{E_0}\right]^{1/2}}$$

The angular for Coulomb potential is

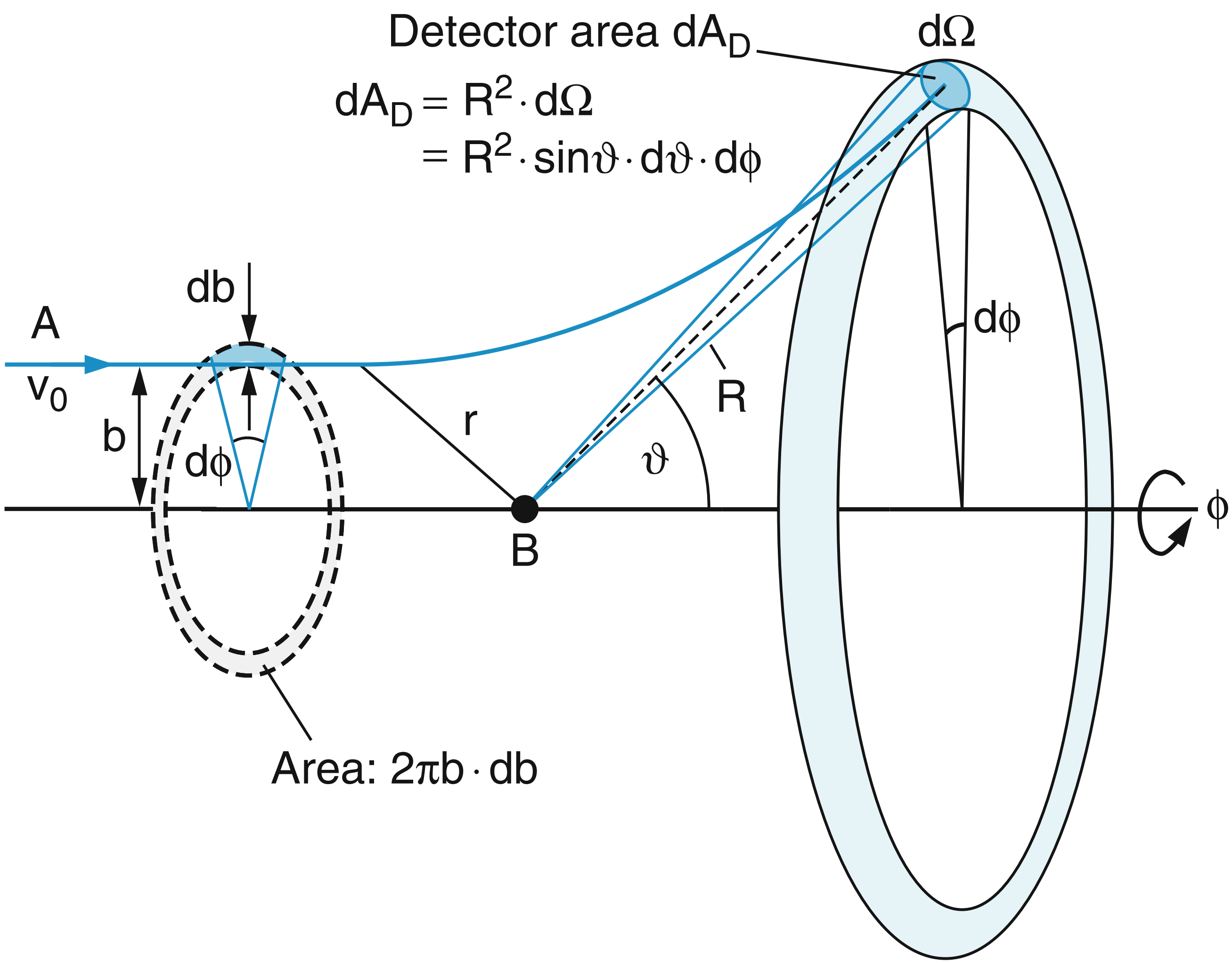
$$\vartheta = 2\cot^{-1}\left(\frac{4\pi\varepsilon_0}{qQ}\mu v_0^2 b\right)$$

$$E_{\mathrm{pot}} = \frac{qQ}{4\pi\varepsilon_0 r}$$

Classical scattering



The cross section



The cross section

Let us assume a parallel beam of incident particles A with particle flux density $\dot{N}_A = n_A v_A$ that passes through a layer of particles B in rest with density n_B.

All particles A passing through an annular ring with radius b and width db around an atom B are deflected by the angle θ andθ ±dθ/2, assuming a spherically symmetric interaction potential.

The angular ring

$$d\dot{N}_A = \dot{N}_A \, dA = n_A v_A 2\pi b \, db$$

particles A pass per second

The cross section

One particle B therefore scatters the fraction

$$\frac{\mathrm{d}\dot{N}_{\mathrm{A}}\left(\vartheta \pm \frac{1}{2}\mathrm{d}\vartheta\right)}{\dot{N}_{\mathrm{A}}} = 2\pi b\,\mathrm{d}b = 2\pi b\frac{\mathrm{d}b}{\mathrm{d}\vartheta}\mathrm{d}\vartheta$$

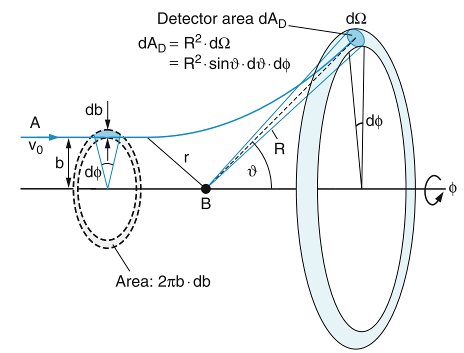
of all particles A, incident per second and unit area onto the target, into the range of deflection angles $\theta \pm \mathrm{d}\theta/2$.

The detector with area

$$A_{\mathrm{D}} = R^2\mathrm{d}\Omega = R^2 \sin\vartheta\,\mathrm{d}\vartheta\,\mathrm{d}\phi$$

receives the fraction

$$\frac{\mathrm{d}\dot{N}_{\mathrm{A}}(\vartheta,\phi)}{\dot{N}_{\mathrm{A}}}\frac{\mathrm{d}\phi}{2\pi} = b\frac{\mathrm{d}b}{\mathrm{d}\vartheta}\mathrm{d}\vartheta\,\mathrm{d}\phi,$$



The fraction of all incident particles A, scattered by all atoms B with density n_B in the volume V = AΔx is then:

$$\frac{\mathrm{d}\dot{N}_{\mathrm{A}}(\vartheta, \mathrm{d}\Omega)}{\dot{N}_{\mathrm{A}}} = n_{\mathrm{B}} A \Delta x b \frac{\mathrm{d}b}{\mathrm{d}\vartheta} \mathrm{d}\vartheta\, \mathrm{d}\phi .$$

We define a differential cross section to be the ratio of scattered particles with per target and per unit solid angle to the number of incoming particles per unit area,

$$\frac{d\sigma}{d\Omega} = \frac{\text{scattered particles}}{\text{incident particles per unit area} \times \text{target particles}} \frac{1}{d\Omega}$$

Therefore,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\mathrm{d}\dot{N}_{\mathrm{A}}(\vartheta, \mathrm{d}\Omega)}{\dot{N}_{\mathrm{A}} n_{\mathrm{B}} A \Delta x \mathrm{d}\Omega}$$

The cross section

Therefore

$$\frac{d\sigma}{d\Omega} = b\frac{db}{d\vartheta}\frac{1}{\sin\vartheta}$$

with

$$d\Omega = \sin\vartheta \, d\vartheta \, d\phi,$$

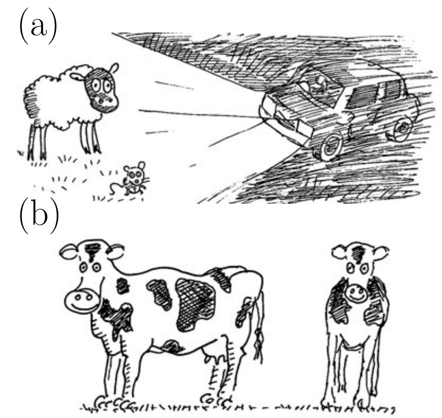


Fig. 20.7 Scattering cross-sections. (a) $\sigma_{\text{sheep}} > \sigma_{\text{field mouse}}$. (b) $\sigma_{\text{cow, side}} > \sigma_{\text{cow, front}}$.

The relationship between b and θ for the Rutherford scattering yields

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{1}{4E}\right)^2 \frac{1}{\sin^4(\theta/2)}$$

The cross section

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If N incident particles strike a foil of thickness t containing n scattering centers per unit volume, the average number dN of particles Ω scattered into the solid angle dΩ around Ω is given by

$$dN = Nnt\frac{d\sigma}{d\Omega}d\Omega$$

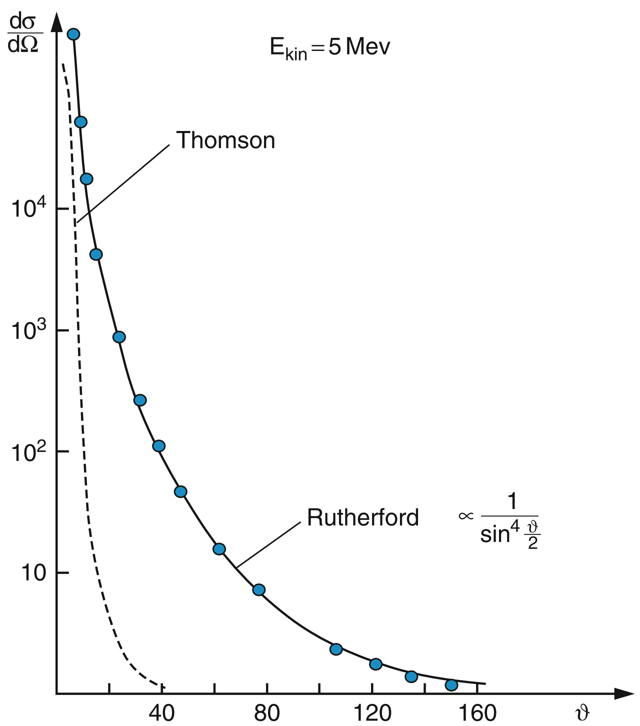
Therefore,

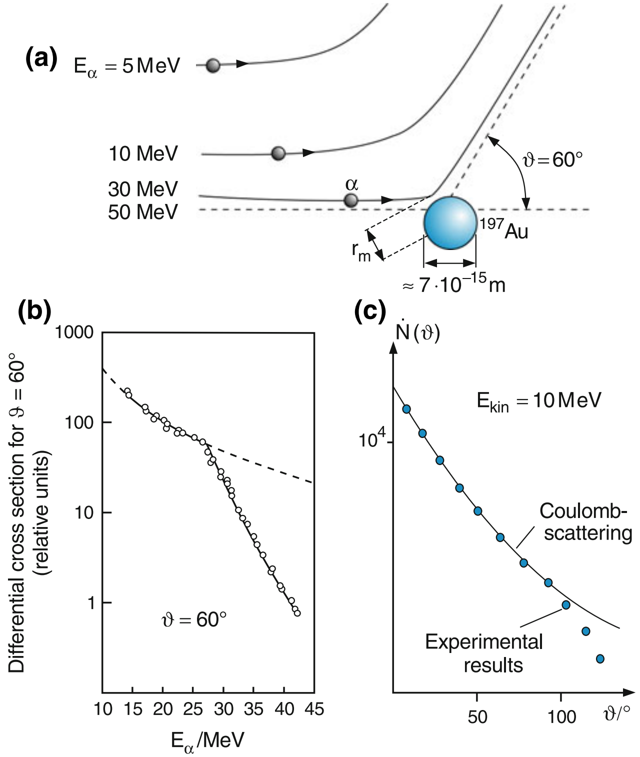
$$\frac{dN}{N} = nt\left(\frac{Z_1 Z_2 e^2}{16\pi\varepsilon_0 E}\right)^2 \frac{1}{\sin^4(\theta/2)}d\Omega$$

This is the Rutherford result explaining the Geiger-Marsden experiment

The Geiger-Marsden experiment

Number of particles scattered at a given angle in Rutherford scattering is calculable and well understood, since it is defined by the well understood electromagnetic force.





The key points

Impact parameter: b

Scattering angle: θ

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{1}{4E}\right)^2 \frac{1}{\sin^4(\theta/2)}$$

Differential cross-section: the ratio of the number of particles scattered into an element of solid angle dΩ in the direction θ per unit area (unit 1 barn=10^{-28} m^2)

The important quantities in Rutherford formula:

1. impact parameter
2. Scattering angle.
3. charges
4. Initial kinetic energy

The integral scattering cross section is obtained by integration over dΩ, where the integration limits are $\theta(b = 0) = \pi$ and $\theta(b_{max}) = \theta_{min}$:

$$\sigma_{\text{int}} = \int_{\Omega} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \mathrm{d}\Omega = \int_{\vartheta=\pi}^{\vartheta_{\min}} \int_{\phi=0}^{2\pi} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \sin\vartheta \, \mathrm{d}\vartheta \, \mathrm{d}\phi ,$$

where θ_{min} is the smallest detectable deflection angle. The integration over φ gives 2π. Therefore, we have

$$\sigma_{\text{int}} = 2\pi \int_{\vartheta=\pi}^{\vartheta_{\min}} \frac{b}{\sin\vartheta} \left| \frac{\mathrm{d}b}{\mathrm{d}\vartheta} \right| \sin\vartheta \, \mathrm{d}\vartheta$$

$$= 2\pi \int_{b=0}^{b_{\max}} b \, \mathrm{d}b = \pi b_{\max}^2 .$$

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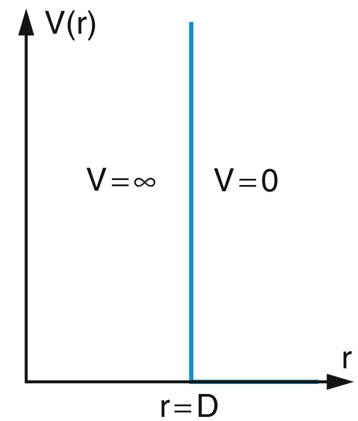
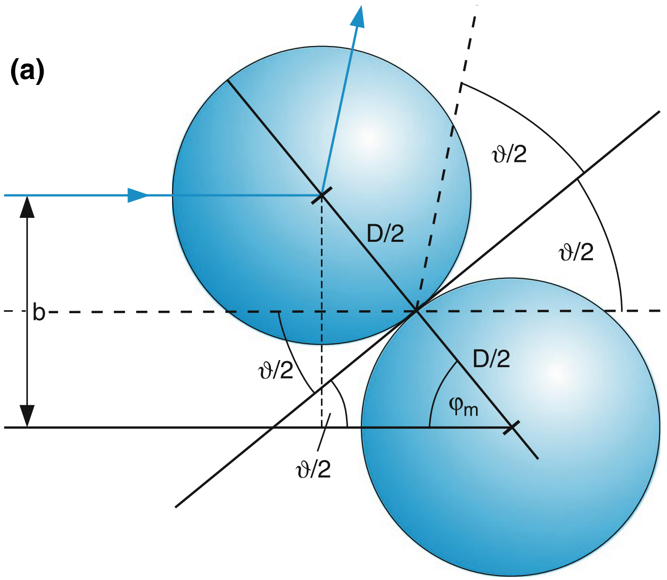
Collisions of hard spheres A and B with equal diameters D. The potential energy in this case is:

$$E_{\text{pot}}(r) = \begin{Bmatrix} \infty & \text{for } r \le D \\ 0 & \text{for } r > D \end{Bmatrix}.$$



It is seen that at the closest approach $\sin\varphi_m = b/D$, which implies that a collision can only take place for $b \le D$. For the scattering angle θ we find $\theta/2 = \pi/2 - \varphi_m$.



The impact parameters for b ≤ D are therefore

$$b = D \sin \varphi_{\mathrm{m}} = D \cos(\vartheta/2).$$

Then the derivative db/dθ becomes

$$\left|\frac{\mathrm{d}b}{\mathrm{d}\vartheta}\right| = \frac{D}{2} \sin \vartheta/2$$

and the differential scattering cross section is:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{b}{\sin\vartheta}\frac{\mathrm{d}b}{\mathrm{d}\vartheta} = \frac{D\cos(\vartheta/2) D\sin(\vartheta/2)}{2\sin\vartheta} = \frac{D^2}{4}$$

$$\Rightarrow \sigma_{\mathrm{int}} = \int \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\mathrm{d}\Omega = 4\pi\frac{D^2}{4} = \pi D^2.$$

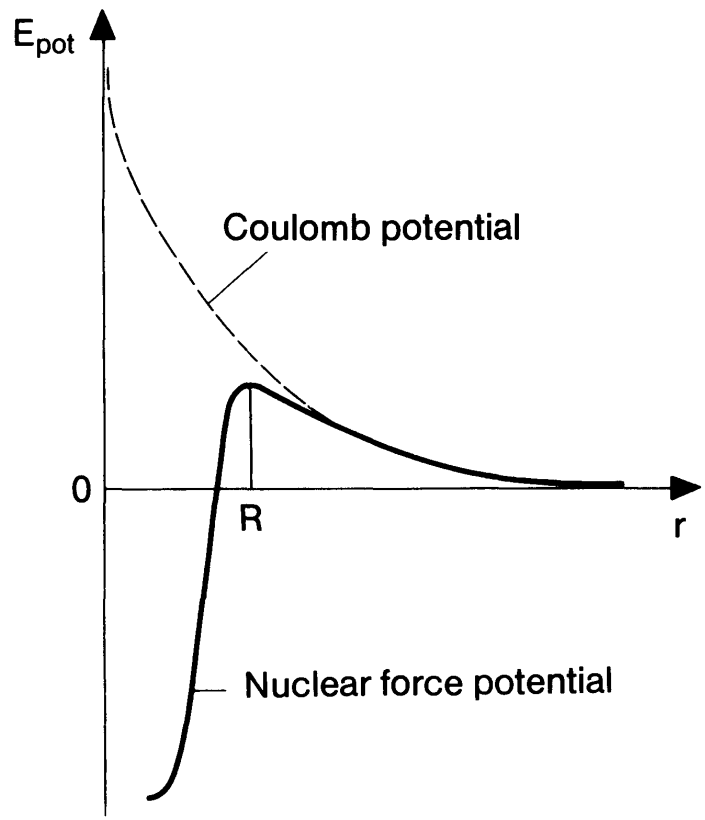
The deflection function θ(b) for hard spheres is

$$\vartheta(b) = \pi - 2\varphi_{\mathrm{m}} = \pi - 2\arcsin(b/2D).$$

The nuclear radius is defined as the distance at which the effect of the nuclear potential is comparable to that of the Coulomb potential

$$E = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{1}{r}$$

$$r_{\min} = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{1}{E}$$



Empirical nuclear radius

$$R = 1.2A^{1/3}\ \mathrm{fm}$$

The Physics of Atoms and Quanta

2.2, 2.4, 4.3, 4.4, 4.5, 4.6, 4.8