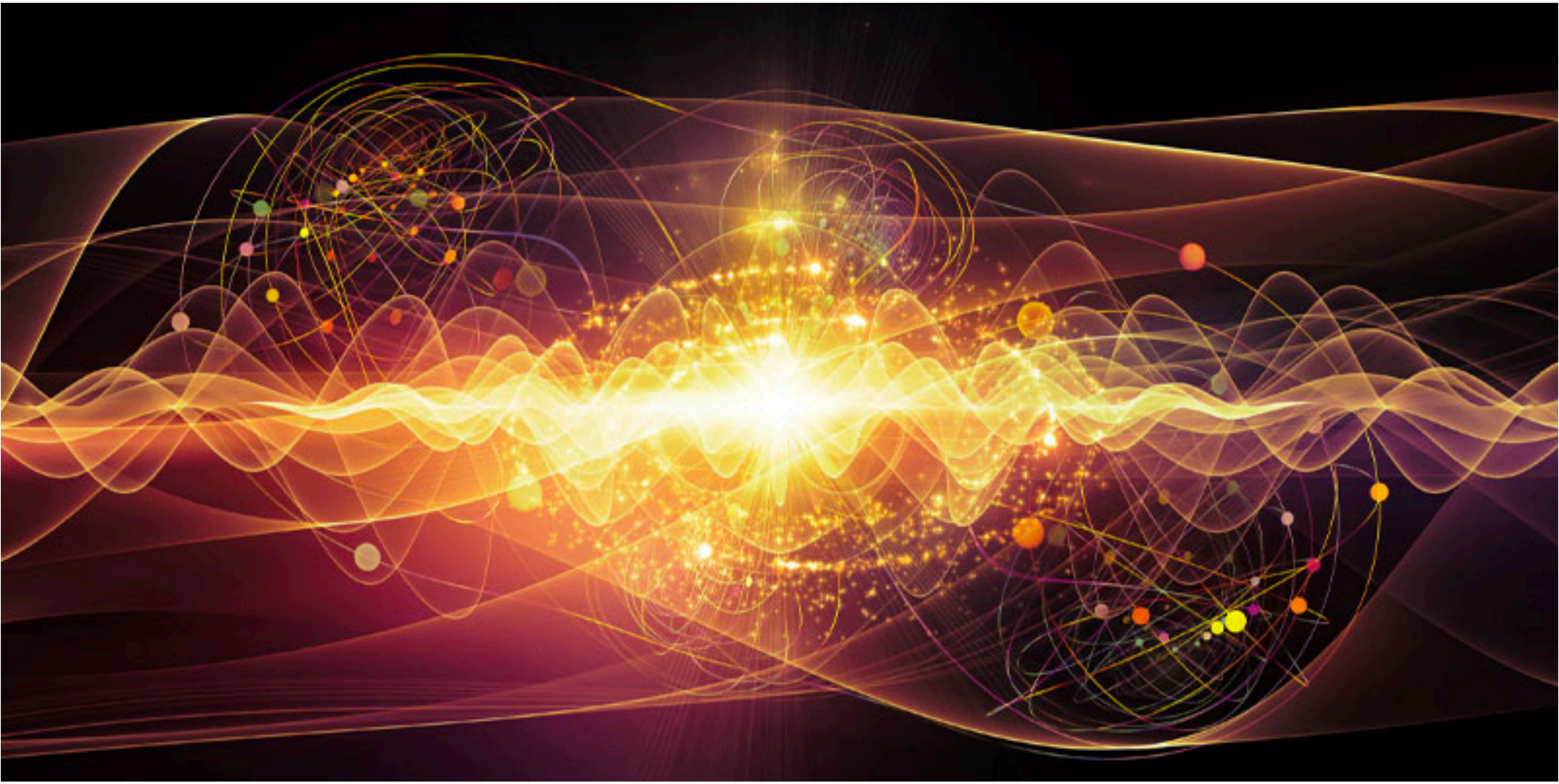
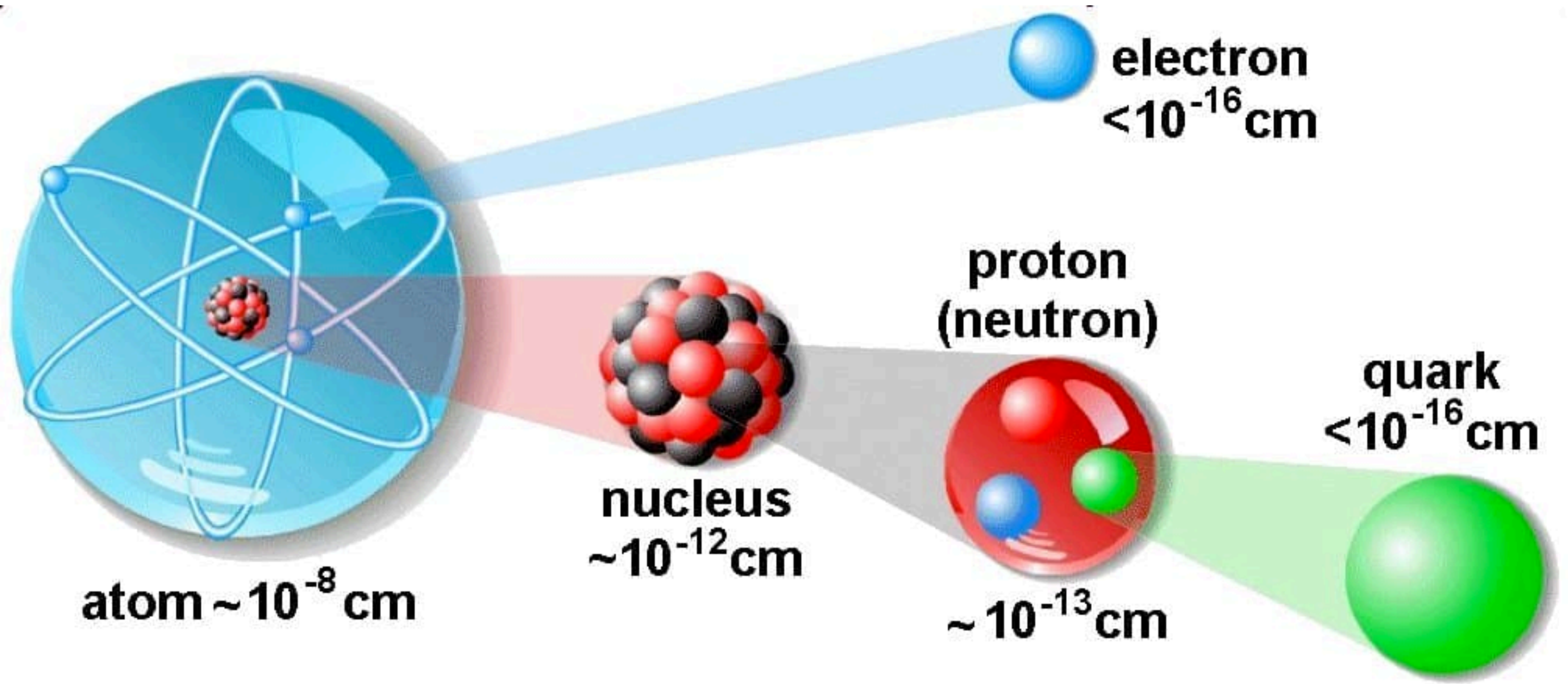
Atomic Physics

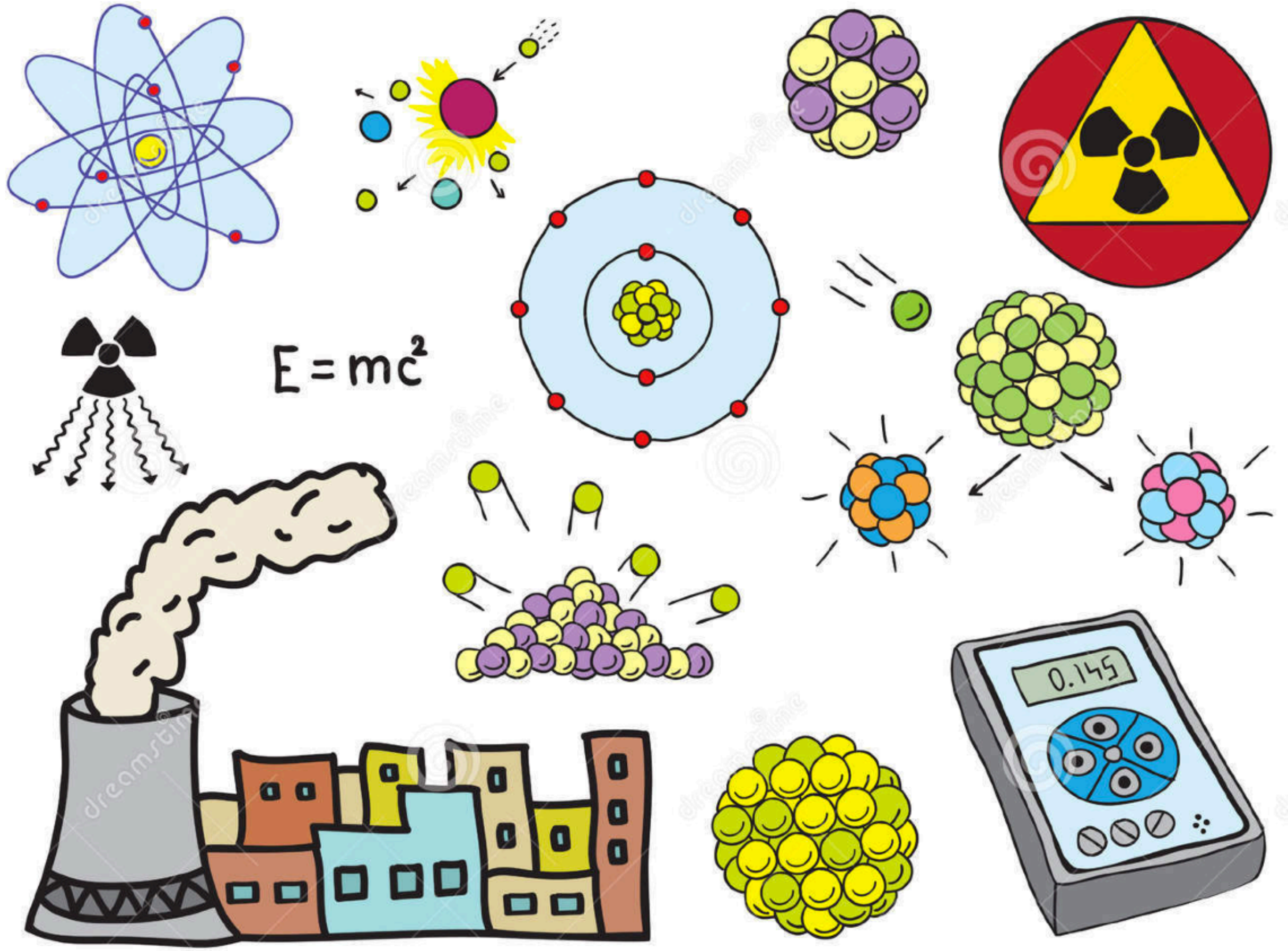
Chapter 7 Nuclear Physics



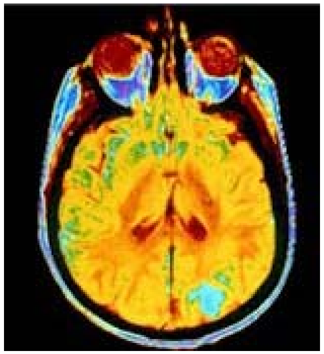
Nuclear structure

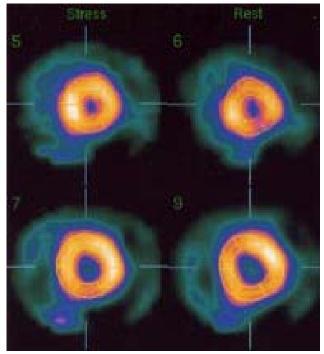


Nuclear physics application

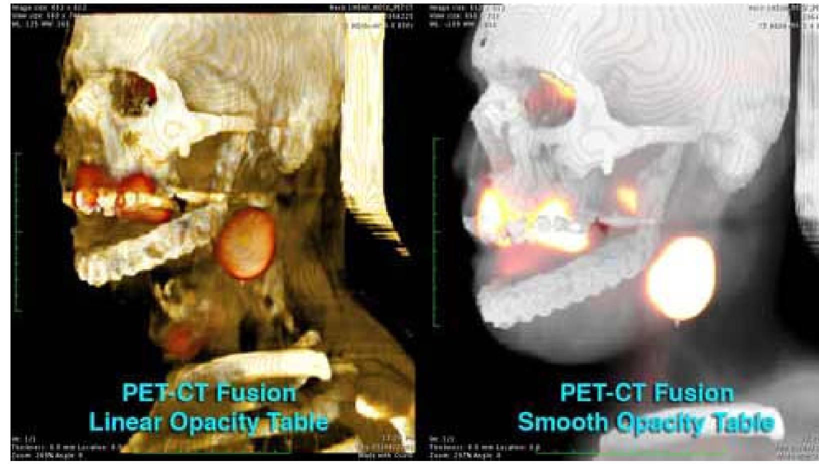


Nuclear Imaging

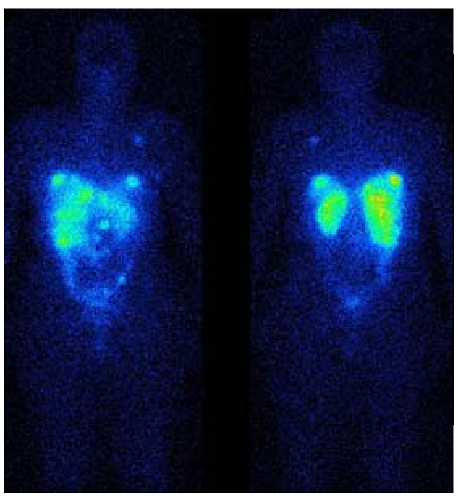




Blood flow with radiopharmaceuticals



Imaging software and analysis

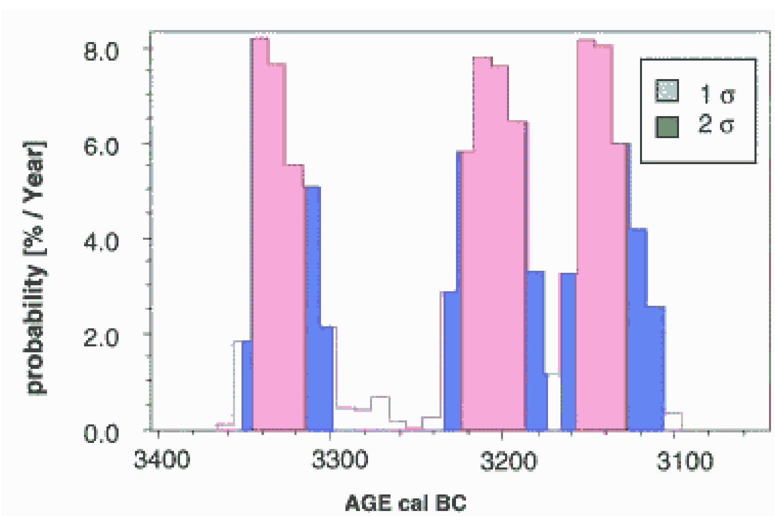


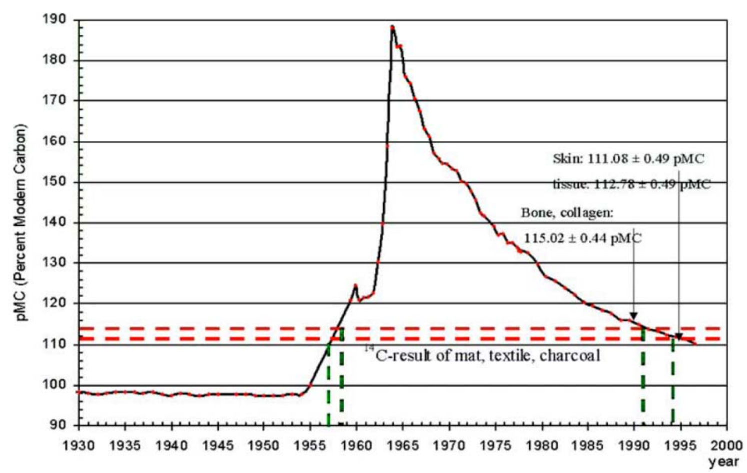
- Gamma Camera
- SPEC & PEP
- Isotopes & Isomer
- Parmaceuticals

Tumor mapping & visualization by radioactive isotope accumulation.

Imaging system development

Dating real and false mummies





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✓ 1896: Henri Becquerel–discovery of radioactivity

Photographic plates blackened when placed near certain minerals (uranium salts). Radioactivity could not be explained by e-m (or gravity), and was one of the unsolved problems.

✓ 1898: Maria and Pierre Curie– discovery of Polonium and Radium (much more radioactive than uranium)

Becquerel and the Curies shared the Nobel Prize in Physics in 1903.

Later, Marie Curie isolates metallic radium and receives the Nobel Price in Chemistry in 1911.

Radioactivity Unit: Bacquerel and Curie

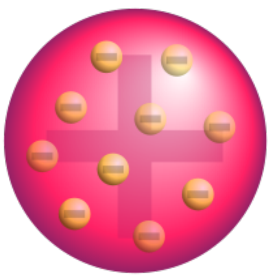
Ernest Rutherford – "the father of nuclear physics"

- ✓ 1899: Rutherford shows 2 types of radiation exits and calls them named α and β.
- ✓ 1900: Villard gives evidence for a 3rd type of radiation coming from radium and calls it γ
- ✓ 1902: Curies show that β radiation is electrons
- ✓ 1904: Rutherford shows α particles are helium

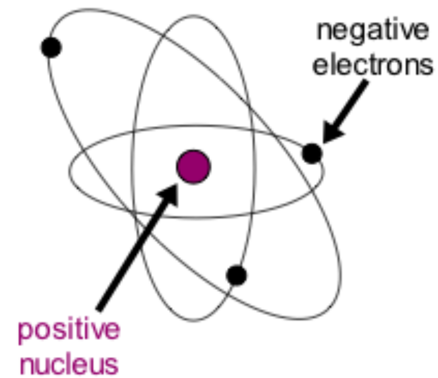
Ernest Rutherford was awarded the Nobel Prize in Chemistry in 1908 "for his investigations into the disintegration of the elements, and the chemistry of radioactive substances".

"I have dealt with many different transformations with various periods of time, but the quickest that I have met was my own transformation in one moment from a physicist to a chemist." E. Rutherford (Nobel banquet 1908).

✓ 1911: Rutherford proposed the existence of a massive nucleus as a small central part of an atom

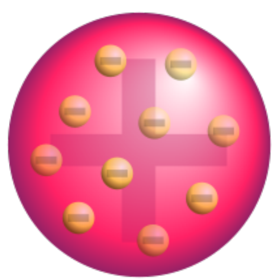


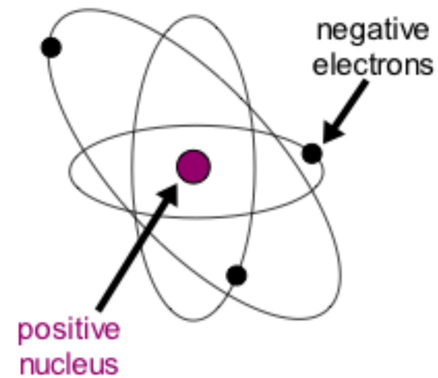
J.J. Thompson's
Plum Pudding Model (1904)



Rutherford's
Planetary Model (1911)

✓ 1911: Rutherford proposed the existence of a massive nucleus as a small central part of an atom



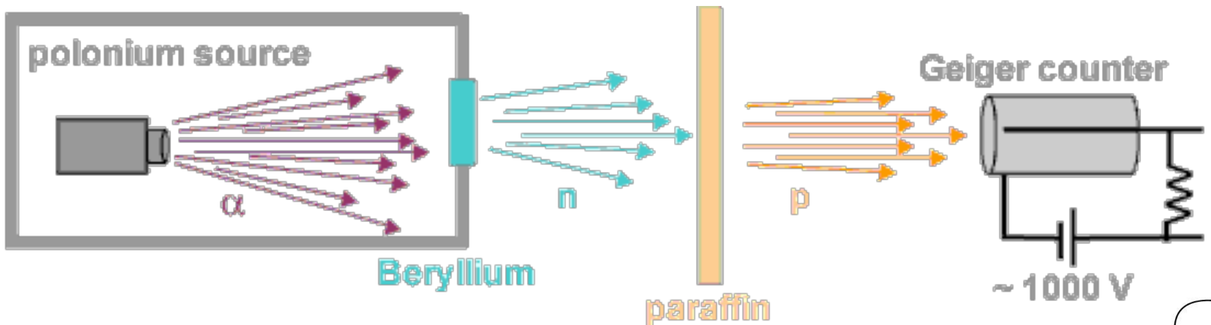


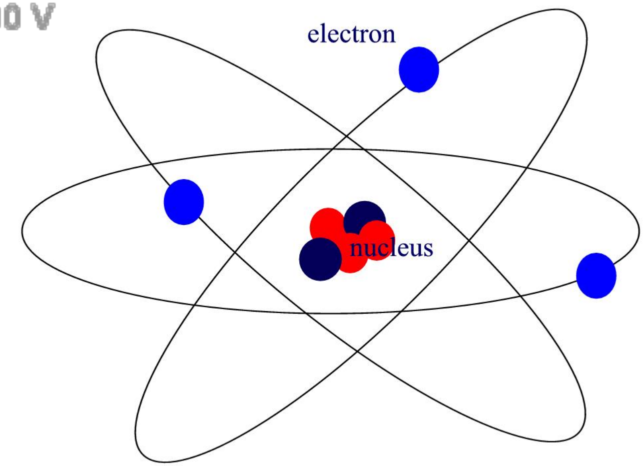
J.J. Thompson's
Plum Pudding Model (1904)

Rutherford's
Planetary Model (1911)

The history of nuclear physics

✓1932: Nobel Prize in Physics for James Chadwick for a discovery of the neutron



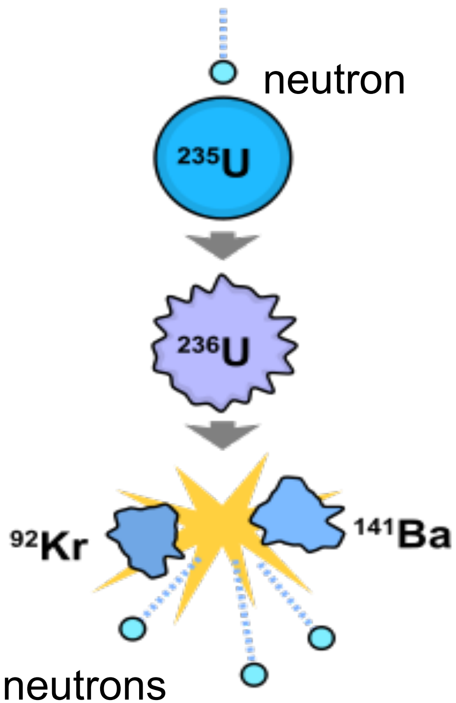


Nucleus = protons + neutrons
(short range nuclear force)

The history of nuclear physics

✓ Chadwick's discovery made it possible to create elements heavier than Uranium in the lab. Later, Enrico Fermi discovered nuclear reactions (slow neutrons) which led to a revolutionary discovery of nuclear fission (Otto Hanh and Fritz Strassmann).

✓ Chadwick's discovery was crucial for the fission of uranium 235. Unlike α particles, neutrons do not need to overcome Coulomb barrier and thus can penetrate and split the nuclei of the heaviest elements. The release of neutrons sustains the fission reaction.



Isotope	Natural Abundance (atomic percentage)	Half-Life (years)
U^{234}	0.0054	2.440×10^5
U^{235}	0.7200	7.040×10^8
U^{238}	99.2746	4.488×10^9

Uranium-235 has the distinction of being the only naturally occurring fissile isotope.

Uranium-238 cannot fission with low energy neutrons (stable nuclear shell structure)

The history of nuclear physics

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✓1914: J. Chadwick shows spectrum of β radiation is continuous, contrary to the fundamental principle of energy conservation

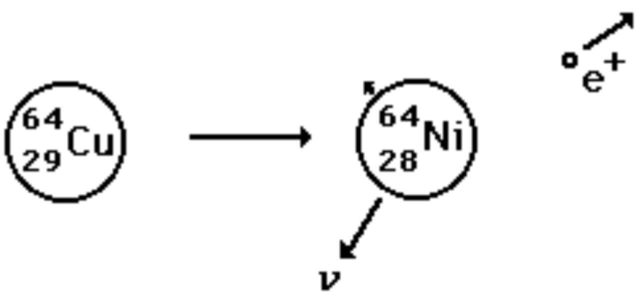
✓1930:W.Pauli proposed a neutrino to explain the continuous spectrum of β decay.

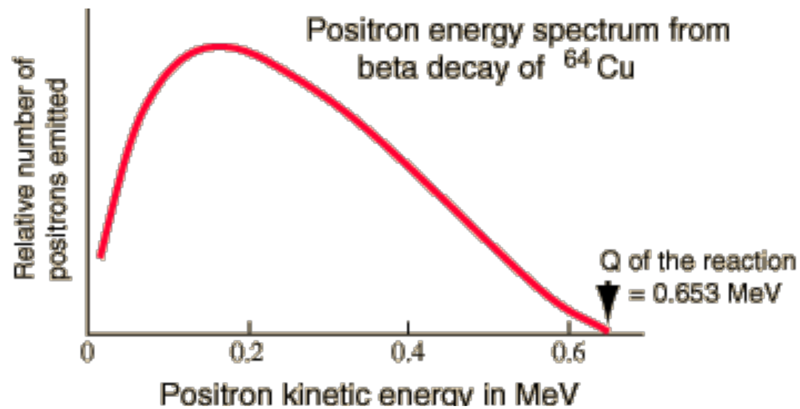
"I have done a terrible thing, I have postulated a particle that cannot be detected."
W. Pauli

Wolfgang Pauli was awarded the Nobel Prize in Physics in 1945 "for the discovery of the Exclusion Principle, also called the Pauli Principle".

The history of nuclear physics

✓ 1933: E. Fermi used the neutrino to explain neutron β decay (model of weak interactions)





Enrico Fermi was aworded the Nobel Prize in Physics in 1938 "for his demonstrations of the existence of new radioactive elements produced by neutron irradiation, and for his related discovery of nuclear reactions brought about by slow neutrons".

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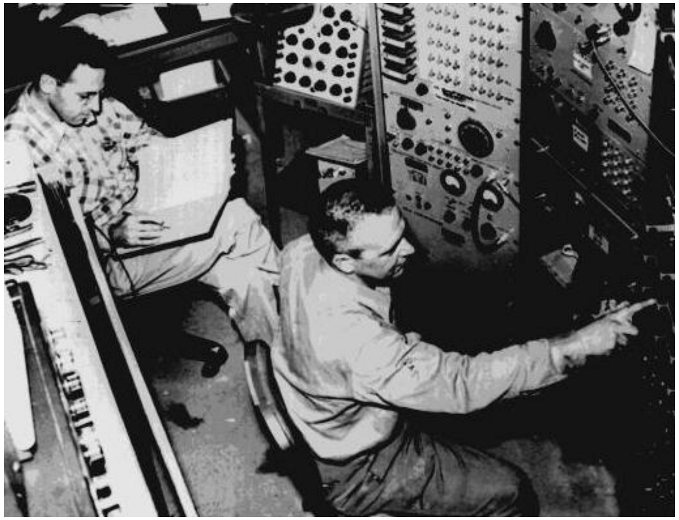
✓ 1956: F. Reines and C. Cowan detection of a neutrino via inverse beta decay reaction

$$\bar{\nu}_e + p \rightarrow n + e^+$$



From then on Reines dedicated his career to the study of the neutrino's properties and interactions, including the discovery of neutrinos emitted from SN1987A by the Irvine-Michigan-Brookhaven Collaboration. This discovery helped to inaugurate the field of neutrino astronomy.

***F. Reines was awarded the Nobel Prize in Physics in 1995* for his co-detection of the neutrino with Clyde Cowan in the neutrino experiment"**

The history of nuclear physics

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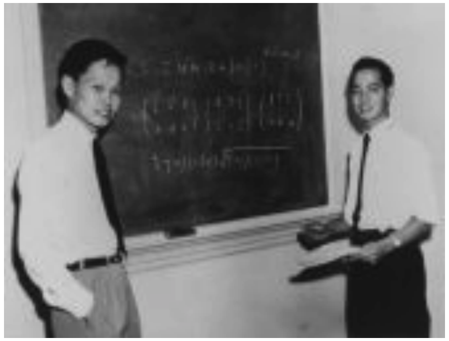
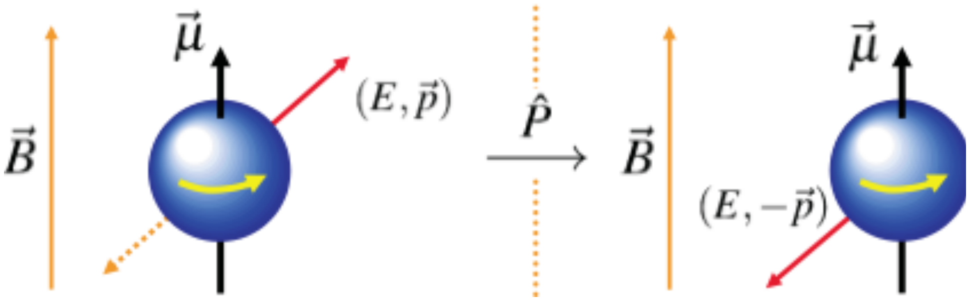
✓ 1957: Lee and Yang – proposed the concept of parity violation in weak interactions (Nobel Prize in Physics)



Implications: if parity is not conserved in weak interactions, it means that the Universe sometimes distinguishes between left and right

✓ 1958: C.S. Wu experimentally confirmed parity violation in weak interactions (β decay of polarized cobalt-60 nuclei)



Observed electrons emitted preferentially in direction opposite to to applied field: If parity were conserved, expect equal rate of electrons in directions along and opposite to the nuclear spin.

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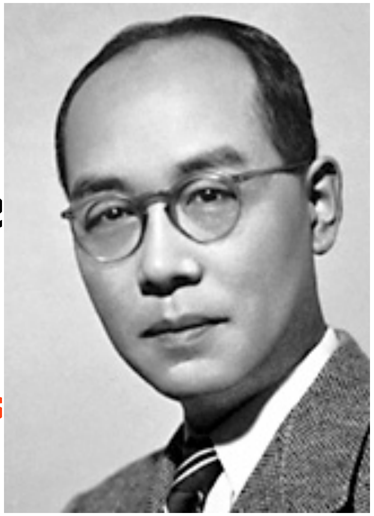
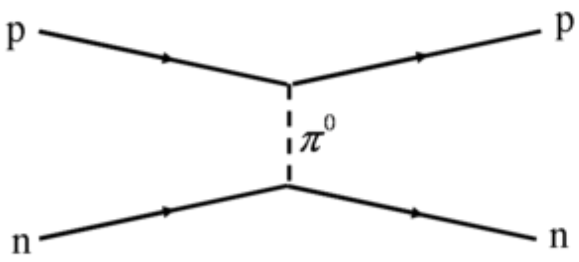


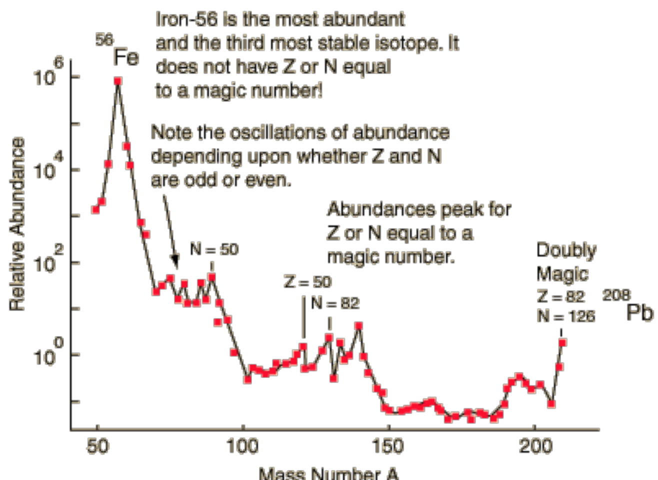
✓1935: Hideki Yukawa proposed the force between nucleons arises from meson exchange

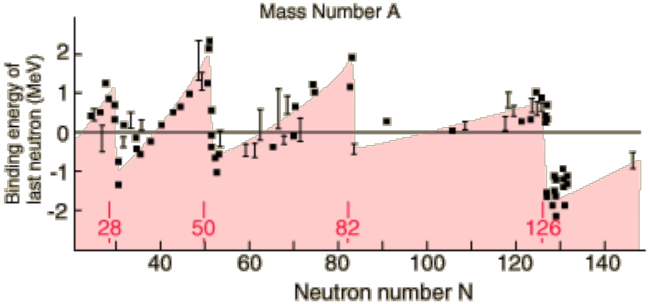
Awarded the Nobel Prize in Physics in 1947 *"for his prediction of the existence of mesons on the basis of theoretical work on nuclear forces*

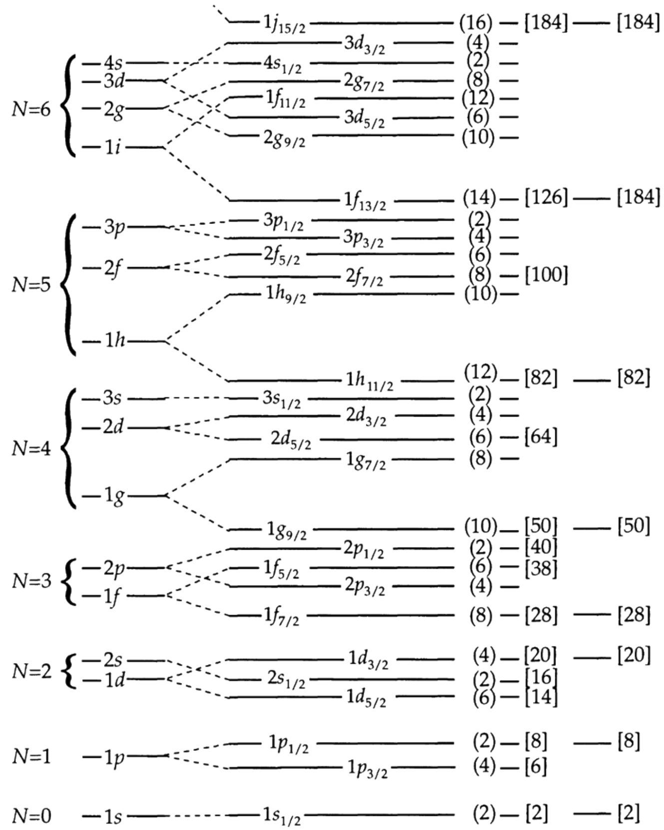


The meson-exchange concept (where hadrons are treated as elementary particles) continues to represent the best working model for a quantitative NN potential.

✓ 1949: M. Meyer and J. Jensen used shell model with spin-orbit interaction to explain magic number

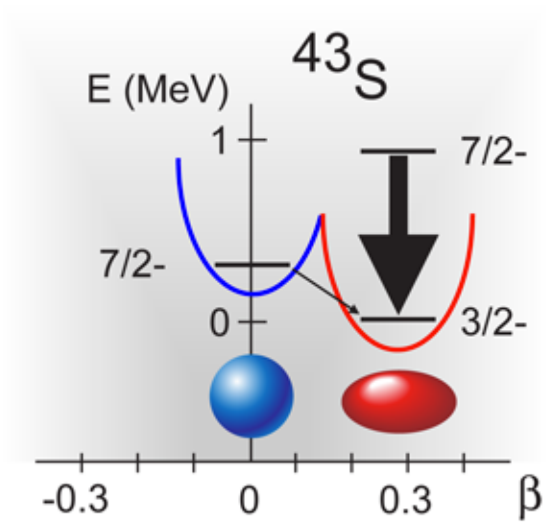


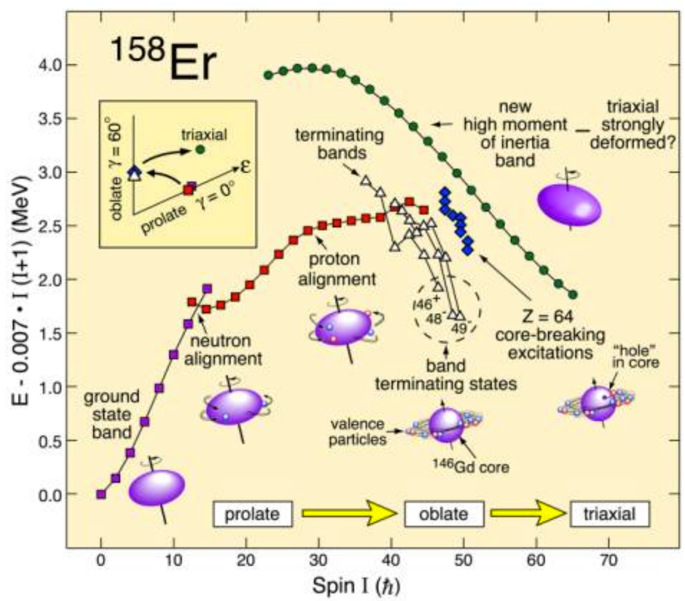


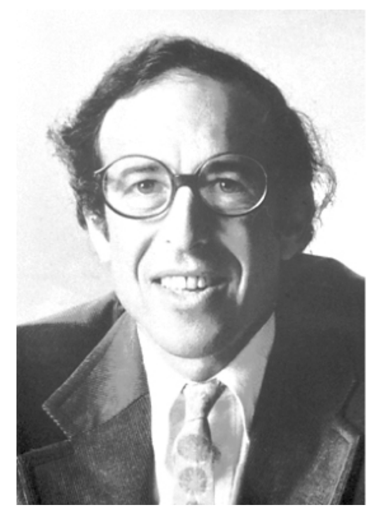


1963 Mayer and Jensen are awarded Nobel prize in physics "for their discoveries concerning nuclear shell structure".

- ✓ 1951: Collective model
  (Bohr, Mottelson, Rainwater)

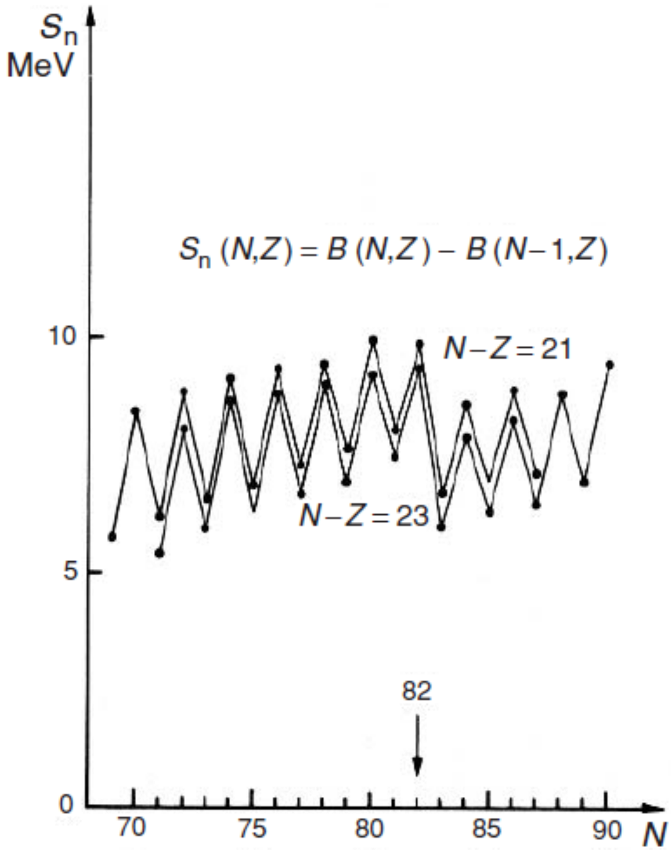


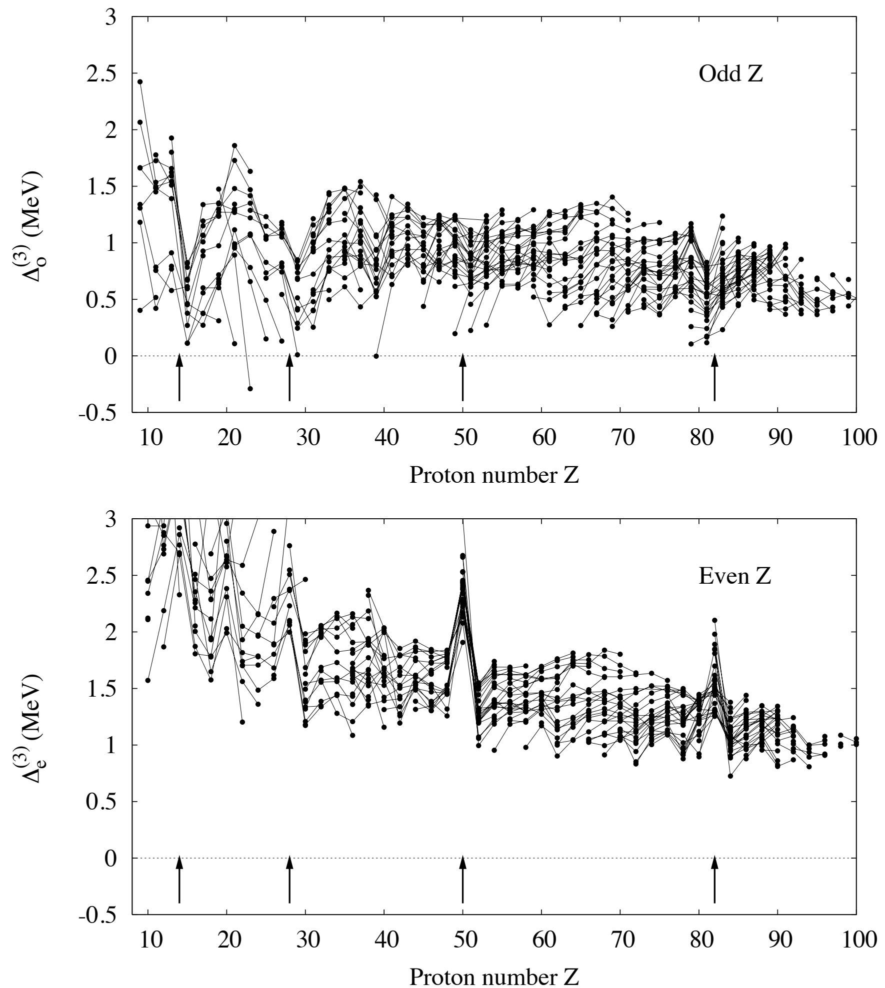




The Nobel Prize in Physics 1975 was awarded jointly to Aage Niels Bohr, Ben Roy Mottelson and Leo James Rainwater "for the discovery of the connection between collective motion and particle motion in atomic nuclei and the development of the theory of the structure of the atomic nucleus based on this connection".

- ✓ 1957: Nuclear superfluidity (Bohr, Mottelson)





✓ Nuclear reaction

decay, fusion, fission, heavy ion collision, ...

✓ Nuclear structure

nuclear basic properties:

nuclear size, nuclear binding, nuclear shape

✓ Models of nuclear structure theory

Collective models

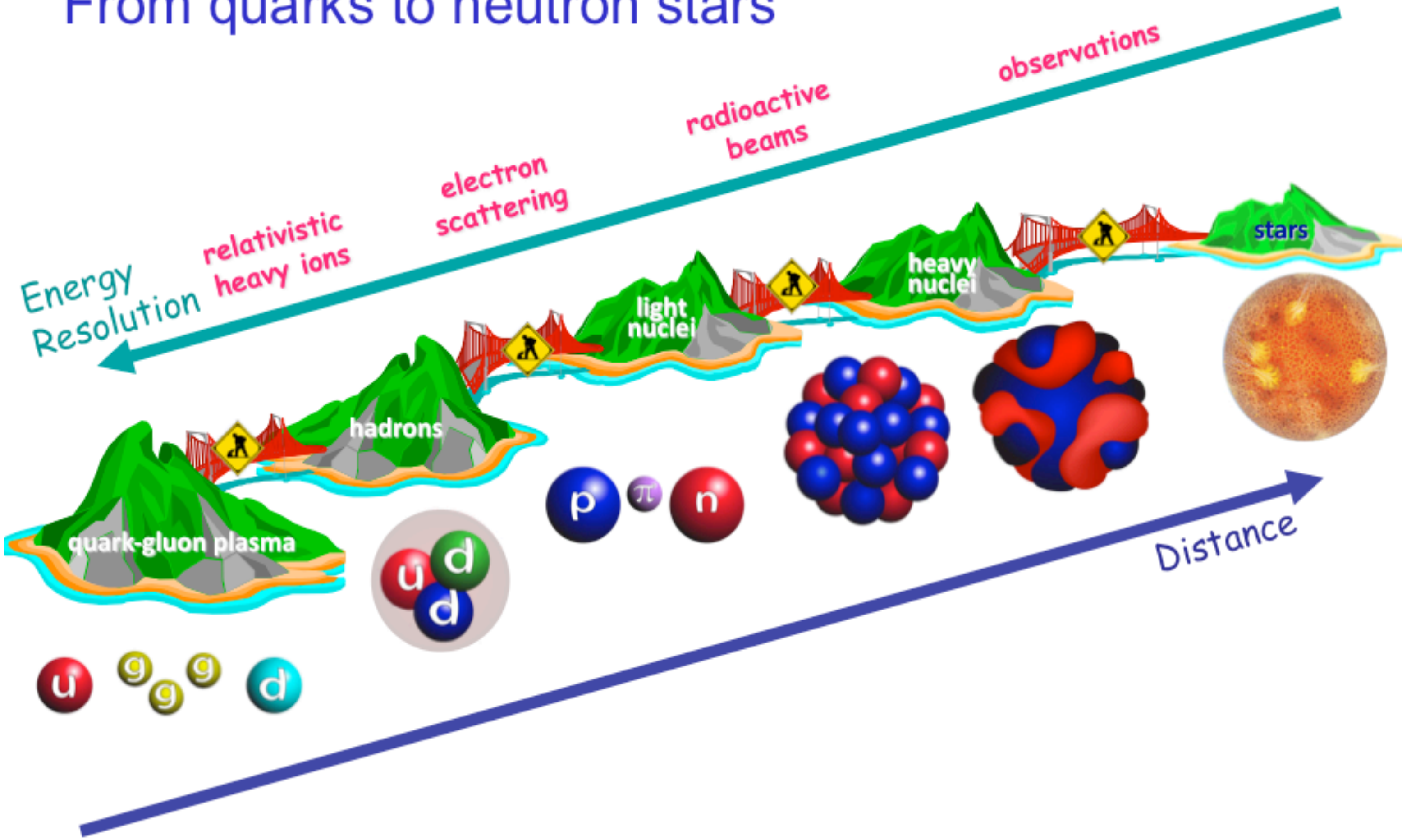
The degrees of freedom are some bulk property of nucleus as a whole

Microscopic models

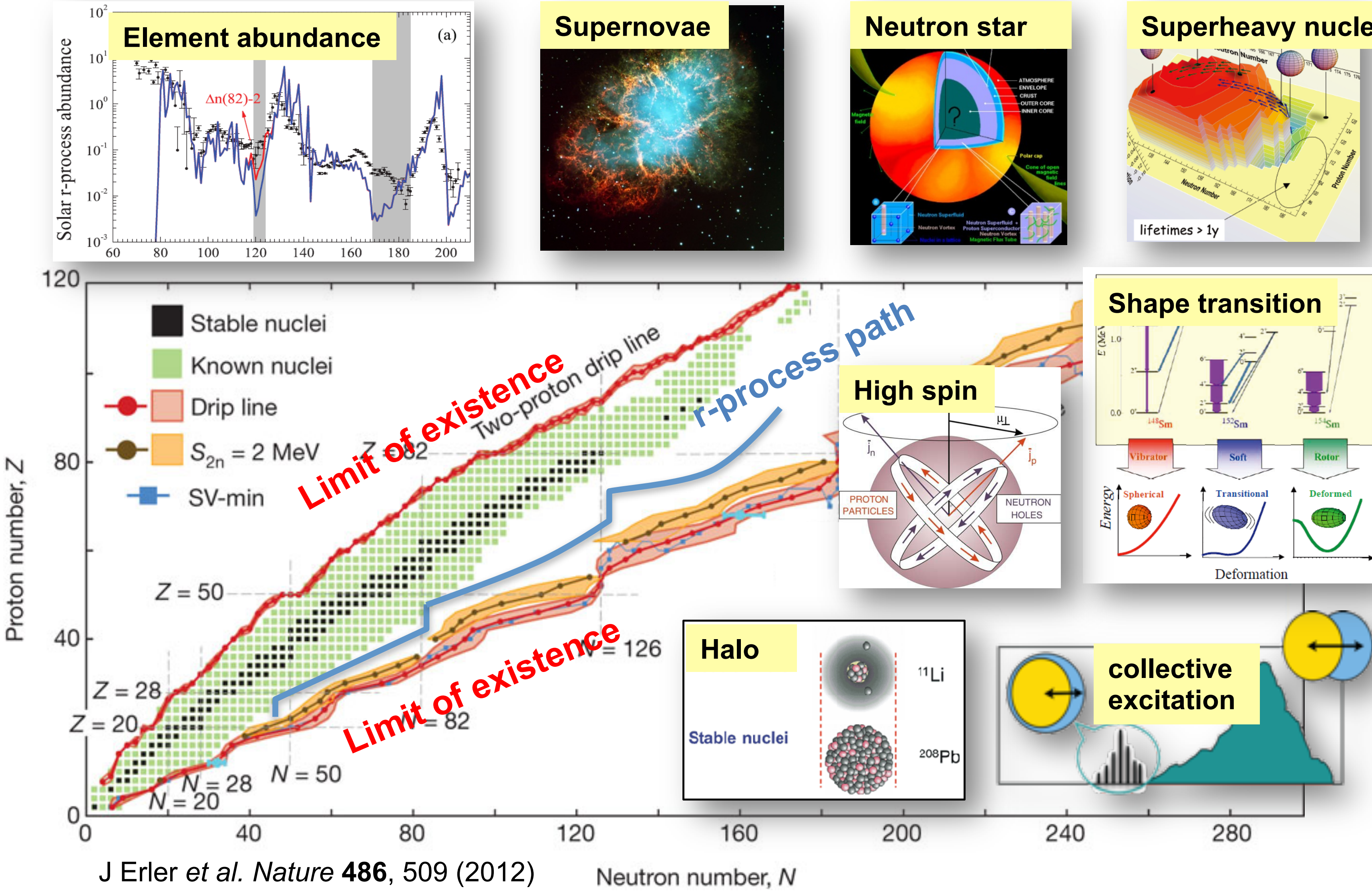
The degrees of freedom are those of the constituent particles of the nucleus

Present nuclear physics

From quarks to neutron stars



Hot topics in nuclear structure theory



J Erler *et al.* *Nature* **486**, 509 (2012)

Some nomenclature

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$^A_Z\mathrm{X}_N$	Isotope notation
X	Chemical symbol, *e.g.* Ca, Pb
A	Atomic mass number (sum of n's and p's in the nucleus)
Z	Atomic number (or, proton number), the number of p's in the nucleus
N	Neutron number, the number of n's in the nucleus
Examples	$^3_2\mathrm{He}_1$, $^{40}_{20}\mathrm{Ca}_{20}$, $^{208}_{82}\mathrm{Pb}_{126}$
Variants	^{40}Ca, Calcium-40, Ca-40

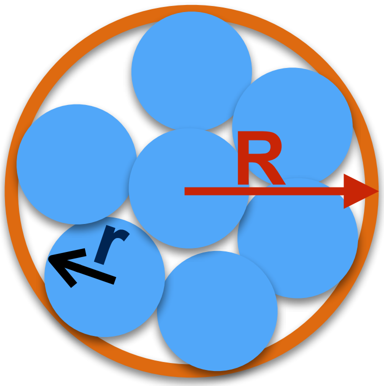
Note that, once X (which encodes Z) and A are given, the rest of the information is redundant, since $A = Z + N$. The full form is usually used only for emphasis.

isotope	Same Z, different N *e.g.* ^{40}Ca and ^{41}Ca Mnemonic: From Greek *isos* (same) *topos* (place) (coined by F. Soddy 1913) *i.e.* same place in the periodic table
isotone	Different Z, same N, *e.g.* ^{13}C and ^{12}B Mnemonic: isoto**P**e and isoto**N**e (coined by K. Guggenheimer 1934)
isobar	Different Z, and N, but same A, *e.g.* ^{12}C and ^{12}B Mnemonic: From Greek *isos* (same) *baros* (weight)

Nuclear Radii

➢ **A nucleons with hard spheres of radius r**

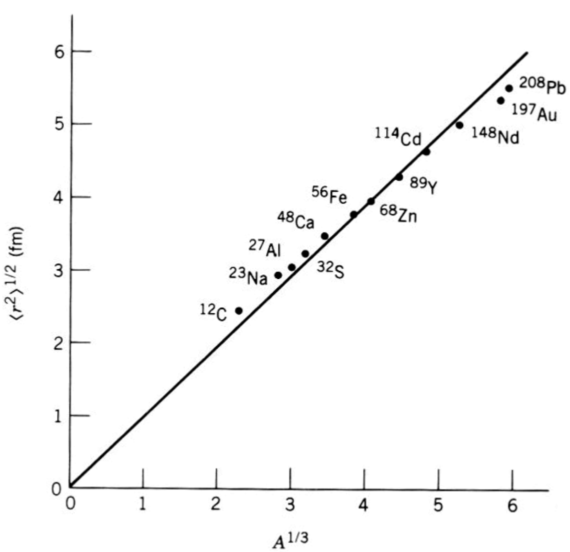
so,

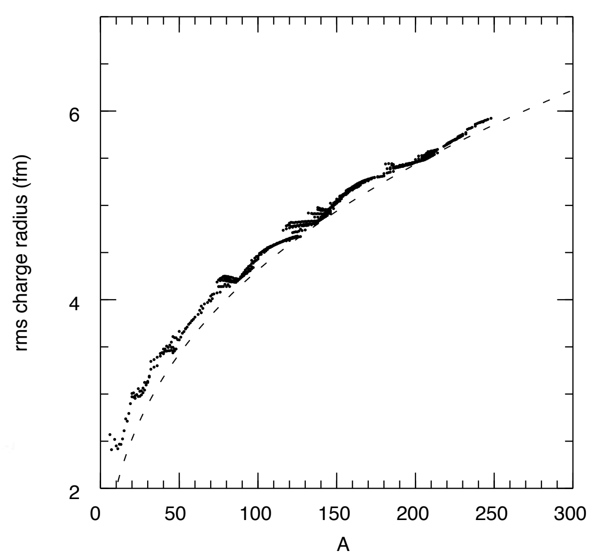


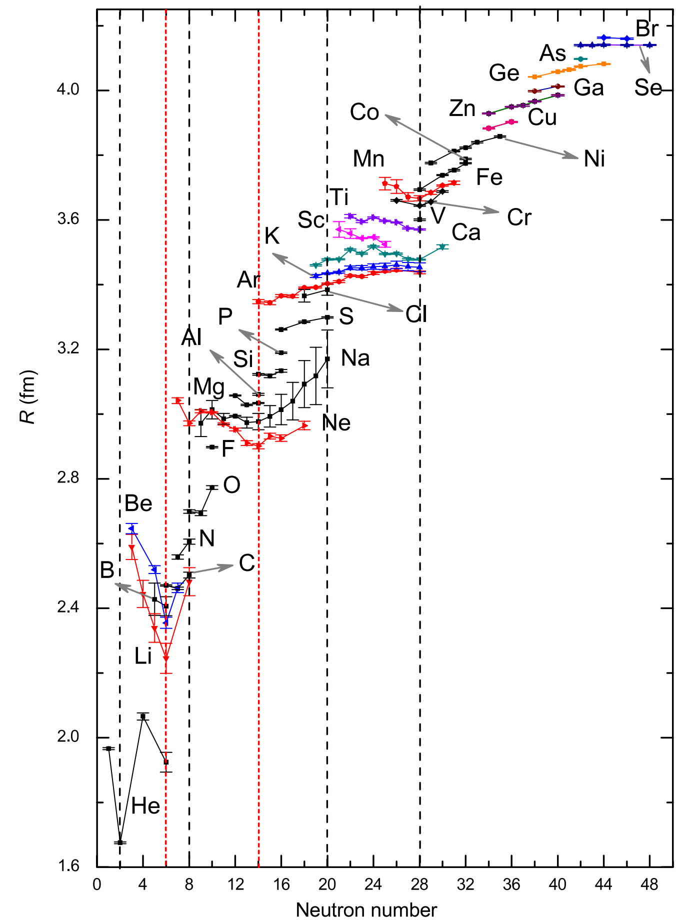
$$\frac{4}{3}\pi R^3 \approx A\frac{4}{3}\pi r^3$$

$$R \approx r_0 A^{1/3}$$

$$r_0 \sim 1.20 - 1.25 \text{ fm}$$







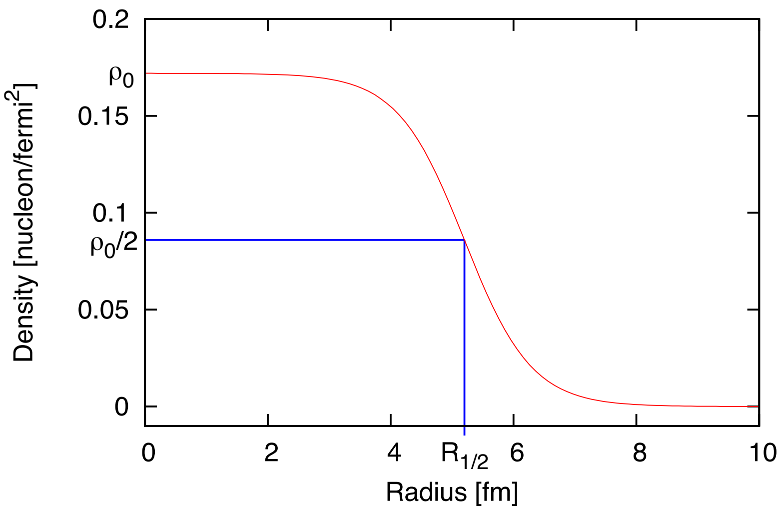
Nuclear density

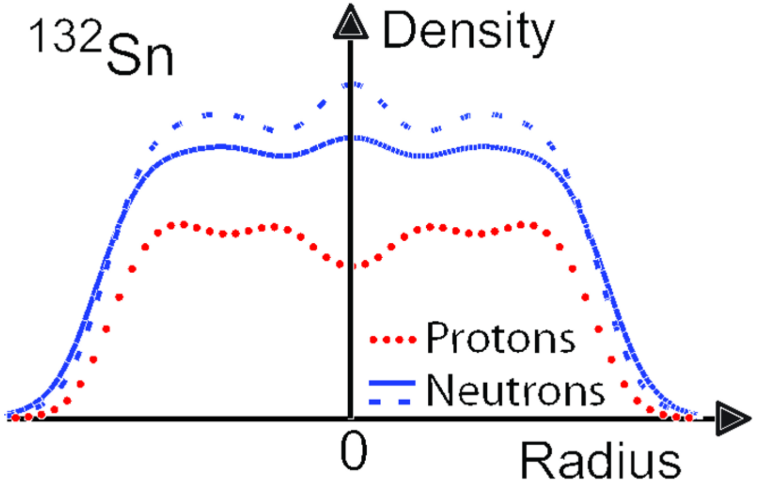
- **Woods-Saxon distribution**

$$\rho(r) = \rho_0 \frac{1}{1 + \exp\left(\frac{r - R_{1/2}}{a}\right)}$$

- **nuclear radii**

$$\rho(R_{1/2}) = \rho_0 \frac{1}{1 + \exp\left(\frac{R_{1/2} - R_{1/2}}{a}\right)} = \rho_0 \frac{1}{1+1} = \frac{1}{2}\rho_0$$





Form factor

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➢ **A uniform charge distribution**

In spherical case

$$
\begin{aligned}
F(q) &= \int_0^{2\pi} \mathrm{d}\phi \int_0^{\infty} r^2 \mathrm{d}r\ \rho_p(r) \int_0^{\pi} \sin\theta\ \mathrm{d}\theta\ e^{iqr\cos\theta} \quad [\text{note } F(\vec{q}) \longrightarrow F(q)] \\
&= 2\pi \int_0^{\infty} r^2 \mathrm{d}r\ \rho_p(r) \int_0^{\pi} \sin\theta\ \mathrm{d}\theta\ e^{iqr\cos\theta} \quad [\text{did the integral over } \phi] \\
&= 2\pi \int_0^{\infty} r^2 \mathrm{d}r\ \rho_p(r) \int_{-1}^{1} \mathrm{d}\mu\ e^{iqr\mu} \quad [\text{change of variable } \mu = \cos\theta] \\
&= 2\pi \int_0^{\infty} r \mathrm{d}r\ \rho_p(r) \left(\frac{2}{q}\right) \sin qr \quad [\text{did the integral over } \mu] \\
&= \frac{4\pi}{q} \int_0^{\infty} r \mathrm{d}r\ \rho_p(r) \sin qr \quad [\text{in final form}]
\end{aligned}
$$

Form factor

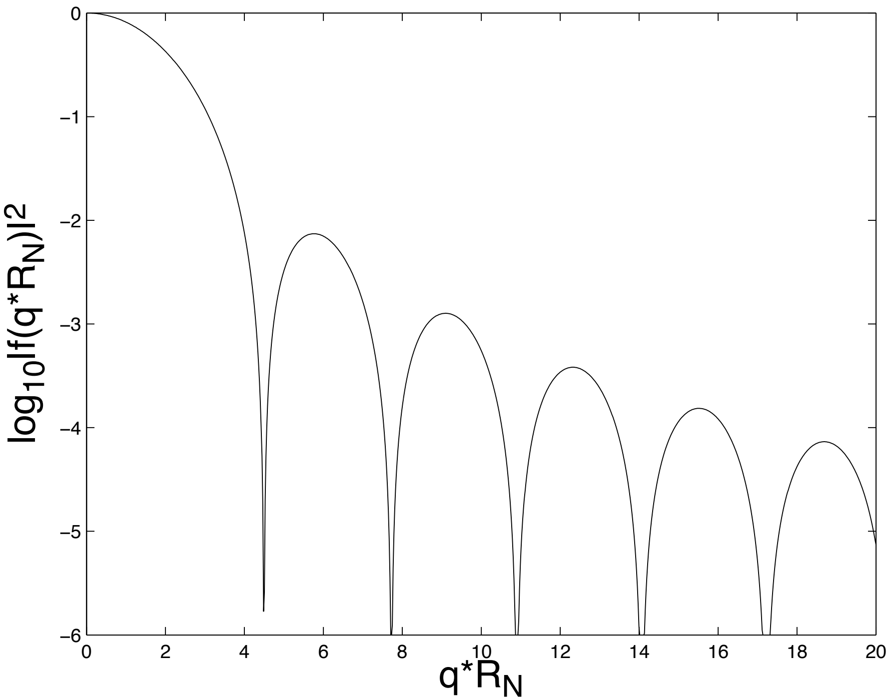
➢ **A uniform charge distribution**

$$\rho_p(r) = \frac{3}{4\pi R_N^3}\Theta(R_N - r)$$

➢ **Form factor from in uniform charge distribution**

$$F(q) = \frac{3}{(qR_N)^3}\int_0^{(qR_N)} \mathrm{d}z\ z\sin z$$

$$= \frac{3[\sin(qR_N) - qR_N\cos(qR_N)}{(qR_N)^3}$$



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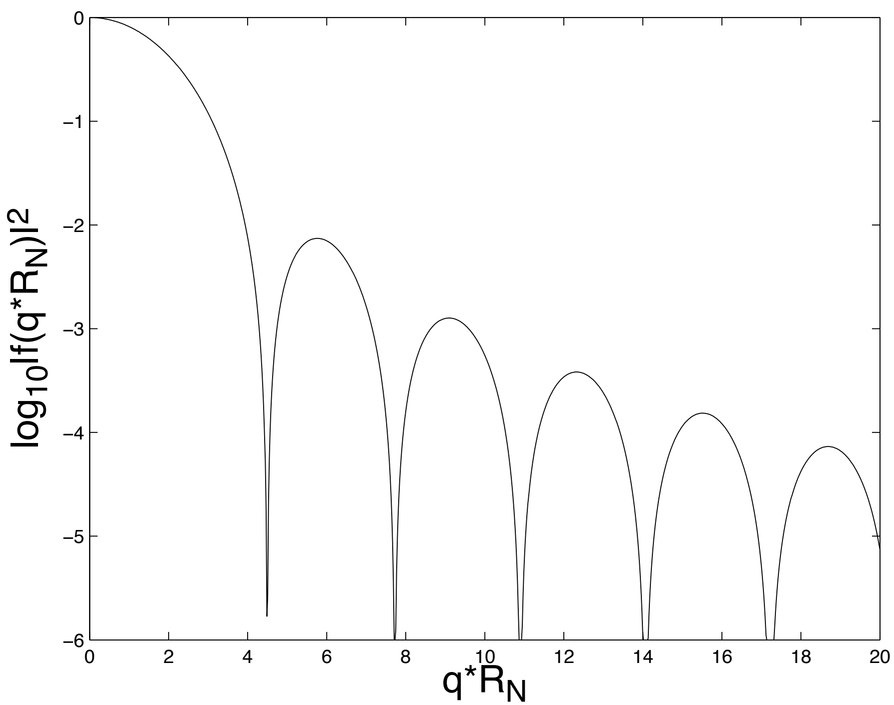

Form factor

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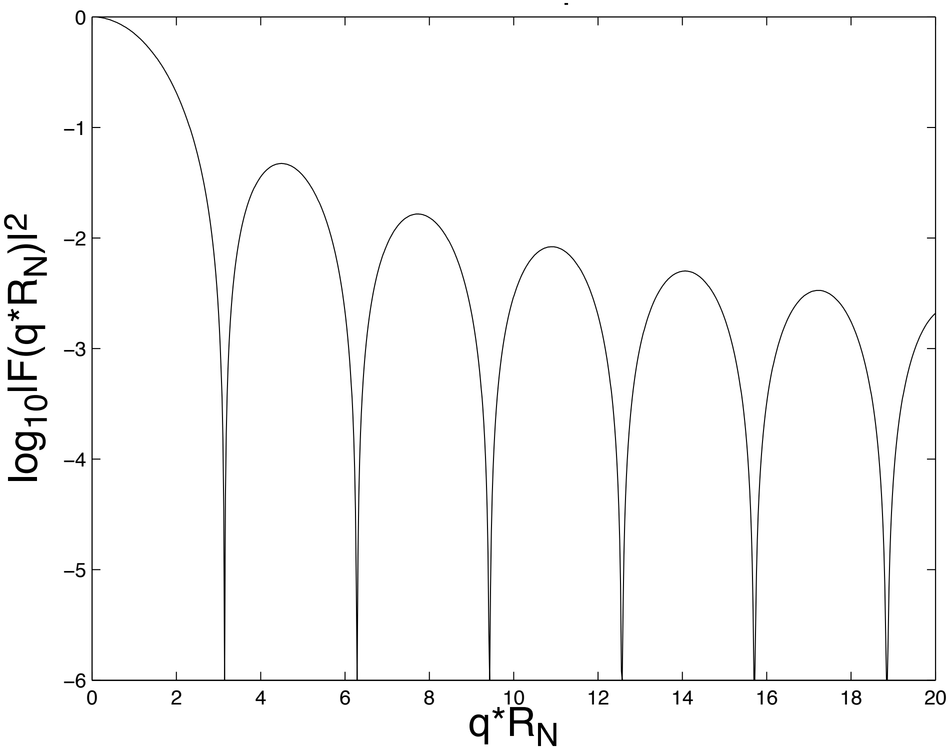
- **A delta charge distribution**

$$\rho_p(r) = \delta(R_N - r)/4\pi R_N^2$$

- **Form factor from in delta charge distribution**

$$F(q) = \sin(qR_N)/qR_N$$



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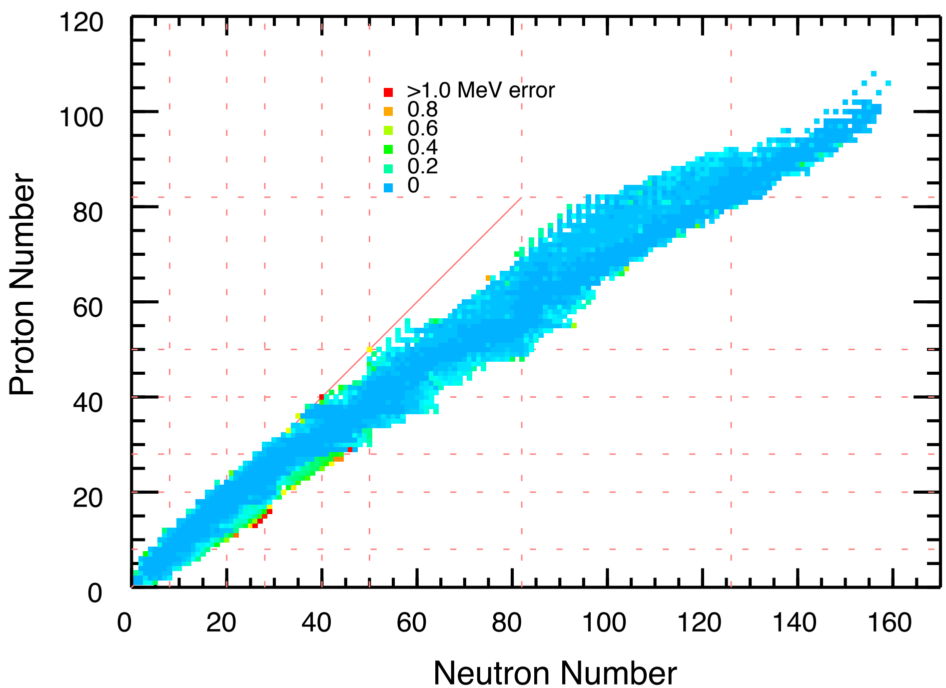
- **Mass excess**

$$\Delta(N,Z) \equiv M(N,Z) - uA,$$

- **Atomic Mass Unit**

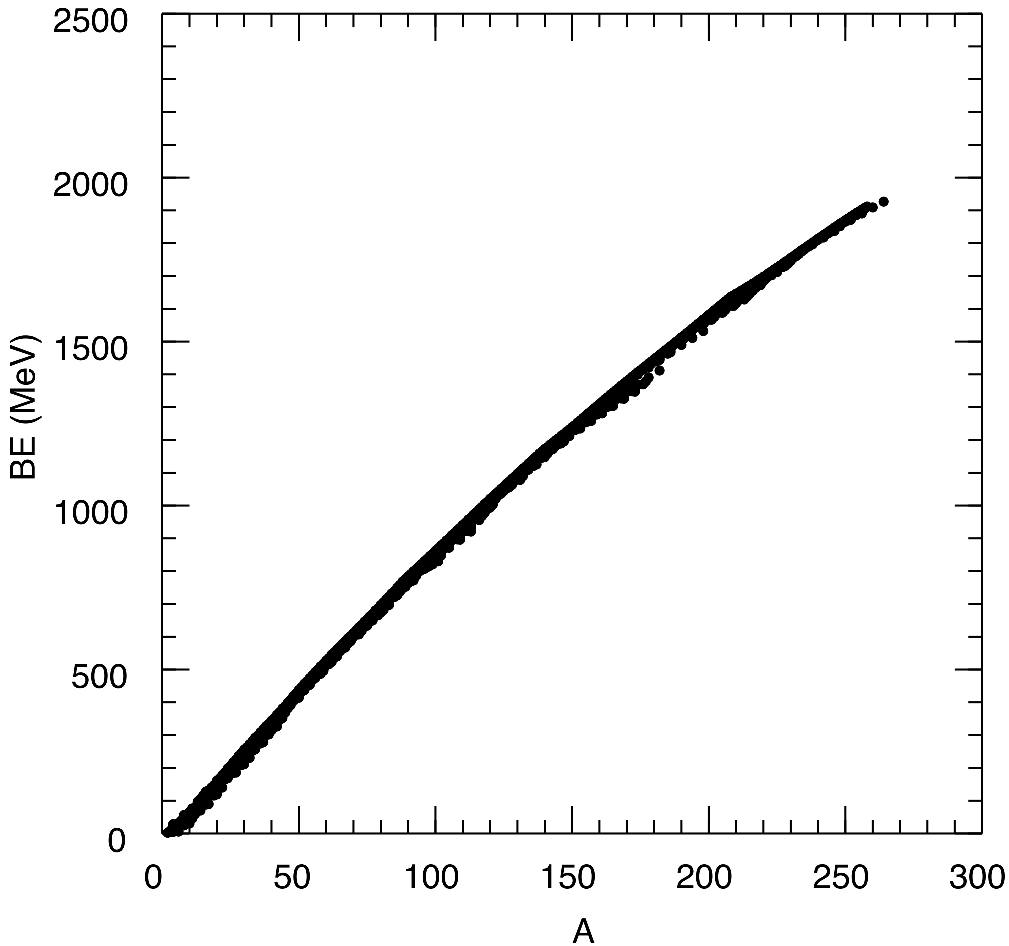
$$u = M(^{12}\mathrm{C})/12 = 931.49386\,\mathrm{MeV}/c^2$$

Nuclei with measured masses



➢ Nuclear binding energy

$$B_N(Z,A) = \left\{Zm_p + Nm_n - \left[m(^AX) - Zm_e\right]\right\}c^2$$



Nuclear binding energy

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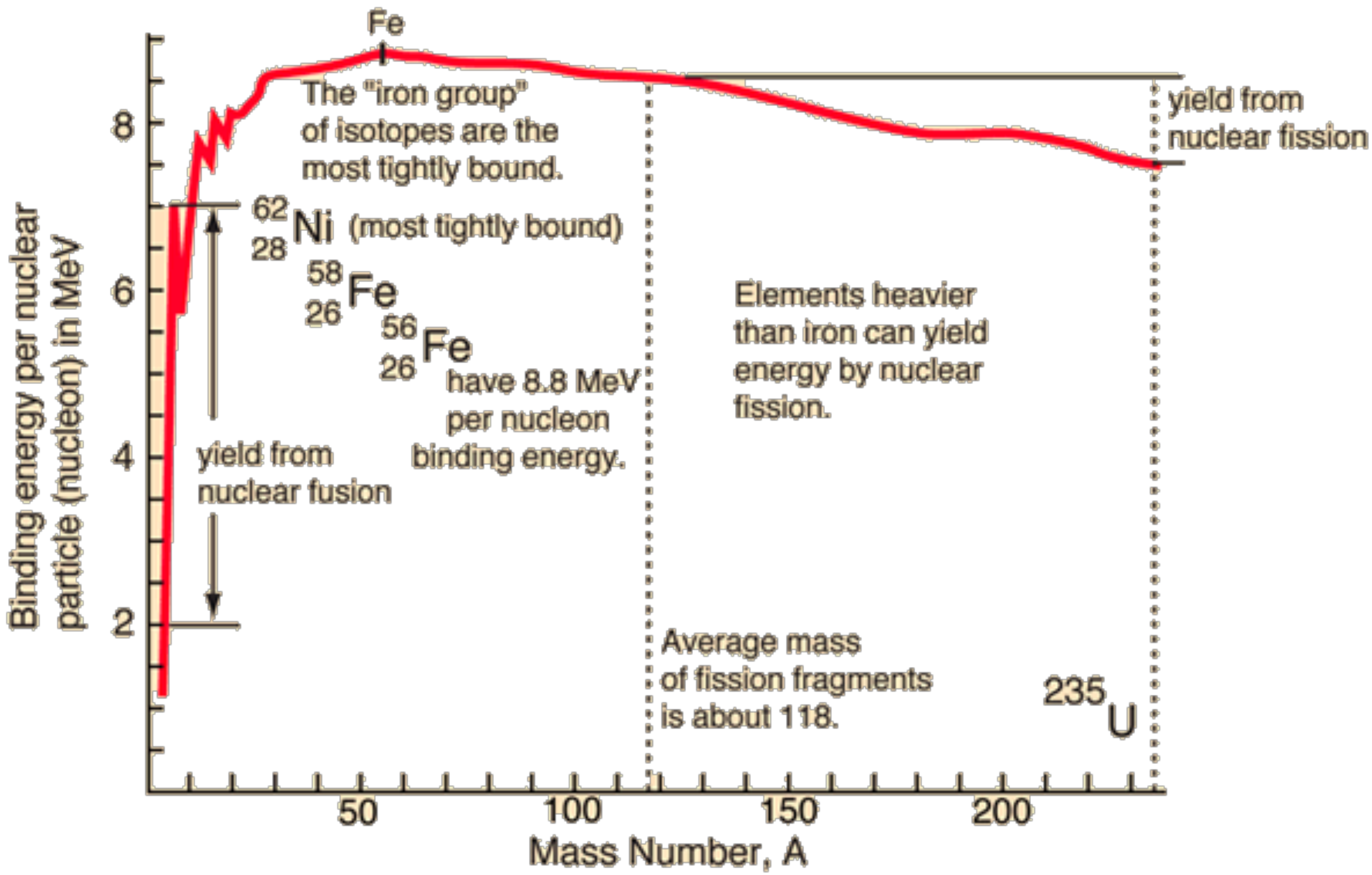
➢ **Binding energy per nucleon**



Nuclear mass table

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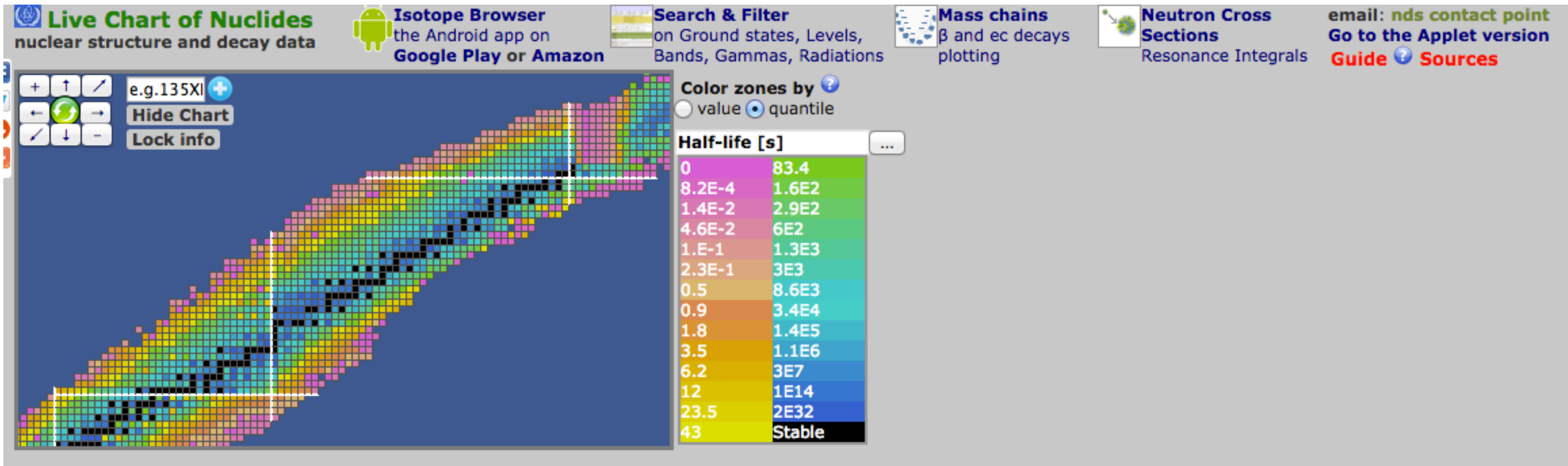
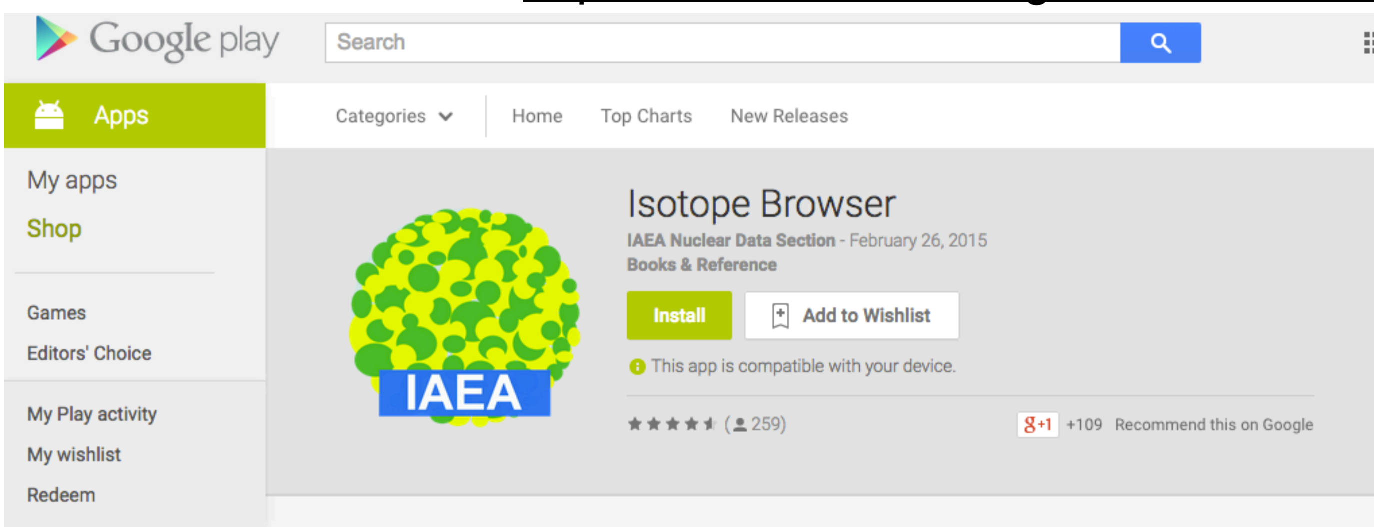
Element	Z	N	A	Atomic mass M_A (u)	Mass excess $M_A - A$ (μu)	Mass defect ΔM_A (μu)	Binding energy E_B (MeV)	E_B/A (MeV/A)
n	0	1	1	1.008665	8665	0	—	—
H	1	0	1	1.007825	7825	0	—	—
D	1	1	2	2.014102	14102	−2388	2.22	1.11
T	1	2	3	3.016049	16049	−9106	8.48	2.83
He	2	1	3	3.016029	16029	−8286	7.72	2.57
He	2	2	4	4.002603	2603	−30377	28.30	7.07
He	2	4	6	6.018886	18886	−31424	29.27	4.88
Li	3	3	6	6.015121	15121	−34348	32.00	5.33
Li	3	4	7	7.016003	16003	−42132	39.25	5.61
Be	4	3	7	7.016928	16928	−40367	37.60	5.37
Be	4	5	9	9.012182	12182	−62442	58.16	6.46
Be	4	6	10	10.013534	13534	−69755	64.98	6.50
B	5	5	10	10.012937	12937	−69513	64.75	6.48
B	5	6	11	11.009305	9305	−81809	76.20	6.93
C	6	6	12	12.000000	0	−98940	92.16	7.68
N	7	7	14	14.003074	3074	−112356	104.7	7.48
O	8	8	16	15.994915	−5085	−137005	127.6	7.98
F	9	10	19	18.998403	−1597	−158671	147.8	7.78
Ne	10	10	20	19.992436	−7564	−172464	160.6	8.03
Na	11	12	23	22.989768	−10232	−200287	186.6	8.11
Mg	12	12	24	23.985042	−14958	−212837	198.3	8.26
Al	13	14	27	26.981539	−18461	−241495	225.0	8.33
Si	14	14	28	27.976927	−23073	−253932	236.5	8.45
P	15	16	31	30.973762	−26238	−282252	262.9	8.48
K	19	20	39	38.963707	−36293	−358266	333.7	8.56
Co	27	32	59	58.933198	−66802	−555355	517.3	8.77
Zr	40	54	94	93.906315	−93685	−874591	814.7	8.67
Ce	58	82	140	139.905433	−94567	−1258941	1172.7	8.38
Ta	73	108	181	180.947993	−52007	−1559045	1452.2	8.02
Hg	80	119	199	198.968254	−31746	−1688872	1573.2	7.91
Th	90	142	232	232.038051	38051	−1896619	1766.7	7.62
U	92	143	235	235.043924	43924	−1915060	1783.9	7.59
U	92	144	236	236.045563	45563	−1922087	1790.4	7.59
U	92	146	238	238.050785	50785	−1934195	1801.7	7.57
Pu	94	146	240	240.053808	53808	−1946821	1813.5	7.56

Nuclear mass table



https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html



- **Nuclear combinations**

$$\sum_i [N,Z]_i \rightarrow \sum_f [N,Z]_f$$

- **Particle conserved**

$$\sum_i N_i = \sum_f N_f \text{ and } \sum_i Z_i = \sum_f Z_f$$

- **Q values**

$$Q = \sum_i M(N_i, Z_i)c^2 - \sum_f M(N_f, Z_f)c^2 = \sum_f B(N_f, Z_f) - \sum_i B(N_i, Z_i)$$

- **One neutron separation energy**

$$S_n = -Q_n = B(N,Z) - B(N-1,Z)$$

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- **One proton separation energy**

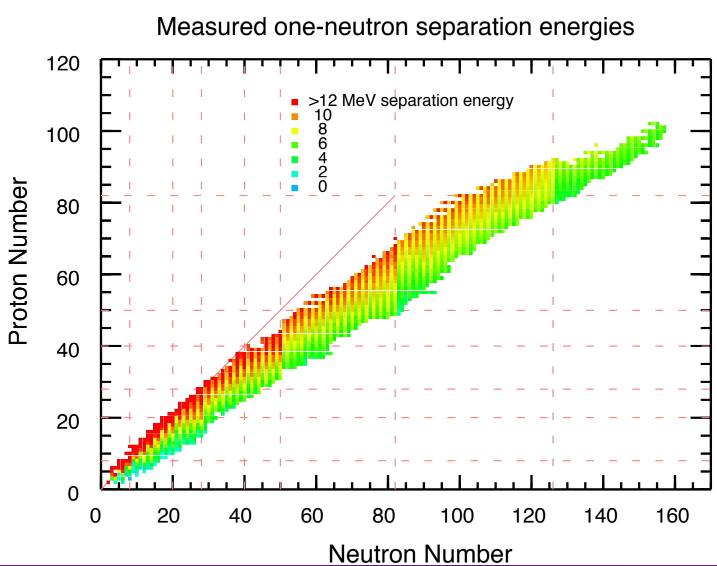
$$S_p = -Q_p = B(N,Z) - B(N,Z-1)$$

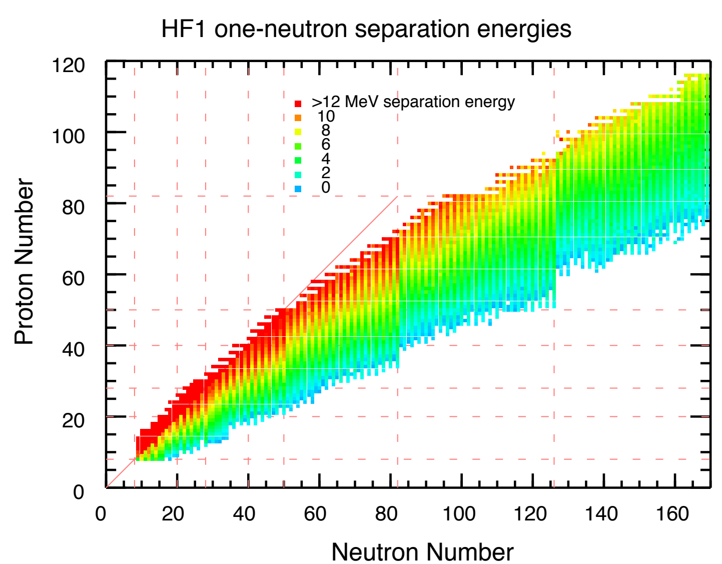
- **Two neutron separation energy**

$$S_{2n} = -Q_{2n} = B(N,Z) - B(N-2,Z)$$

- **Two proton separation energy**

$$S_{2p} = -Q_{2p} = B(N,Z) - B(N,Z-2)$$

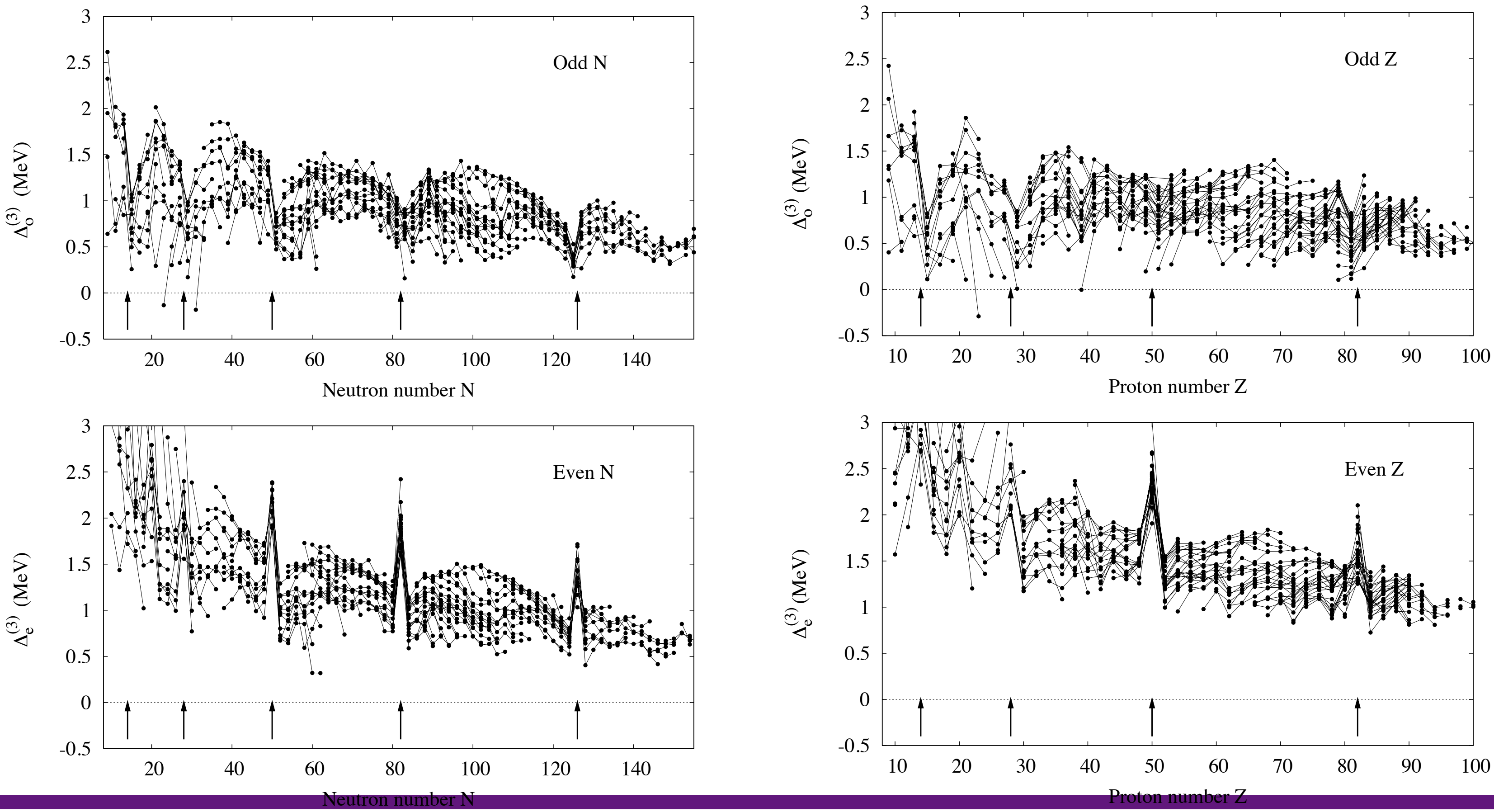




➢ Pairing energy

$$\Delta S_n = B(N,Z) - B(N-1,Z) - [B(N+1,Z) - B(N,Z)]$$

$$= 2B(N,Z) - B(N-1,Z) - B(N+1,Z).$$



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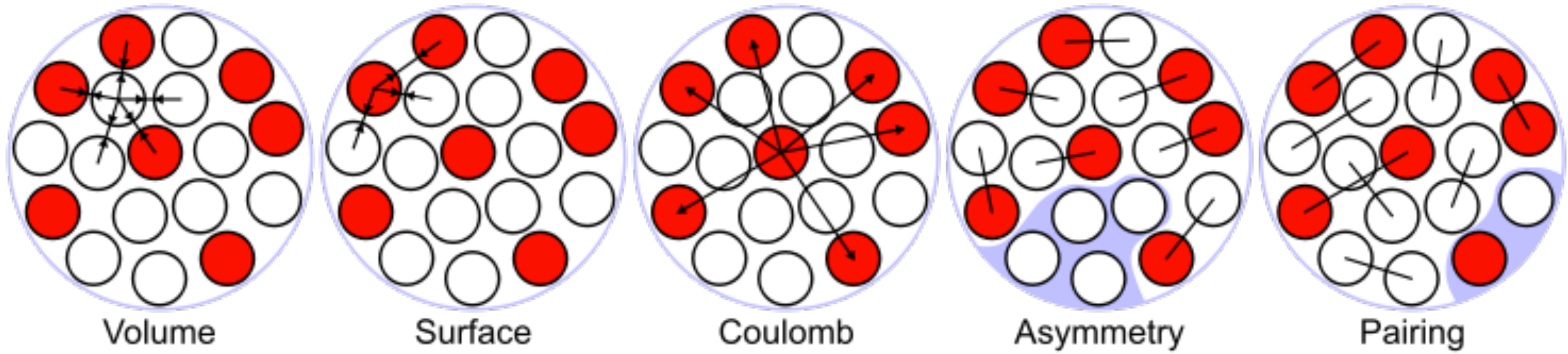
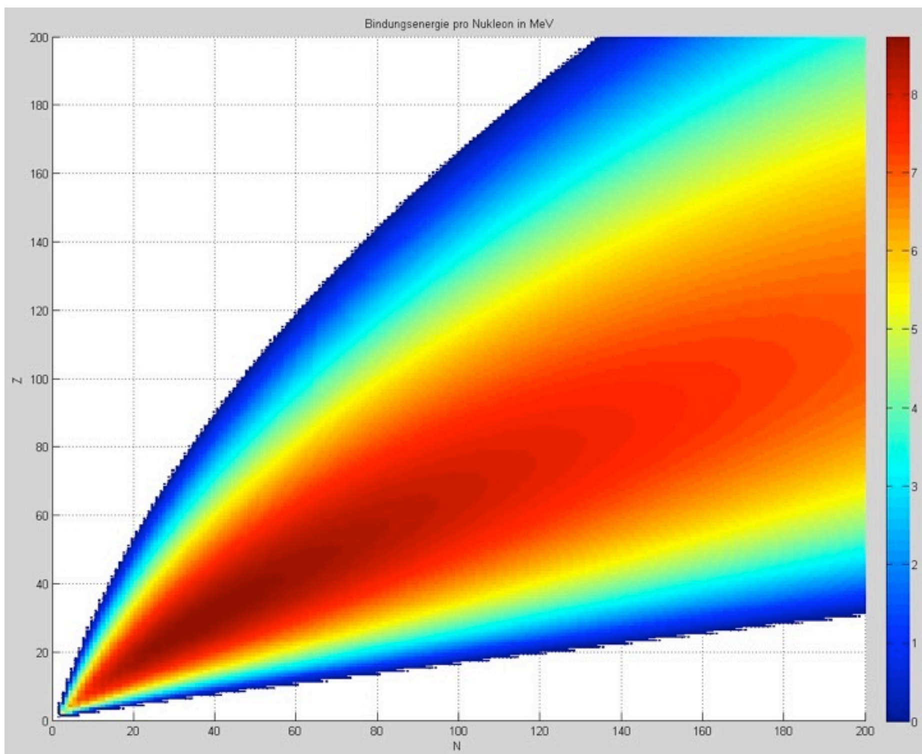
➢ Semi-empirical mass formula

$$
\begin{aligned}
B(Z,A) = \quad & a_V A && \text{("volume" term)} \\
& -a_S A^{2/3} && \text{("surface" term)} \\
& -a_C Z(Z-1)A^{-1/3} && \text{("Coulomb repulsion" term)} \\
& -a_{\text{sym}}\frac{(A-2Z)^2}{A} && \text{("symmetry" term)} \\
& +a_p \frac{(-1)^Z[1+(-1)^A]}{2} A^{-3/4} && \text{("pairing" term)}
\end{aligned}
$$

a_i	[MeV]	Description	Source
a_V	15.5	Volume attraction	Liquid Drop Model
a_S	16.8	Surface repulsion	Liquid Drop Model
a_C	0.72	Coulomb repulsion	Liquid Drop Model + Electrostatics
a_{sym}	23	n/p symmetry	Shell model
a_p	34	n/n, p/p pairing	Shell model

Semi-empirical mass formula

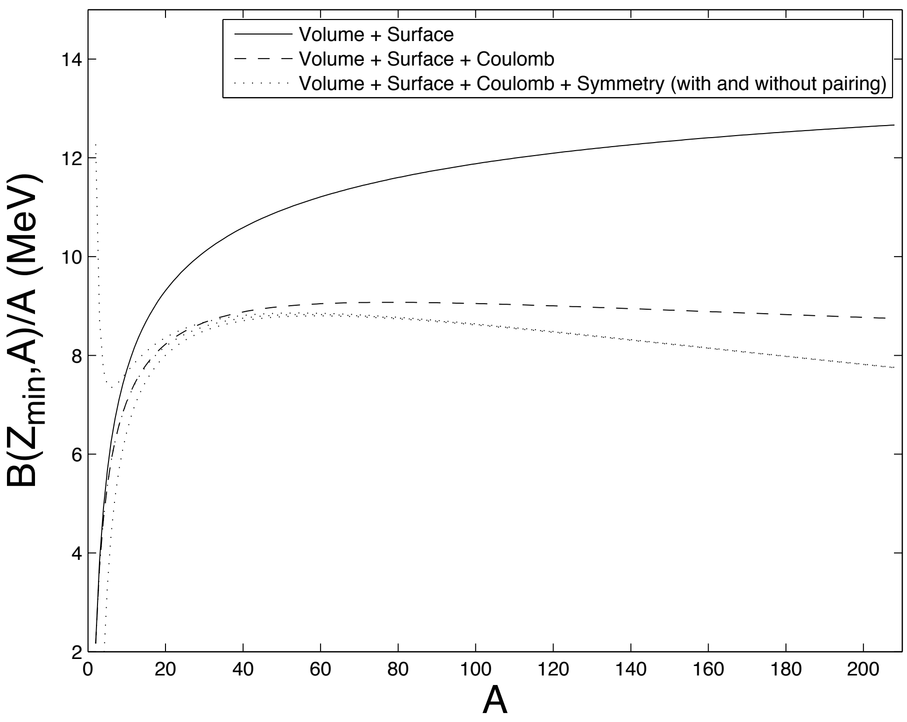




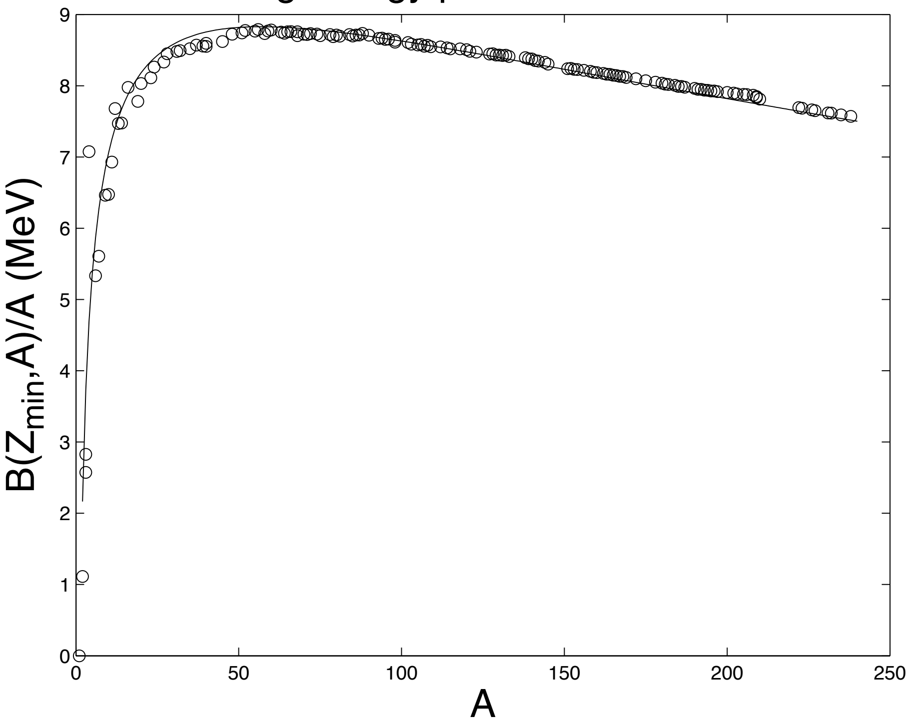
Semi-empirical mass formula

$$B(Z,A) = a_V A - a_S A^{2/3} - a_C Z(Z-1)A^{-1/3} - a_{sym}\frac{(A-2Z)^2}{A} + a_p \frac{(-1)^Z[1+(-1)^A]}{2} A^{-3/4}$$

Binding energy per nucleon *vs.* A



Binding energy per nucleon *vs.* data



Nuclear stability

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- α decay

$$^{A}Z \rightarrow\ ^{(A-4)}(Z-2)\ +\ ^{4}\mathrm{He}.$$

- Electron capture

$$e^- +\ ^{A}Z \rightarrow\ ^{A}(Z-1) + \nu_e$$

- β⁻ decay

$$^{A}Z \rightarrow\ ^{A}(Z+1) + e^- + \bar{\nu}_e$$

- β⁺ decay

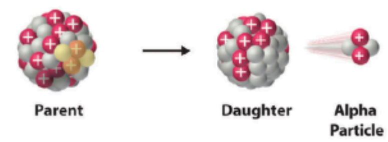
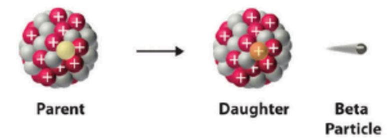
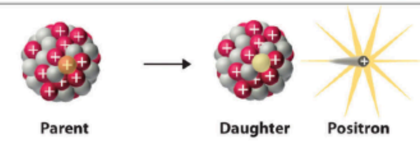
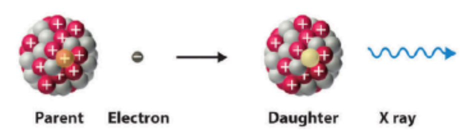
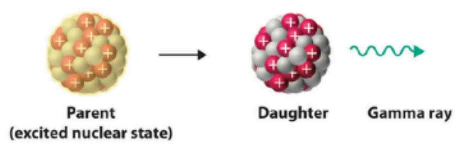
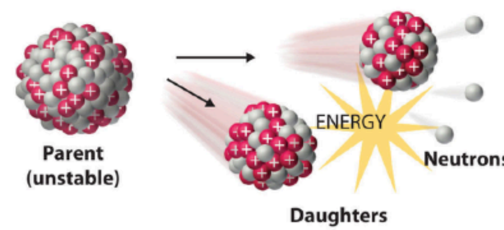
$$^{A}Z \rightarrow\ ^{A}(Z-1) + e^+ + \nu_e$$

- Light fragment emission

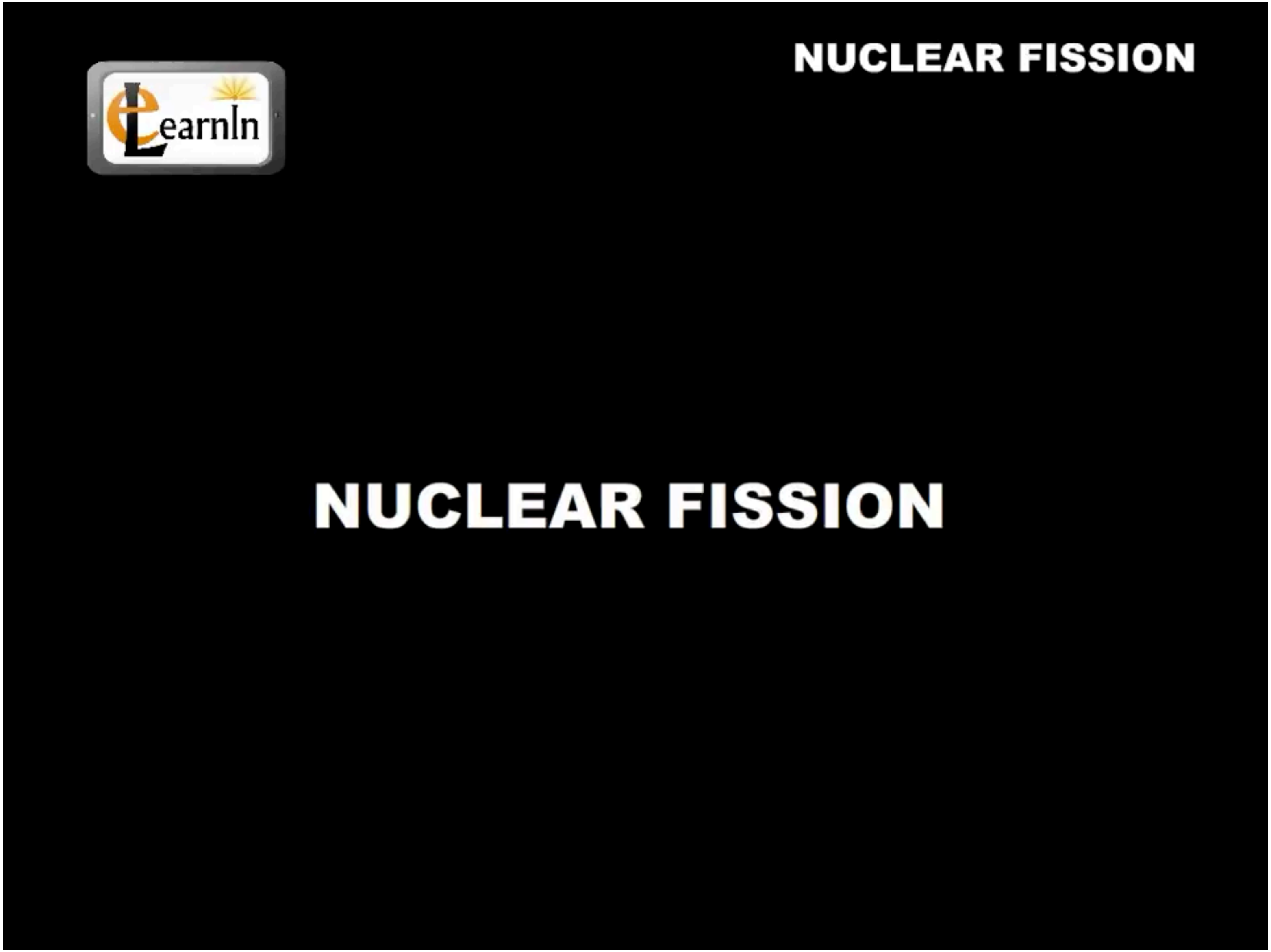
- Fission

Nuclear stability

Nuclear stability

Decay Type	Radiation Emitted	Generic Equation	Model
Alpha decay	${}^{4}_{2}\alpha$	${}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}X' + {}^{4}_{2}\alpha$	
Beta decay	${}^{0}_{-1}\beta$	${}^{A}_{Z}X \longrightarrow {}_{Z+1}{}^{A}X' + {}^{0}_{-1}\beta$	
Positron emission	${}^{0}_{+1}\beta$	${}^{A}_{Z}X \longrightarrow {}_{Z-1}{}^{A}X' + {}^{0}_{+1}\beta$	
Electron capture	X rays	${}^{A}_{Z}X + {}^{0}_{-1}e \longrightarrow {}_{Z-1}{}^{A}X' +$ X ray	
Gamma emission	${}^{0}_{0}\gamma$	${}^{A}_{Z}X^{*} \xrightarrow{\text{Relaxation}} {}^{A}_{Z}X' + {}^{0}_{0}\gamma$	
Spontaneous fission	Neutrons	${}^{A+B+C}_{Z+Y}X \longrightarrow {}^{A}_{Z}X' + {}^{B}_{Y}X' + C{}^{1}_{0}n$	

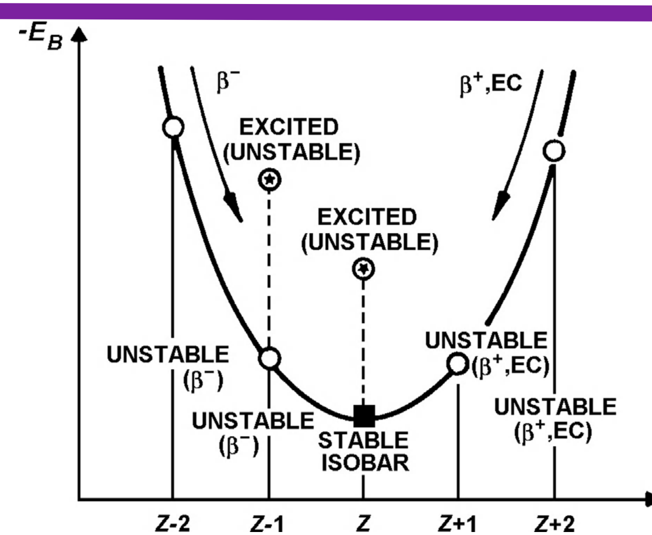
Nuclear stability

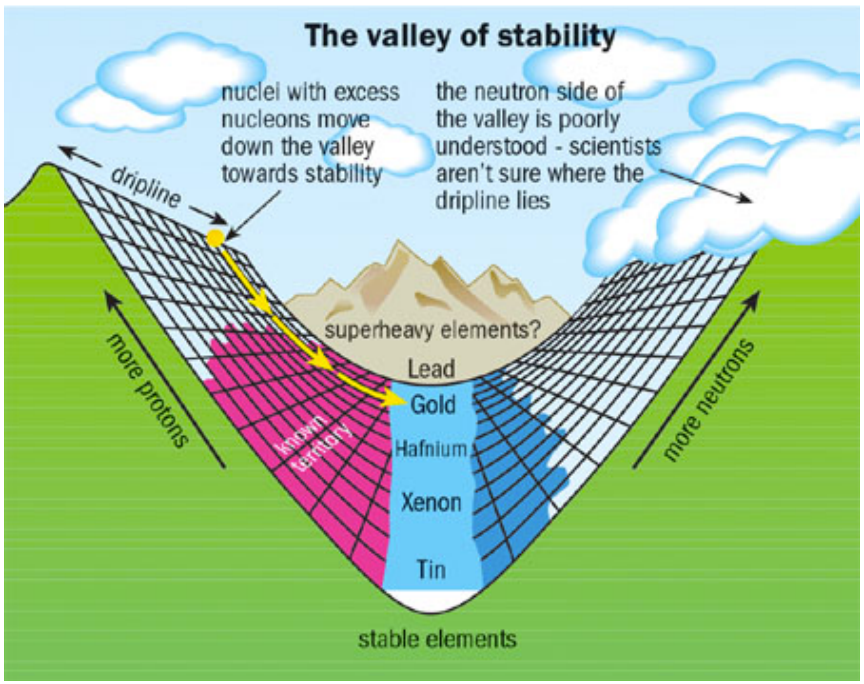


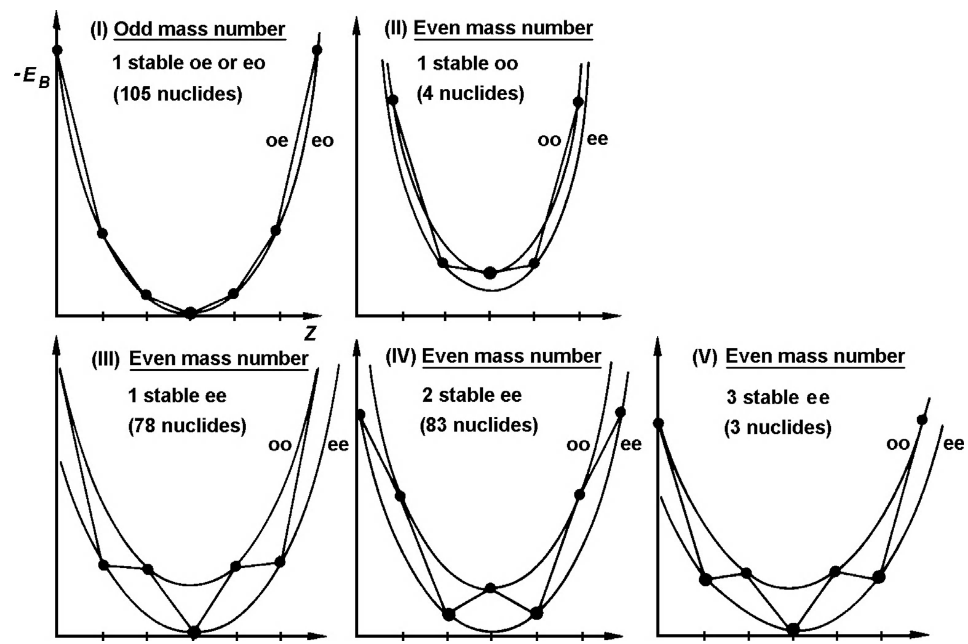
➢ Nuclear stability

$$Z_{\min} = \frac{[m_n - m(^1\mathrm{H})]c^2 + a_C A^{-1/3} + 4a_{\mathrm{sym}}}{2a_C A^{-1/3} + 8a_{\mathrm{sym}} A^{-1}}$$

$$Z_{\min} \approx \frac{A}{2}\frac{1}{1 + (1/4)(a_C/a_{\mathrm{sym}})A^{2/3}}$$

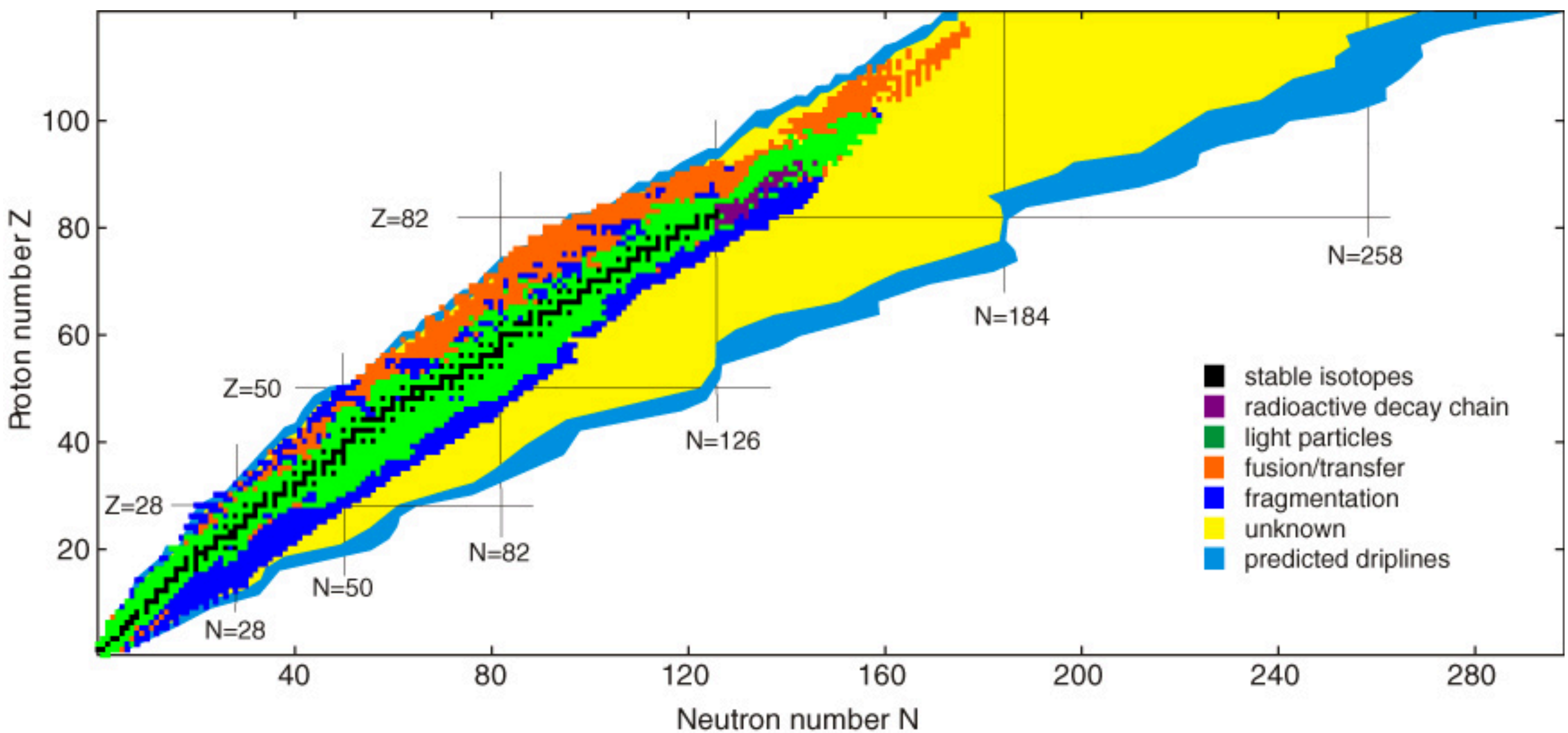






Type of nuclide by stability class	Number of nuclides in class	Running total of nuclides in all classes to this point	Notes
Theoretically stable to according to the Standard Model	90	90	Includes first 40 elements. If protons decay, then there are no stable nuclides.
Theoretically stable to alpha decay, beta decay, isomeric transition, and double beta decay but not spontaneous fission, which is possible for "stable" nuclides ⩾ niobium–93	56	146	Note that spontaneous fission has never been observed for nuclides with mass number < 230.
Energetically unstable to one or more known decay modes, but no decay yet seen. Considered stable until radioactivity confirmed.	106 [*citation needed*]	252	Total is the observationally stable nuclides.
Radioactive primordial nuclides.	34	286	Includes Bi, Th, U
Radioactive nonprimordial, but naturally occurring on Earth.	~61 significant	~347 significant	Cosmogenic nuclides from cosmic rays; daughters of radioactive primordials such as francium, etc.

Nuclear stability



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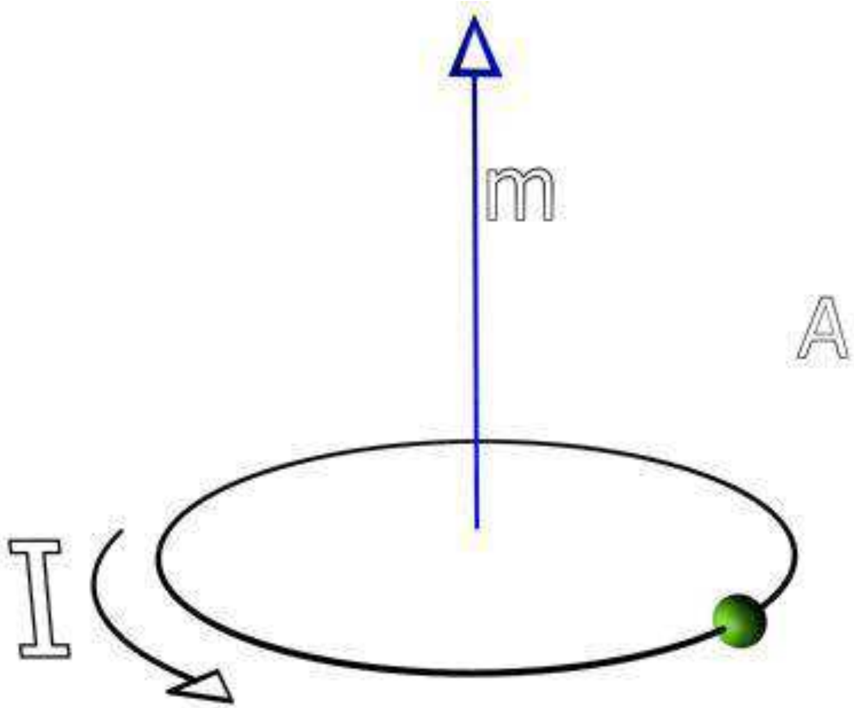
Symbol	meaning
l_i	orbital angular momentum of the i'th nucleon
s_i	intrinsic spin angular momentum of the i'th nucleon
j_i	total angular momentum of the i'th nucleon, *i.e.* $\vec{j_i} = \vec{l_i} + \vec{s_i}$
$\vec{L}$	sum of all orbital angular momenta in a nucleus *i.e.* $\vec{L} = \sum_{i=1}^{A} \vec{l_i}$
$\vec{S}$	sum of all intrinsic spin angular momenta in a nucleus *i.e.* $\vec{S} = \sum_{i=1}^{A} \vec{s_i}$

➢ Nuclear angular momentum

$$\vec{I} = \sum_{i=1}^{A} (\vec{l_i} + \vec{s_i}) = \vec{L} + \vec{S}$$

$$\begin{aligned}
\langle \vec{I}^2 \rangle &= \hbar^2 I(I+1) \\
I &= 0, \frac{1}{2}, 1, \frac{3}{2} \cdots \\
\langle I_z \rangle &= \hbar m_I \\
m_I &: -I \le m_I \le I \\
\Delta m_I &: \text{integral jumps}
\end{aligned}$$

$$\begin{aligned}
I &= n\hbar \quad \text{for odd-odd nuclei} \\
I &= (n + \frac{1}{2})\hbar \quad \text{for odd-A nuclei} \\
I &= 0 \quad \text{for even-even nuclei}
\end{aligned}$$



➢ **Nuclear magnetic moment**

$$\vec{\mu} = \frac{1}{2}\int \mathrm{d}\vec{x}\ \vec{x} \times J(\vec{x})$$

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- **Nuclear magnetic moment**

$$\mu_z = g^l l_z + g^s s_z$$

so

$$\mu = g^l j + (g^s - g^l)\langle s_z \rangle$$

finally

$$\mu = g^l\left(j - \frac{1}{2}\right) + \frac{1}{2}g^s, \quad j = l + \frac{1}{2}$$

$$\mu = g^l \frac{j(j+\frac{3}{2})}{j+1} - \frac{j g^s}{2(j+1)}, \quad j = l - \frac{1}{2}$$

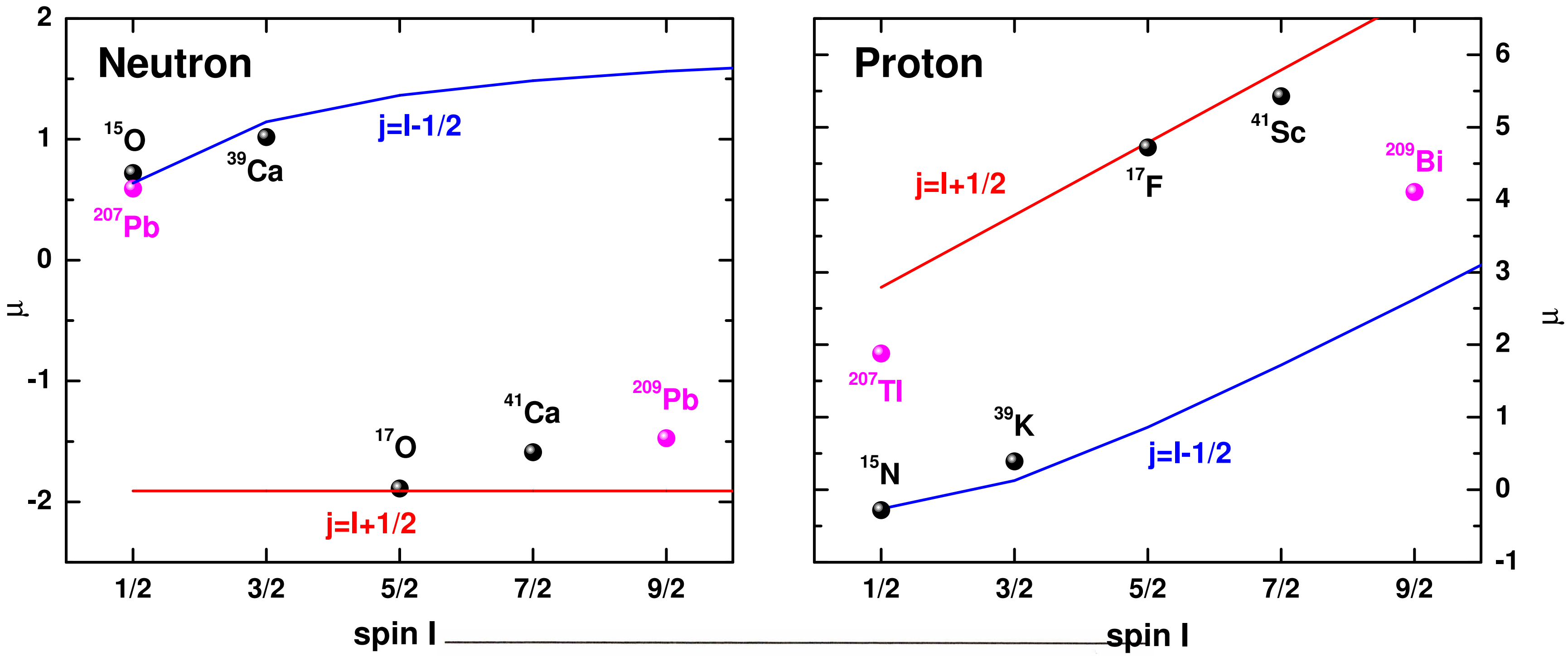
where, we used

$$\langle s_z \rangle = \left\langle \frac{(\vec{j}\cdot\vec{s}) j_z}{j^2} \right\rangle$$

$$= \frac{j}{2j(j+1)}[j(j+1) - l(l+1) + s(s+1)]$$

$$|ljm> = \sum_{m_l m_s} | < l m_l \frac{1}{2} m_s | jm > |l m_l > |\frac{1}{2} m_s >$$

Nuclear magnetic moment



Nuclide	$\mu(\mu_N)$
n	−1.9130418
p	+2.7928456
^{2}H (D)	+0.8574376
^{17}O	−1.89379
^{57}Fe	+0.09062293
^{57}Co	+4.733
^{93}Nb	+6.1705

Nuclear quadrupole moment

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- **Electrostatic potential of nuclei**

$$V(\vec{x}) = \frac{Ze}{4\pi\epsilon_0}\int \mathrm{d}\vec{x}' \, \frac{\rho_p(\vec{x}')}{|\vec{x}-\vec{x}'|}$$

- **Taylor expansion**

$$\begin{aligned} V(\vec{x}) &= \frac{Ze}{4\pi\epsilon_0}\left[\frac{1}{|\vec{x}|}\int \mathrm{d}\vec{x}' \, \rho_p(\vec{x}') + \frac{\vec{x}}{|\vec{x}|^3}\cdot\int \mathrm{d}\vec{x}' \, \vec{x}'\rho_p(\vec{x}') + \right. \\ &\left. \frac{1}{2|\vec{x}|^5}\int \mathrm{d}\vec{x}' \, \left(3(\vec{x}\cdot\vec{x}')^2 - |\vec{x}|^2|\vec{x}'|^2\right)\rho_p(\vec{x}')\cdots\right] \end{aligned}$$

It can be simplified

$$V(\vec{x}) = \frac{Ze}{4\pi\epsilon_0}\left[\frac{1}{|\vec{x}|} + \frac{Q}{2|\vec{x}|^3}\cdots\right]$$

where

$$Q = \int \mathrm{d}\vec{x} \, \left(3z^2 - r^2\right)\rho_p(\vec{x})$$

- **Nuclear quadrupole moment**

$$Q = \int \mathrm{d}\vec{x}\ \psi^*_N(\vec{x})(3z^2 - r^2)\psi_N(\vec{x})$$

- **For j=l+1/2 case**

$$\psi = \frac{u_l(r)}{r} Y_{ll}(\theta, \phi)\chi_{\text{spin}}\chi_{\text{isospin}}$$

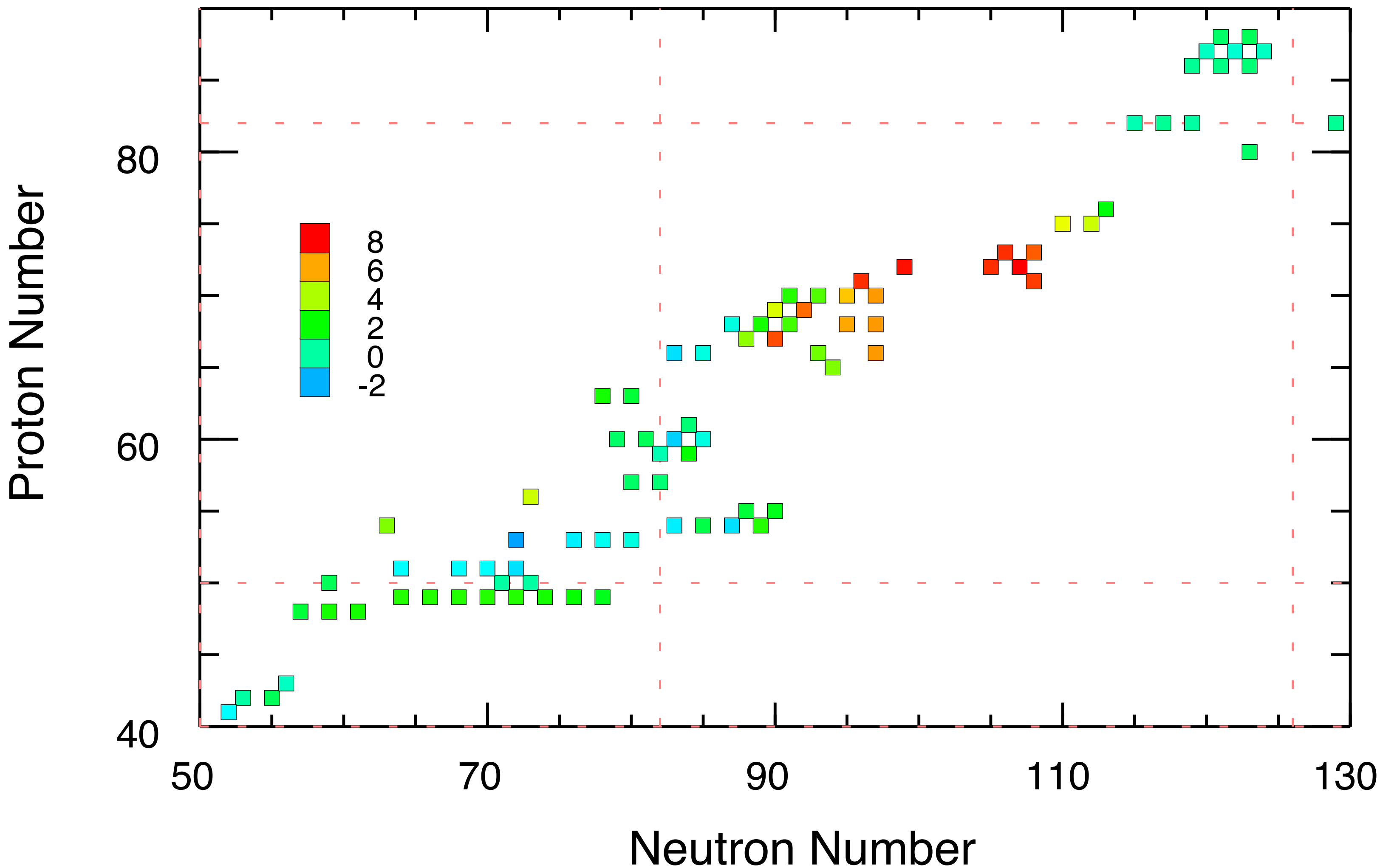
so

$$Q = \int u_l^2(r)|Y_{ll}(\theta, \phi)|^2 r^2 (3\cos^2\theta - 1)\sin\theta dr d\theta d\phi$$

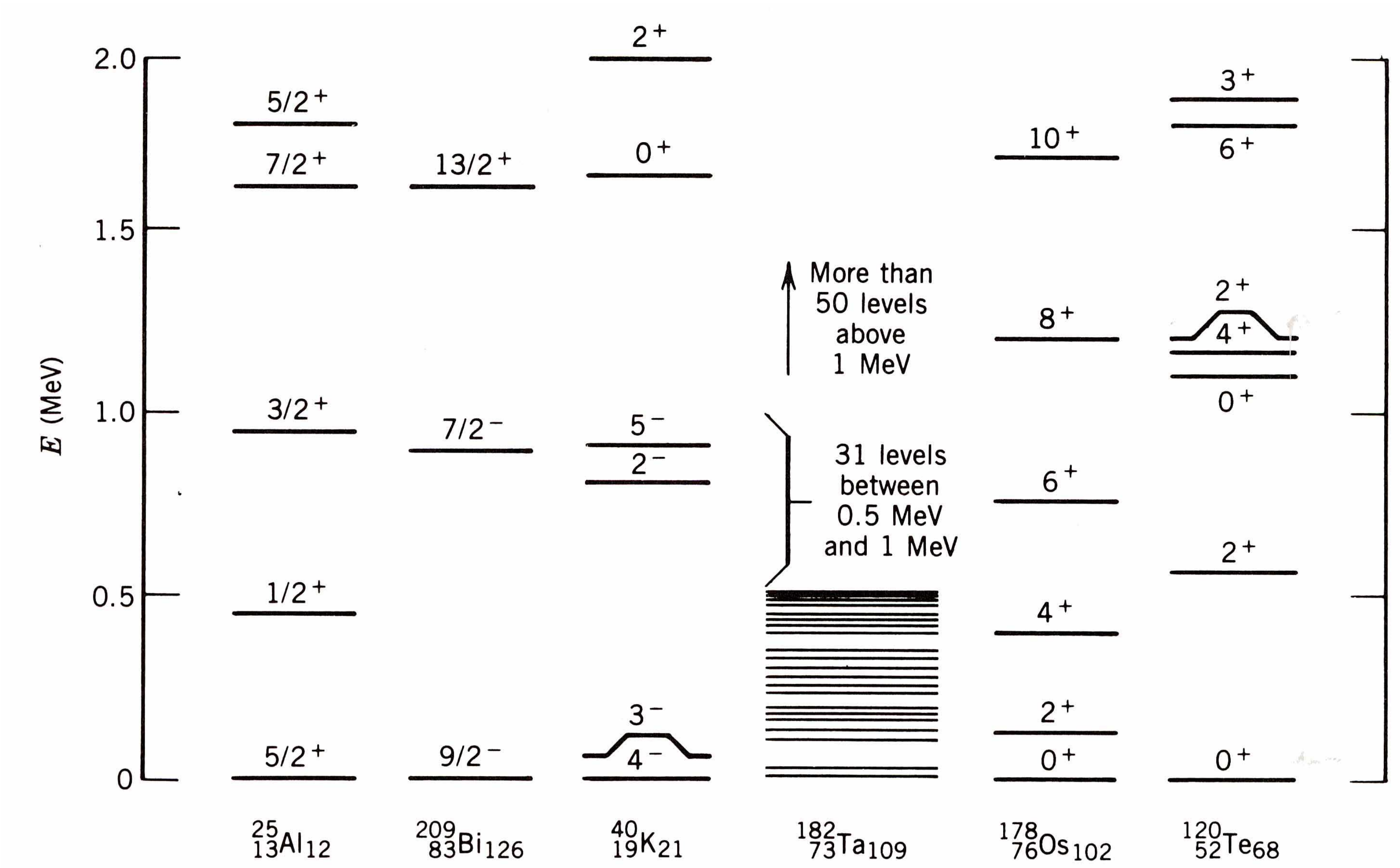
finally

$$Q = -\langle r^2 \rangle \frac{2j-1}{2j+2}$$

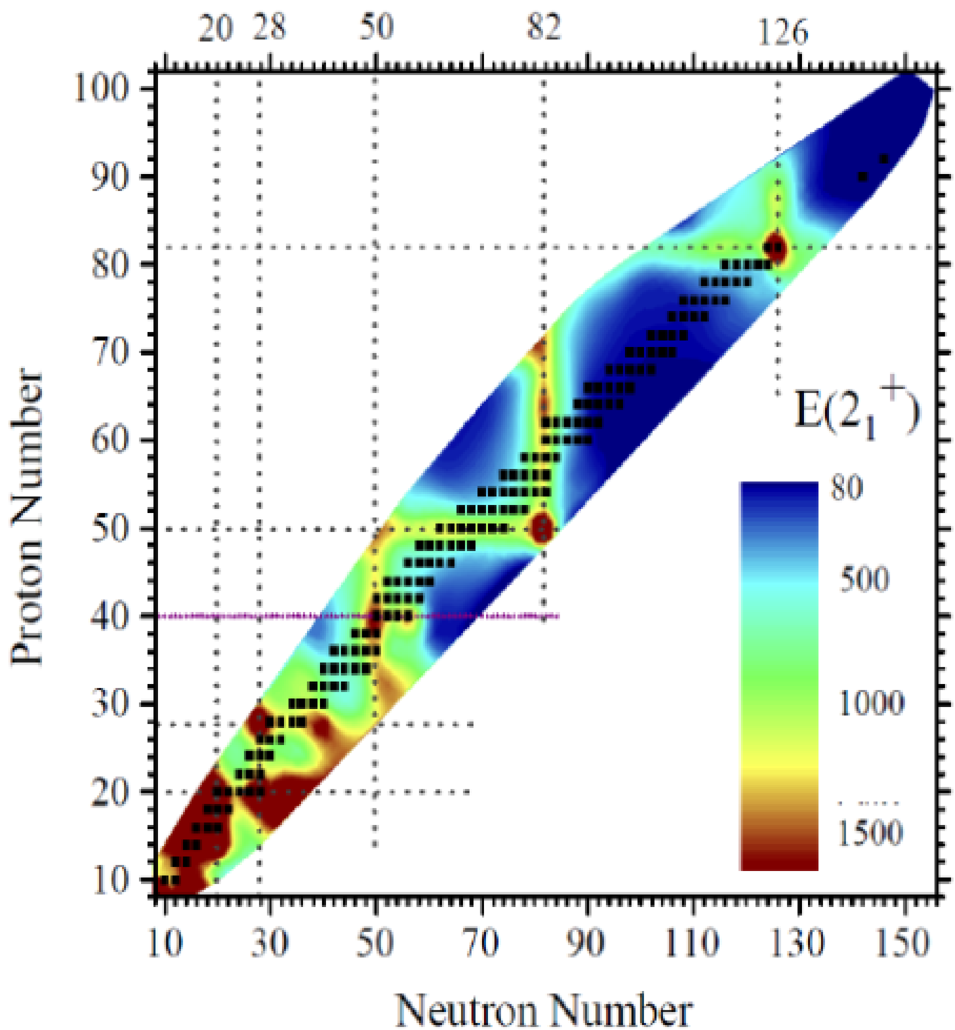
Nuclear quadrupole moment

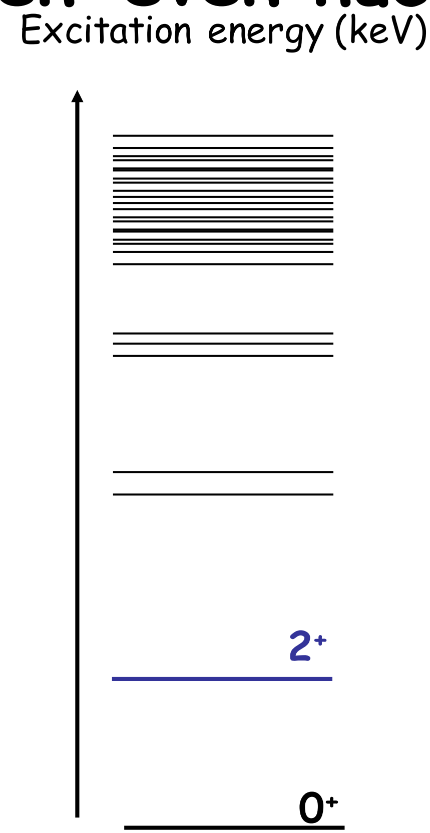


Excited state of nuclei



First excited state of even-even nuclei





Ground state Configuration.
Spin/parity $I^{\pi}=0^{+}$;
E_x = 0 keV

The basic concept of the Fermi-gas model

The theoretical concept of a Fermi-gas may be applied for systems of weakly interacting fermions, i.e. particles obeying Fermi-Dirac statistics leading to the Pauli exclusion principle ➔

• Simple picture of the nucleus:

— Protons and neutrons are considered as moving freely within the nuclear volume. The binding potential is generated by all nucleons

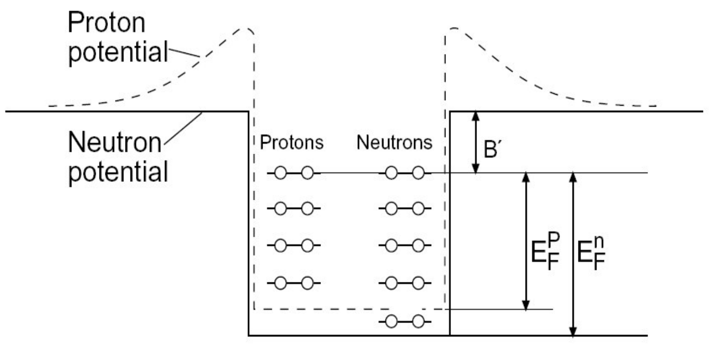
— In a first approximation, these nuclear potential wells are considered as rectangular: it is constant inside the nucleus and stops sharply at its edge

— Neutrons and protons are distinguishable fermions and are therefore situated in two separate potential wells

— Each energy state can be ocupied by two nucleons with different spin projections

— All available energy states are filled by the pairs of nucleons ➔ no free states , no transitions between the states

— The energy of the highest occupied state is the Fermi energy E_F



— The difference *B'* between the top of the well and the Fermi level is constant for most nuclei and is just the average binding energy per nucleon B'/A = 7–8 MeV.

Nuclear mean potential

Single particle equation in mean field

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r})\right)\psi_i(\boldsymbol{r}) = \varepsilon_i\,\psi_i(\boldsymbol{r})$$

Square-well potential

$$V(r) = \begin{cases} -V_0 & \text{for} \quad r \le R \\ \infty & \text{for} \quad r > R \end{cases}$$

Woods-Saxon potential

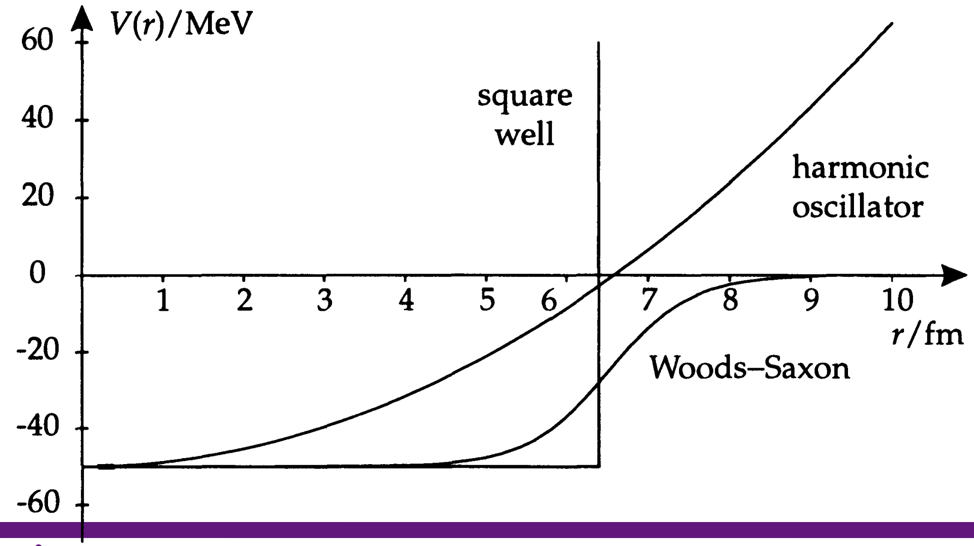
$$V(r) = -\frac{V_0}{1+\exp[(r-R)/a]}$$

where $V_0 \sim$ -50 MeV, R~1.1 fm $A^{1/3}$, $a \sim$ 0.5 fm

Harmonics-oscillator potential

$$V(r) = \tfrac{1}{2} m\omega^2 r^2$$

where ω~ 41 MeV $A^{-1/3}$



Nuclear shell model

Schroedinger equation with harmonics oscillator potential

$$H_{ho}\psi = (-\frac{\boldsymbol{\nabla}^2}{2m} + \frac{1}{2}m\omega^2 r^2)\psi = E\psi$$

Kinetic energy in spherical coordinate

$$-\frac{1}{2m}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]$$

Wave function

$$\psi(r,\theta,\varphi) = R(r)Y(\theta,\varphi)$$

Angular equation

$$\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} + \lambda\right]Y(\theta,\varphi) = 0$$

Solution of angular equation

$$Y_{lm}(\theta,\varphi) = (-)^m\left[\frac{(2l+1)(l-m)!}{4\pi(l+m)!}\right]^{1/2} P_l^m(\cos\theta)e^{im\varphi} \qquad \lambda = l(l+1)$$

Nuclear shell model

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Radial Schoedinger equation

$$\left[\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)+2m\left(E-\frac{m\omega^2r^2}{2}\right)-\frac{\lambda}{r}\right]R(r)=0$$

Variable substitution

$$\frac{1}{2}\left[-\frac{1}{\rho^2}\frac{d}{d\rho}\left(\rho^2\frac{d}{d\rho}\right)+\rho^2+\frac{l(l+1)}{\rho}\right]R(\rho)=\tilde{E}R(\rho)$$

with $\rho = r/b$, $b = \sqrt{\hbar\omega}$ **and** $\tilde{E} = E/\hbar\omega.$

Wave function

$$R_{nl}(r)=\left[\frac{2n!}{b^3\Gamma(n+l+\frac{3}{2})}\right]^{1/2}\left(\frac{r}{b}\right)^l e^{-r^2/2b^2}L_n^{l+1/2}(r^2/b^2)$$

Eigenengery

$$E_{nl}=(2n+l+\frac{3}{2})\hbar\omega$$

Nuclear shell model

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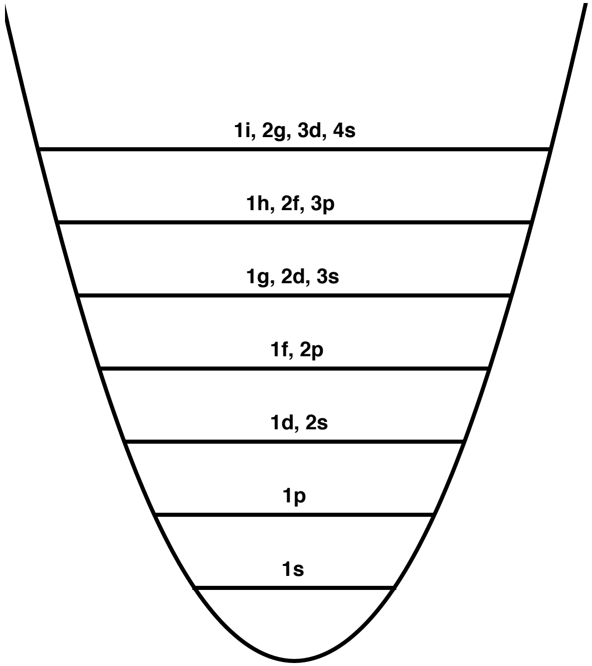
Nuclear parameters

$$\hbar\omega = 41A^{-1/3}\ \text{MeV}, \qquad b = 1.005A^{1/6}\ \text{fm}$$

New quantum number

$$E_{nl} = (N + \frac{3}{2})\hbar\omega \qquad\qquad N = 2n + l,$$



N	E_N	d_N	$\sum_N d_N$	$n(l)$	parity
0	$\frac{3}{2}\hbar\omega$	2	2	1s	+
1	$\frac{5}{2}\hbar\omega$	6	8	1p	-
2	$\frac{7}{2}\hbar\omega$	12	20	1d, 2s	+
3	$\frac{9}{2}\hbar\omega$	20	40	1f, 2p	-
4	$\frac{11}{2}\hbar\omega$	30	70	1g, 2d, 3s	+
5	$\frac{13}{2}\hbar\omega$	42	112	1h, 2f, 3p	-
6	$\frac{15}{2}\hbar\omega$	56	168	1i, 2g, 3d, 4s	+

The total degeneracy at level N

$$(N+1)(N+2)$$

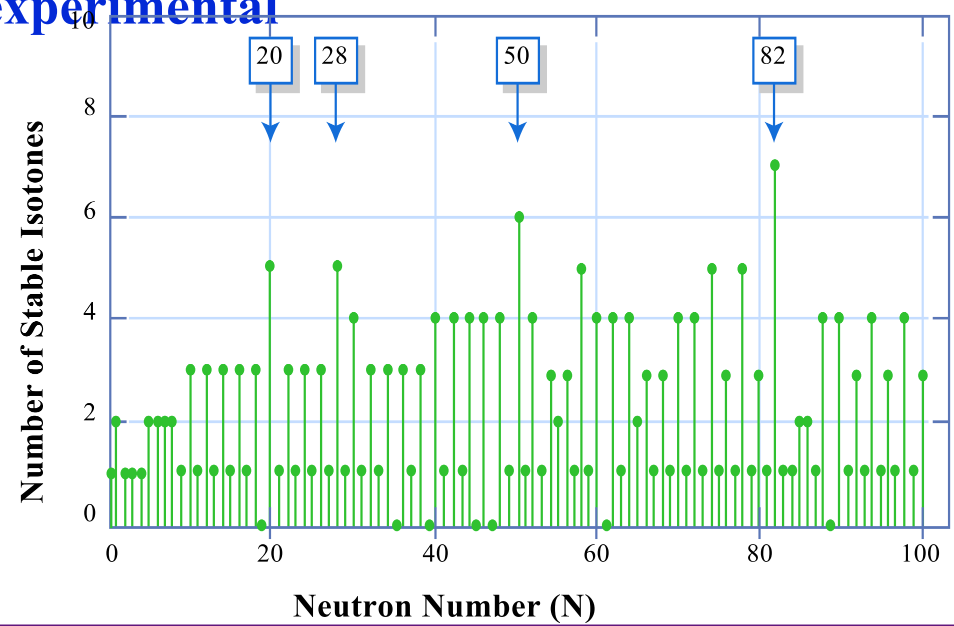
The magic number in harmonics oscillator

$$(N+1)(N+2)(N+3)/3$$

2, 8, 20, 40, 70, 112, ...

The magic number in experimental

2, 8, 20, 50, 82, 126, ...



Nuclear shell model

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Spin-orbit force

$$V_{so} = \zeta_{zo}\mathbf{l}\cdot\mathbf{s}$$

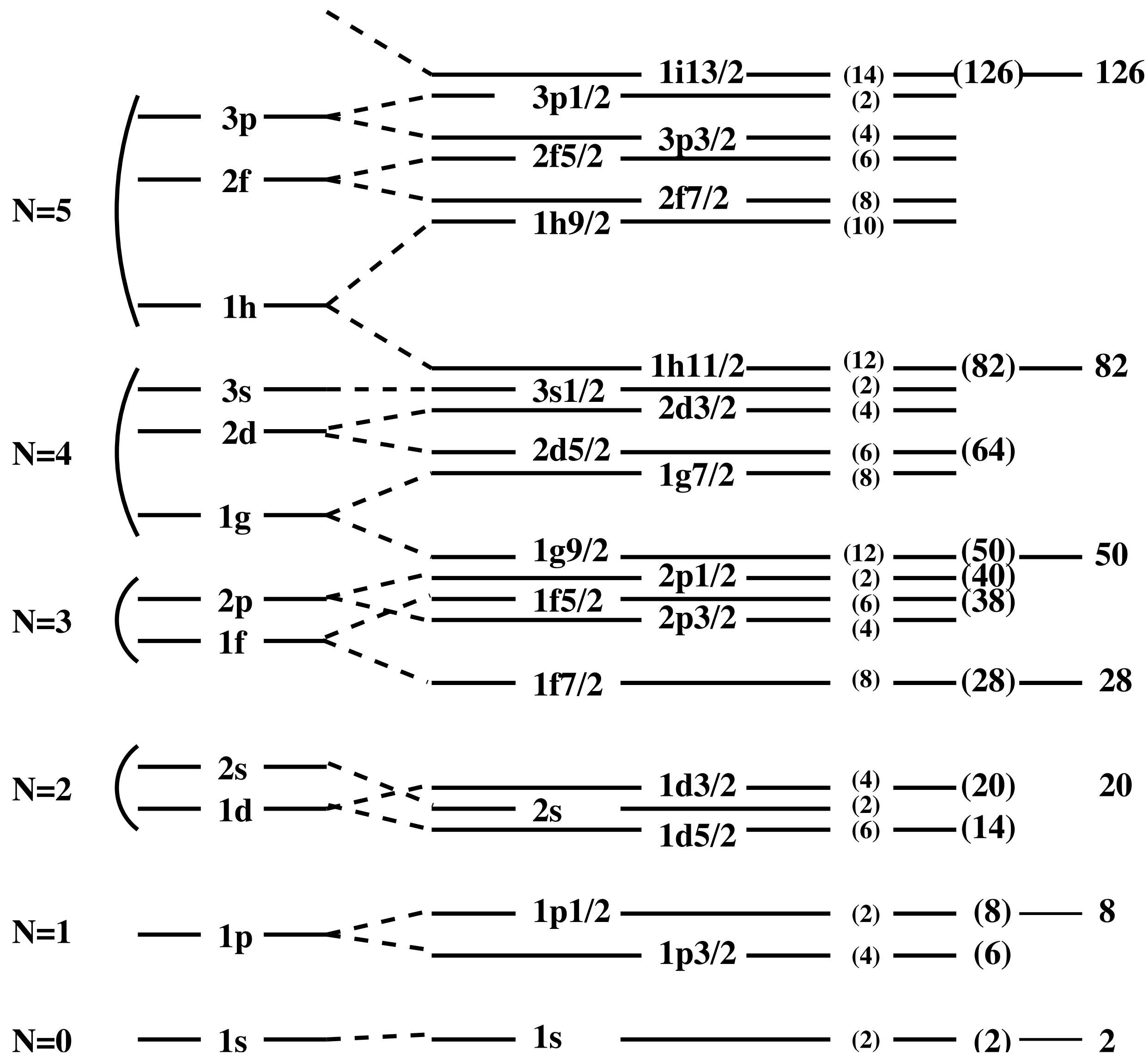
The expecting energy of spin-orbit force

$$\begin{aligned}\langle nljm|V_{so}|nljm\rangle &= \frac{1}{2}\langle\zeta_{so}(r)\rangle_{nl}\left[j(j+1)-l(l+1)-\frac{3}{4}\right]\\ &= \begin{cases} -\frac{1}{2}(l+1)\langle\zeta_{so}(r)\rangle_{nl} & \text{for } j=l-1/2\\ \frac{1}{2}l\langle\zeta_{so}(r)\rangle_{nl} & \text{for } j=l+1/2\end{cases}\end{aligned}$$

where

$$\langle\zeta(r)\rangle_{nl} = \int_0^\infty \zeta(r)R_{nl}^2(r)r^2dr$$

Nuclear shell model



Calculate the total number of possible microstates N for a given electron configuration. As before, we discard the filled (sub)shells, and keep only the partially filled ones. For a given orbital quantum number l, t is the maximum allowed number of electrons, t = 2(2l+1). If there are e electrons in a given subshell, the number of possible microstates is

$$N = \binom{t}{e} = \frac{t!}{e!\,(t-e)!}.$$

m										
1	↑↓		↑		↑	↑↓	↑	↑	↑	↑
0	↑	↑	↑↓	↑↓	↑			↓	↑	↓
-1		↑↓		↓	↓	↑	↑↓	↑	↑	↓
M_L	2	-2	1	-1	0	1	-1	0	0	0
M_S	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$
	$^2D_{\frac{5}{2},\frac{3}{2}}$					$^2P_{\frac{3}{2},\frac{1}{2}}$			$^4S_{\frac{3}{2}}$	