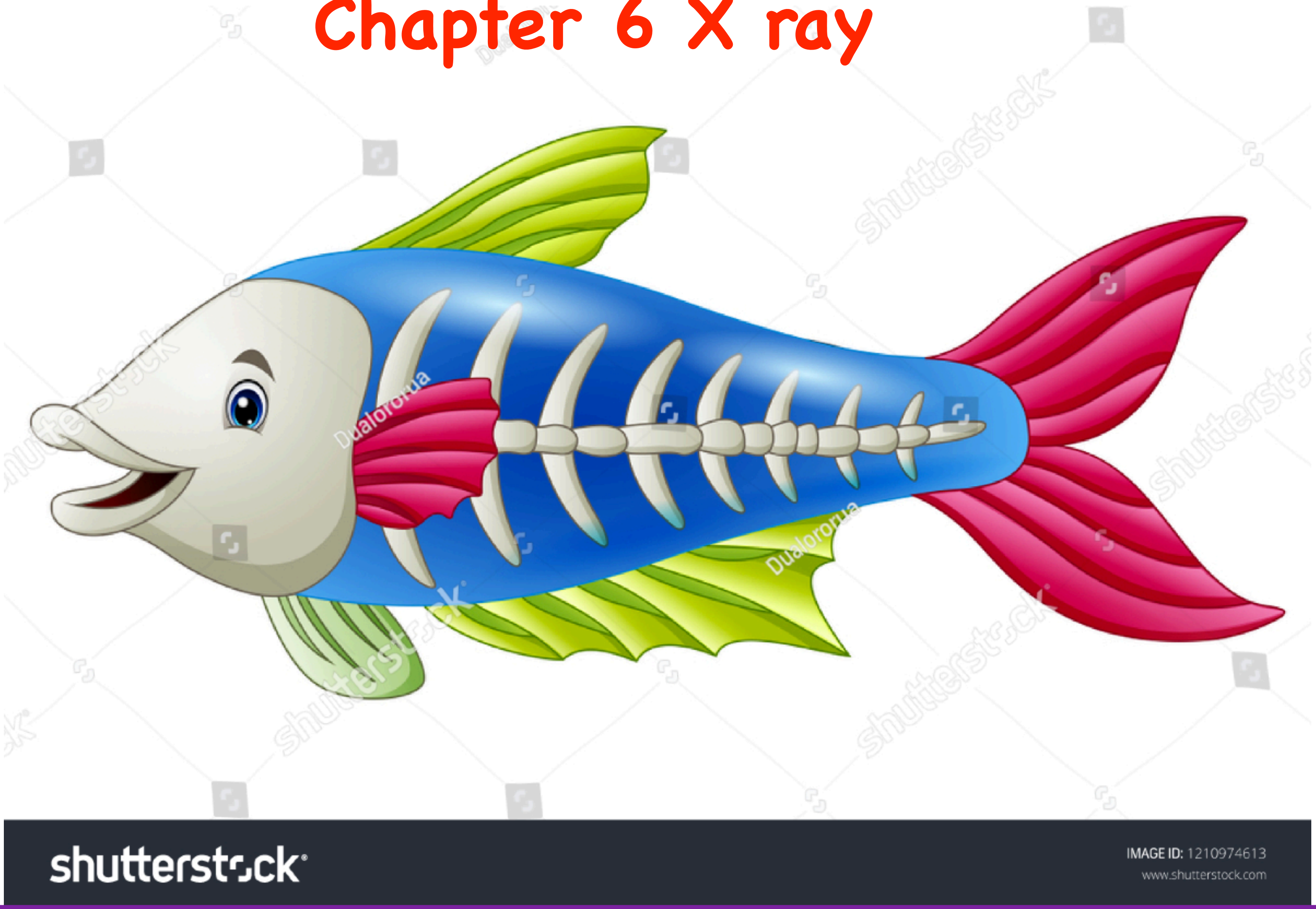
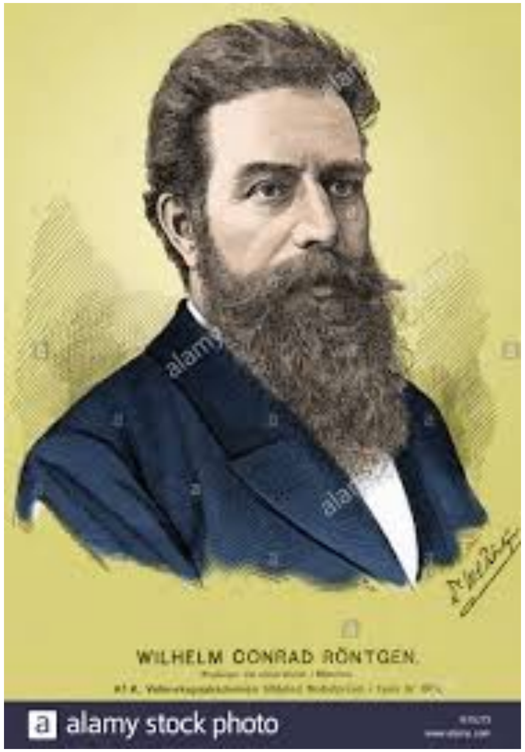
Atomic Physics

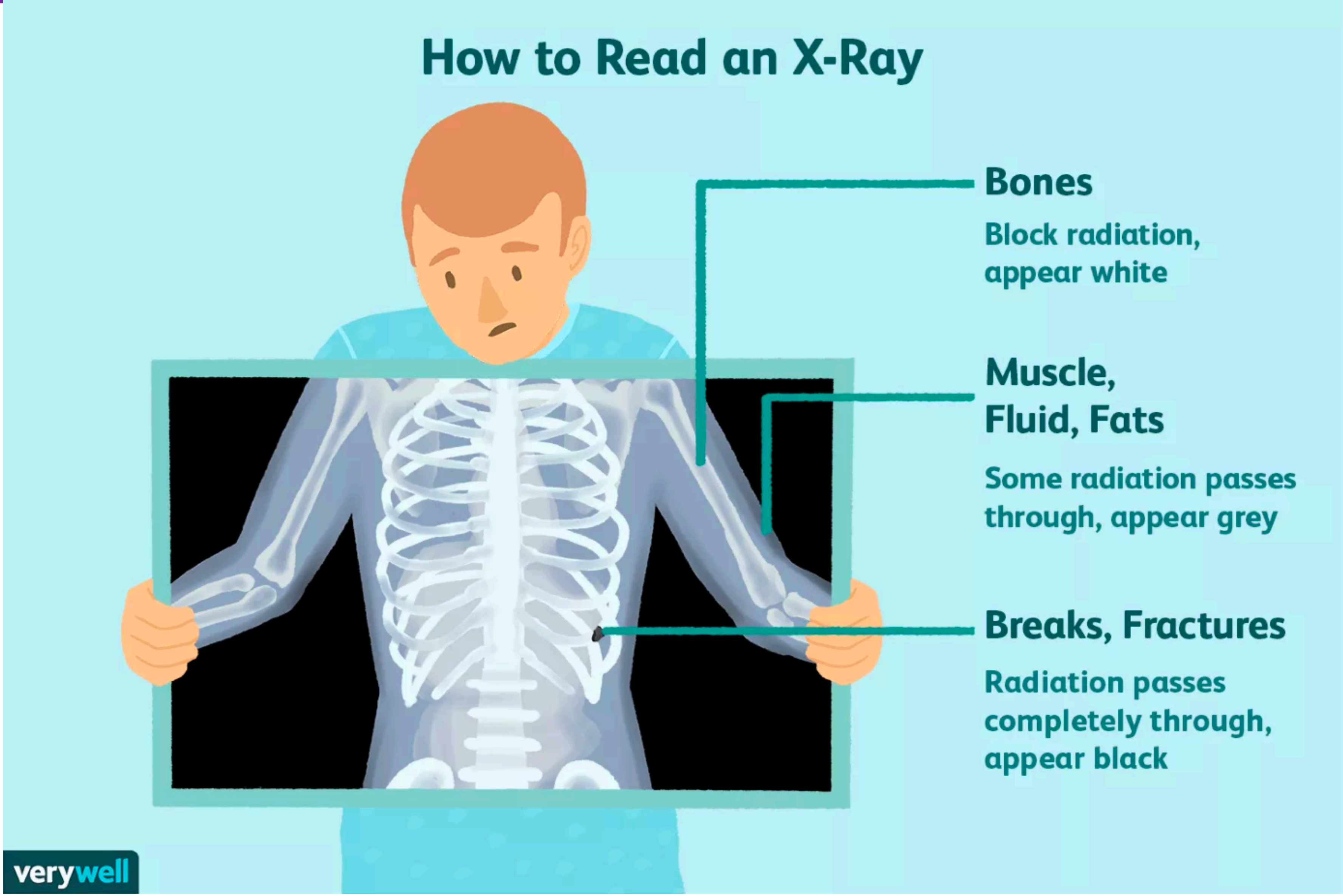
Chapter 6 X ray



6.1 The discovery of X ray

X-rays were discovered in 1895 by the German physicist Wilhelm Roentgen. He found that a beam of high-speed electrons striking a metal target produced a new and extremely penetrating type of radiation.





How to Read an X-Ray

Bones

Block radiation, appear white

Muscle, Fluid, Fats

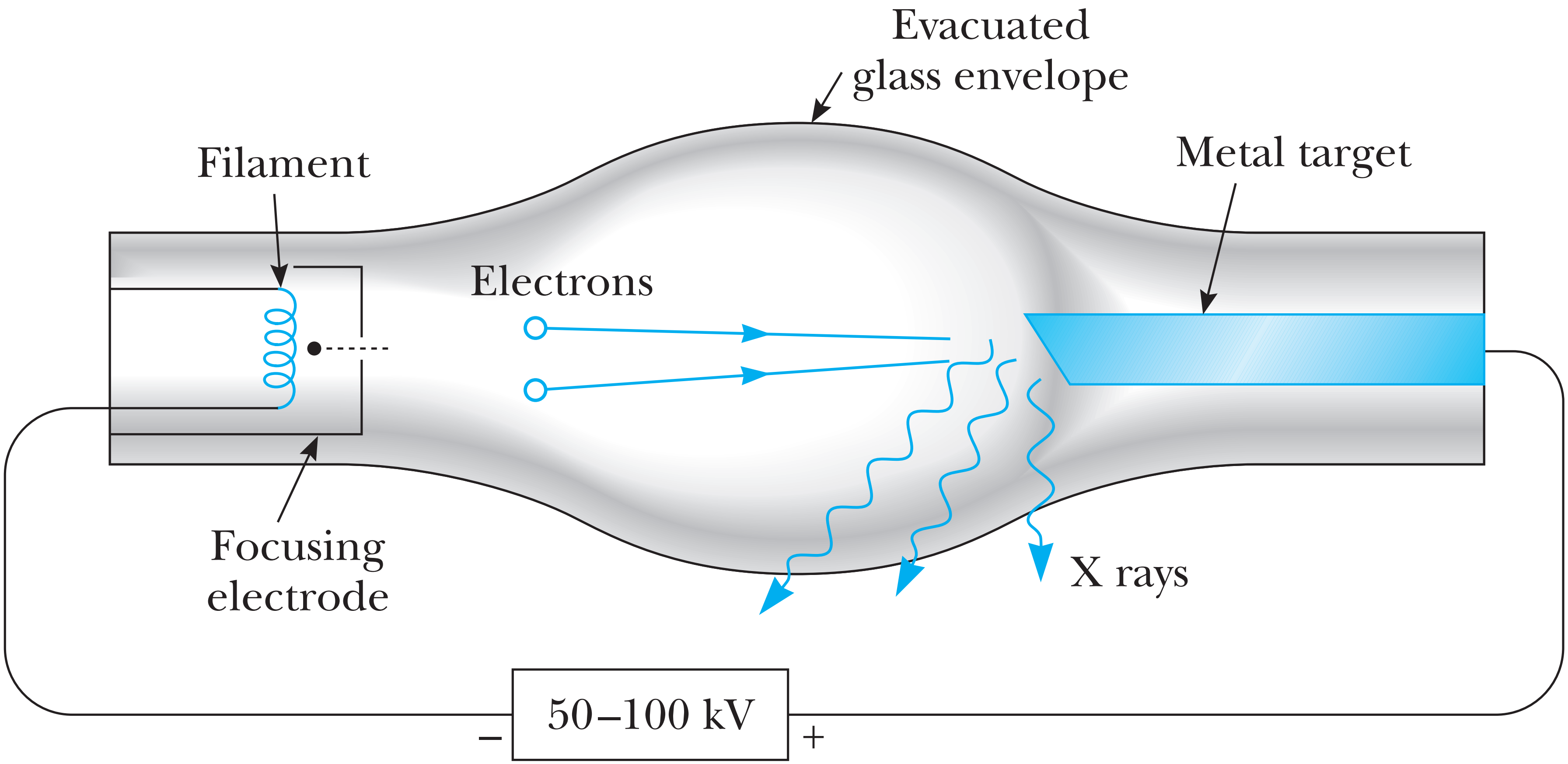
Some radiation passes through, appear grey

Breaks, Fractures

Radiation passes completely through, appear black

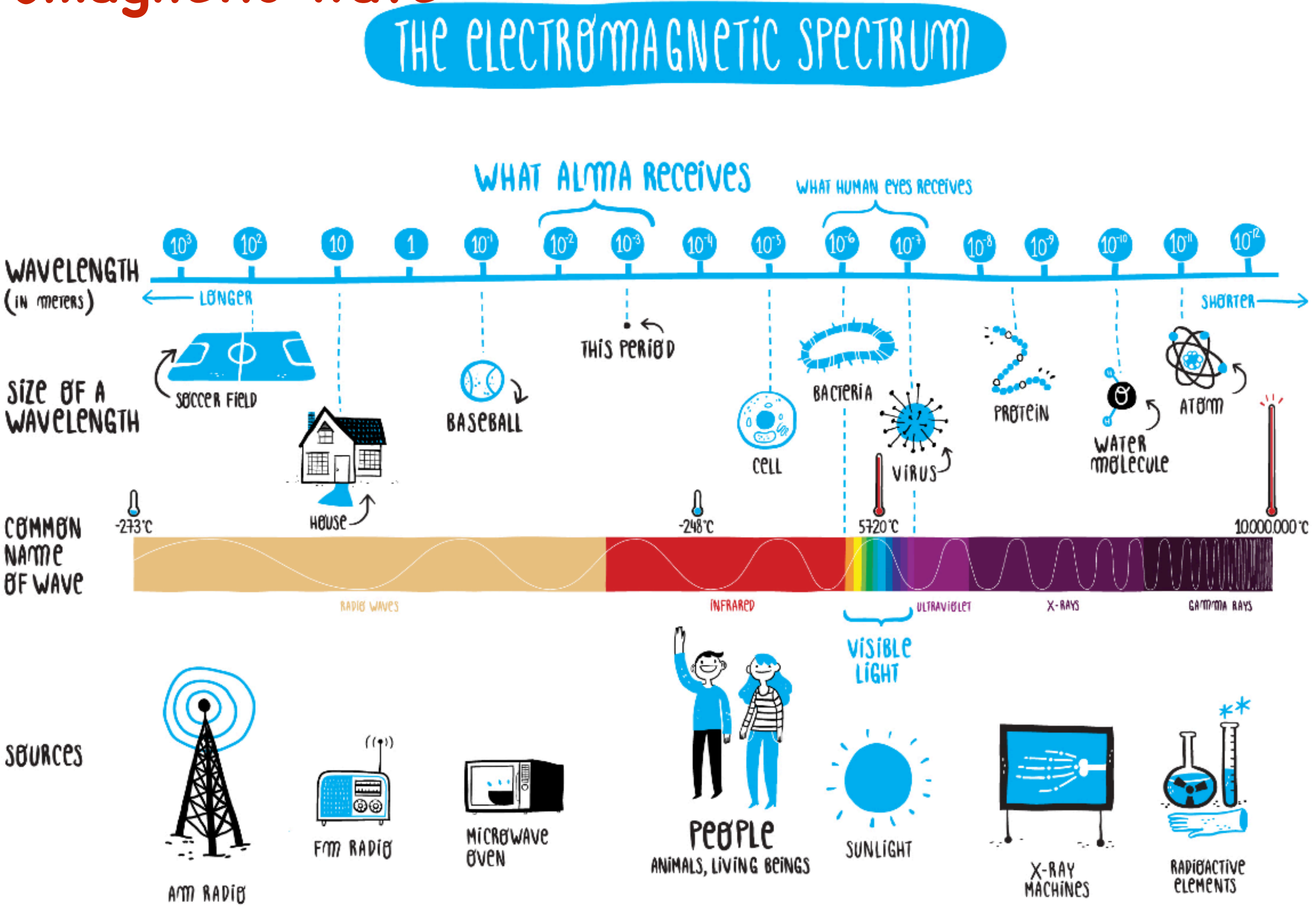
6.1 The discovery of X ray

X-rays Tube



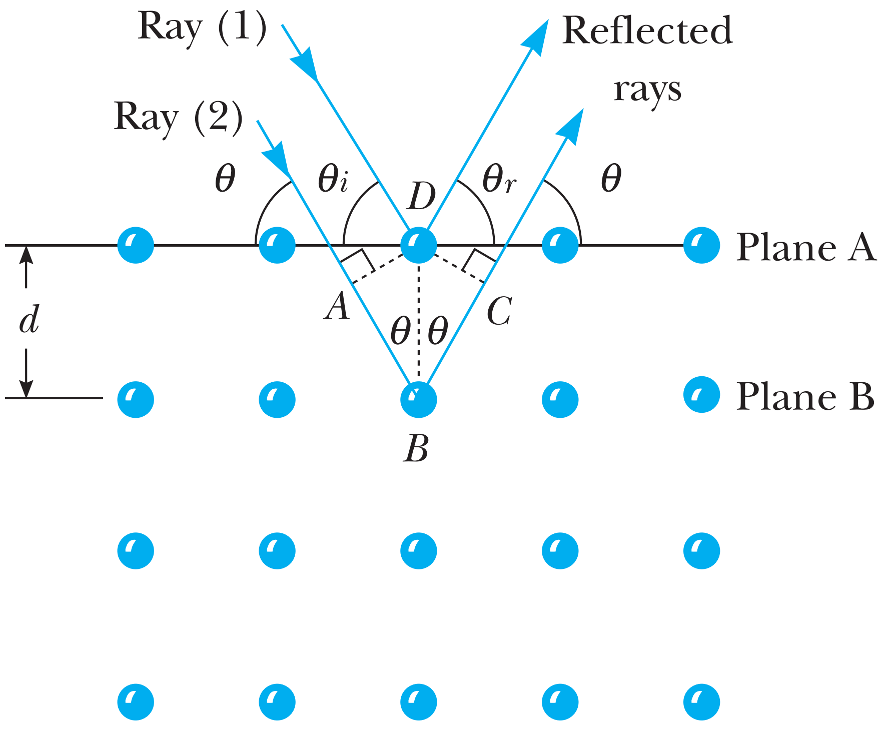
6.1 The discovery of X ray

Electromagnetic wave



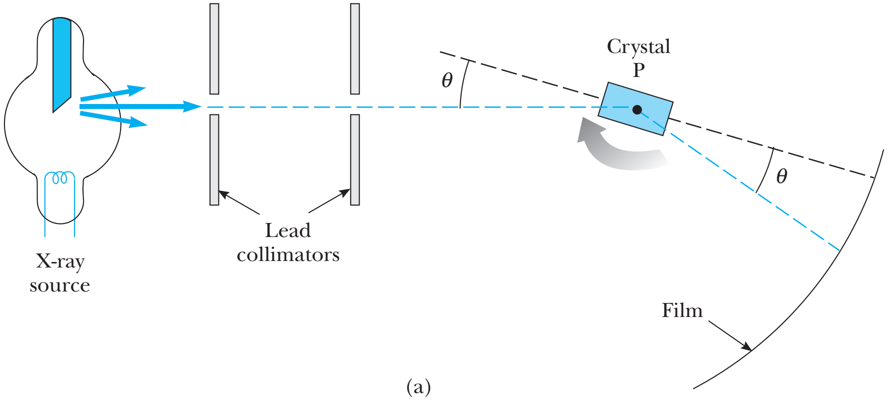
6.1 The discovery of X ray

X ray diffraction



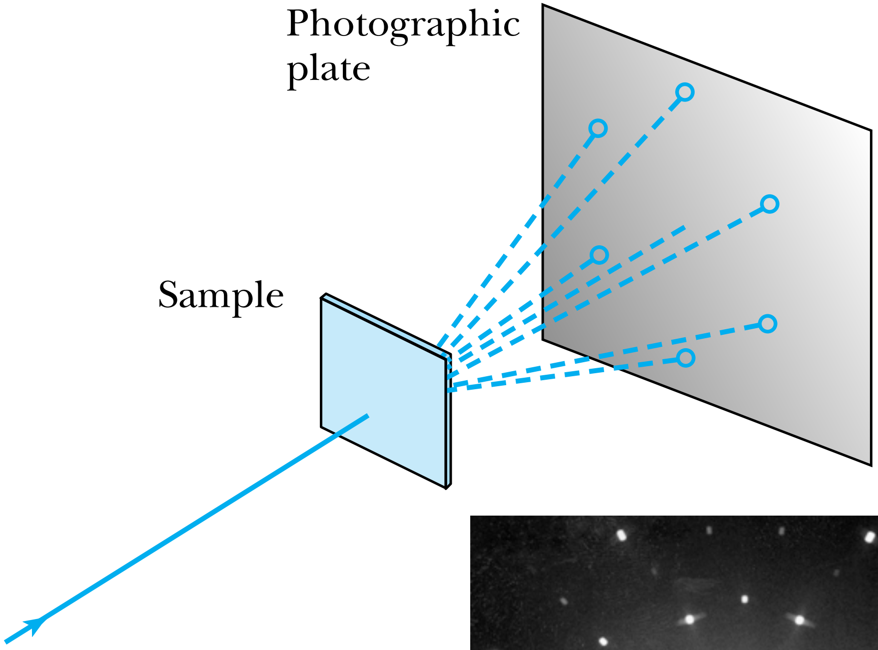
Bragg equation

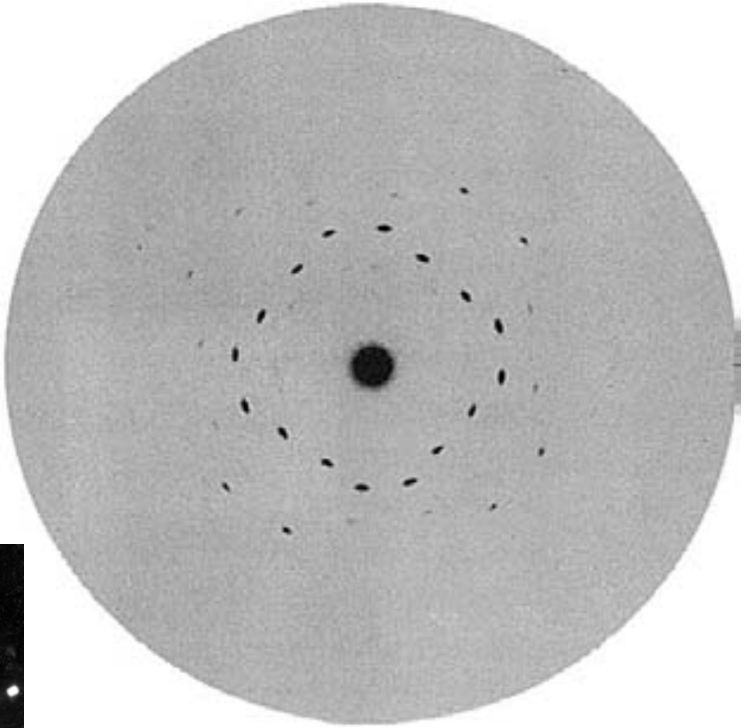
$$n\lambda = 2d\sin\theta \qquad n = 1, 2, 3, \ldots$$

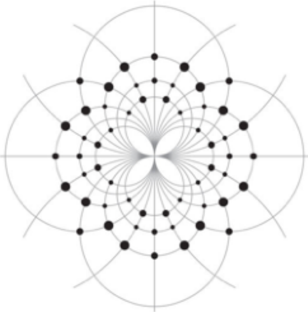


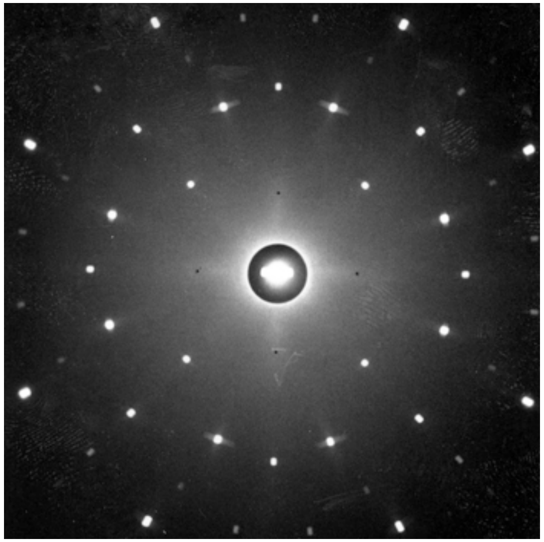
6.1 The discovery of X ray

Laue diffraction transmission method

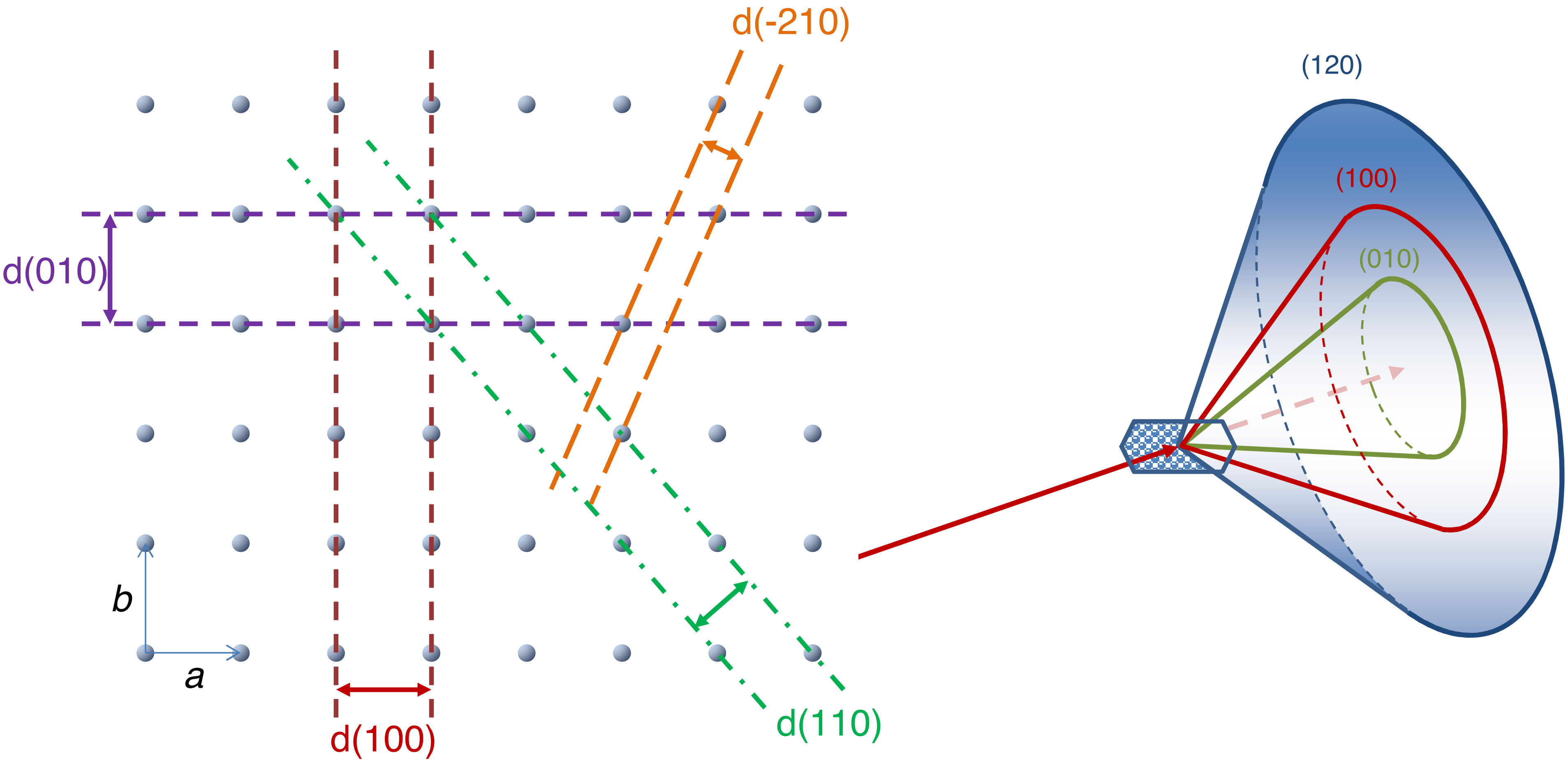








The diffraction of X ray

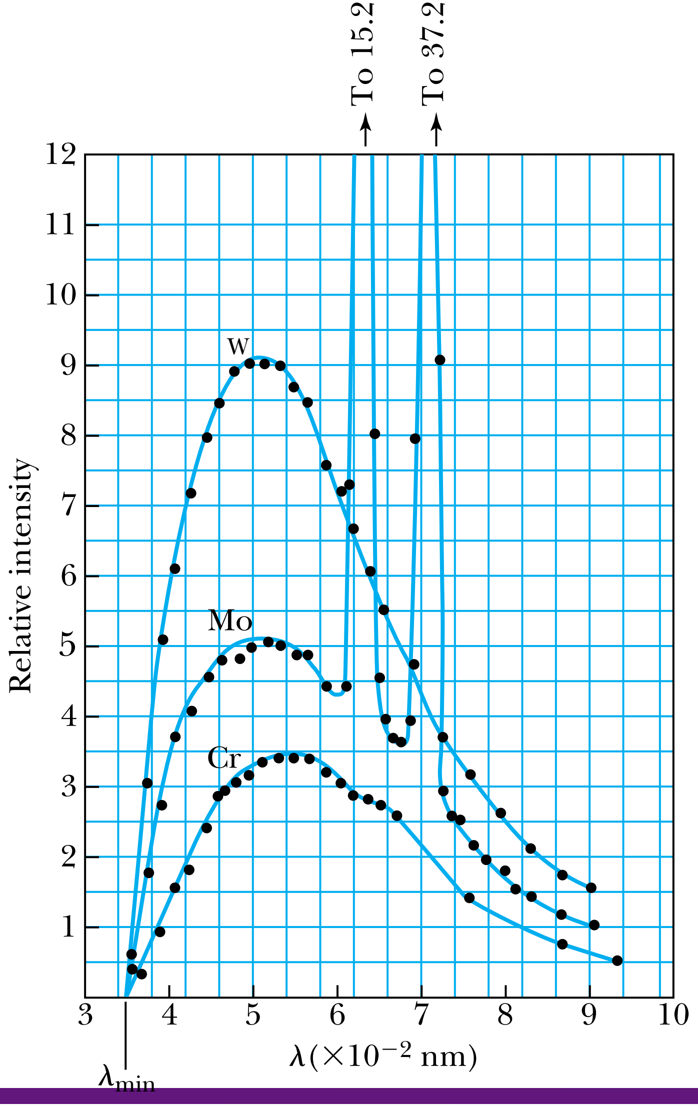


6.1 The discovery of X ray

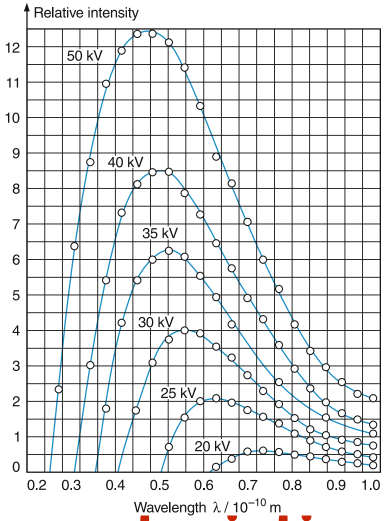
The polarization of X ray

6.2 X ray production

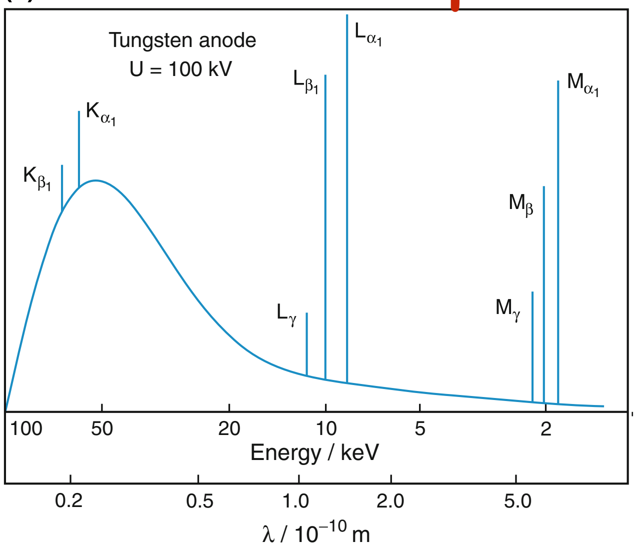
The spectrum of X ray



The continuous spectrum



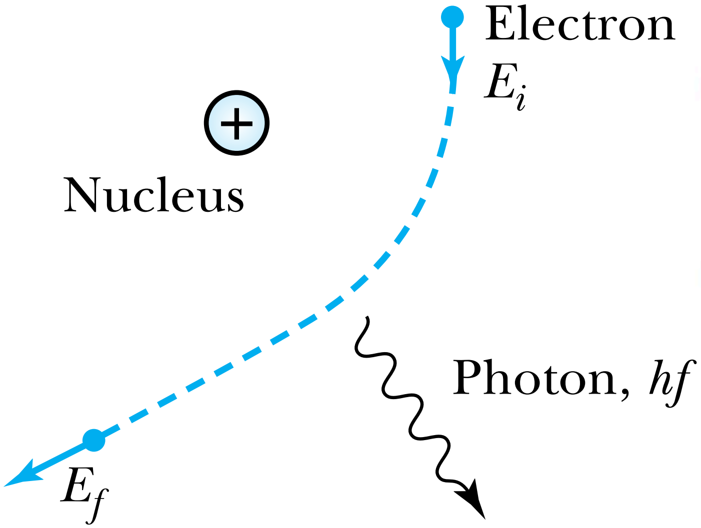
The characteristic spectrum

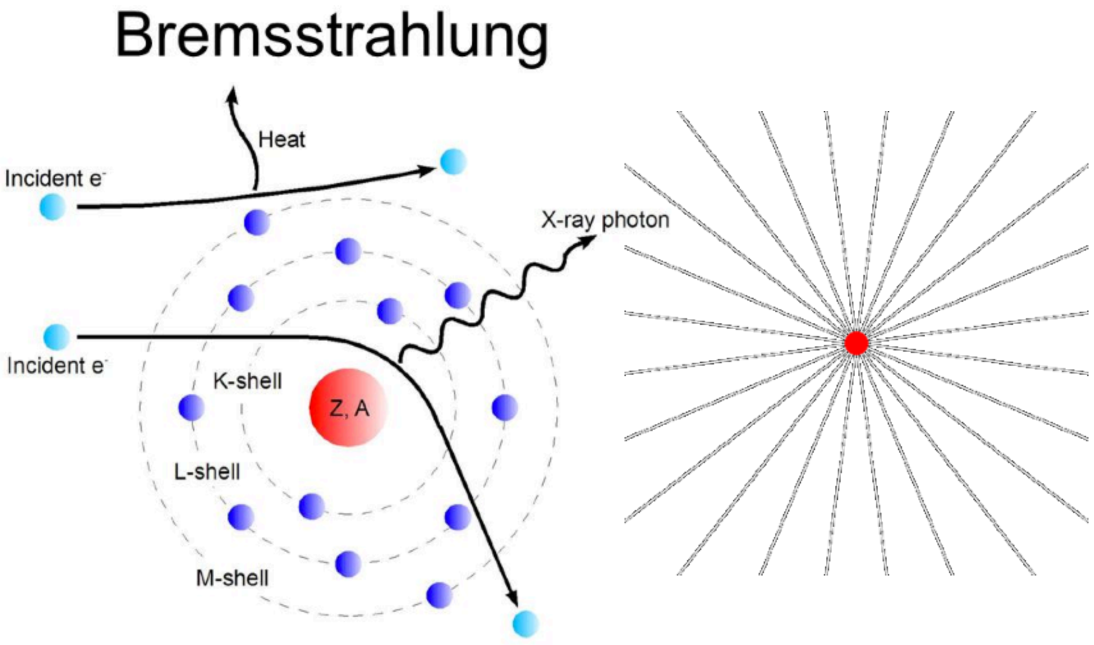


6.2 X ray production

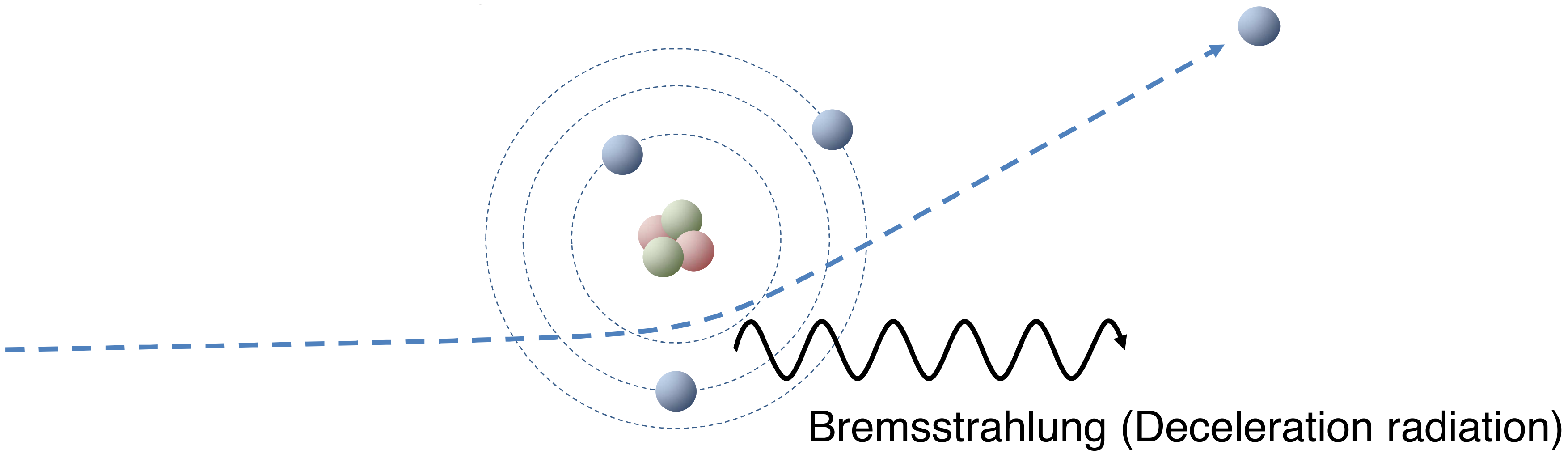
The continuous spectrum:

An energetic electron passing through matter will radiate photons and lose kinetic energy. The process by which photons are emitted by an electron slowing down is called bremsstrahlung, from the German word for "braking radiation."





The continuous spectrum



Electron is deflected and decelerated by the atomic nucleus. (Inelastic scattering).

Deflected electron emits electromagnetic radiation. Wavelength depends on the loss of energy.

6.2 X ray production

The minimum wavelength is due to the inverse photoelectric effect. The conservation of energy requires that the electron kinetic energy equal the maximum photon energy:

$$eV_0 = hf_{\max} = \frac{hc}{\lambda_{\min}}$$

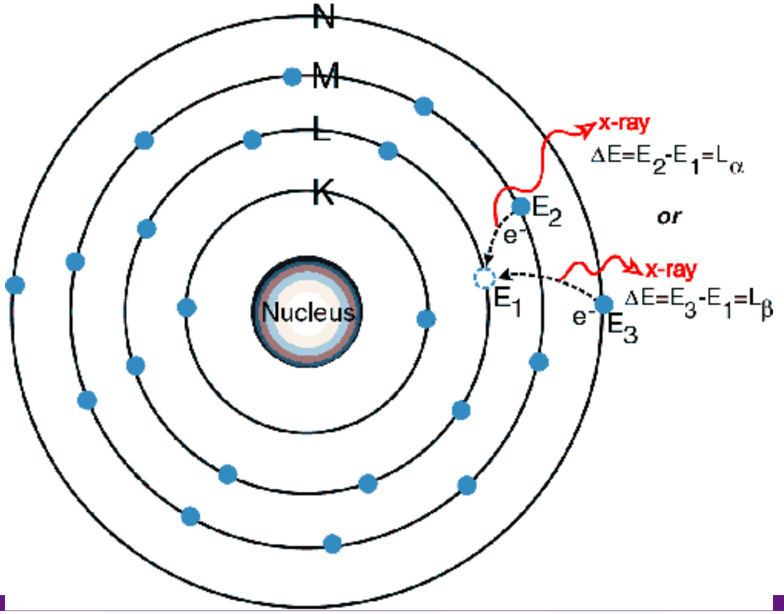
Therefore, the minimum wavelength (Duane-Hunt rule) is

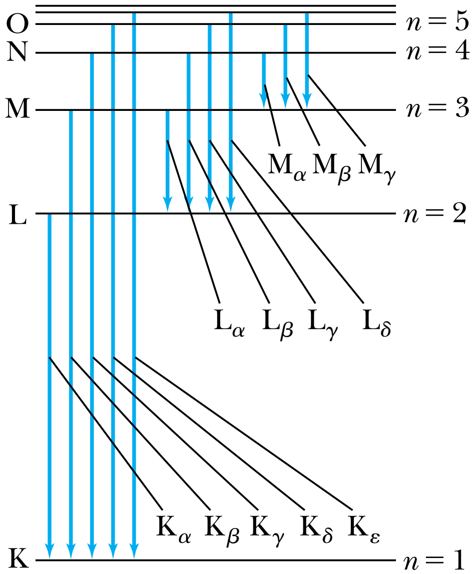
$$\lambda_{\min} = \frac{hc}{e}\frac{1}{V_0} = \frac{1.240 \times 10^{-6}\,\mathrm{V \cdot m}}{V_0}$$

6.2 X ray production

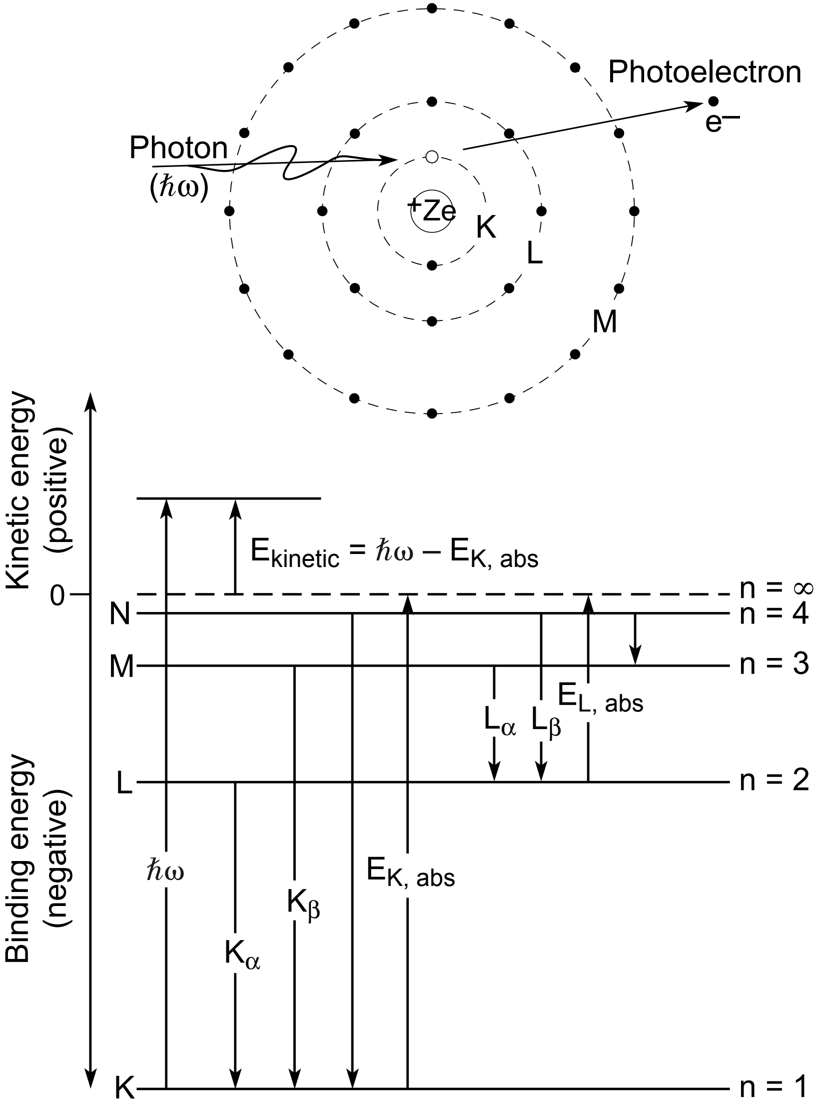
The characteristic spectrum:

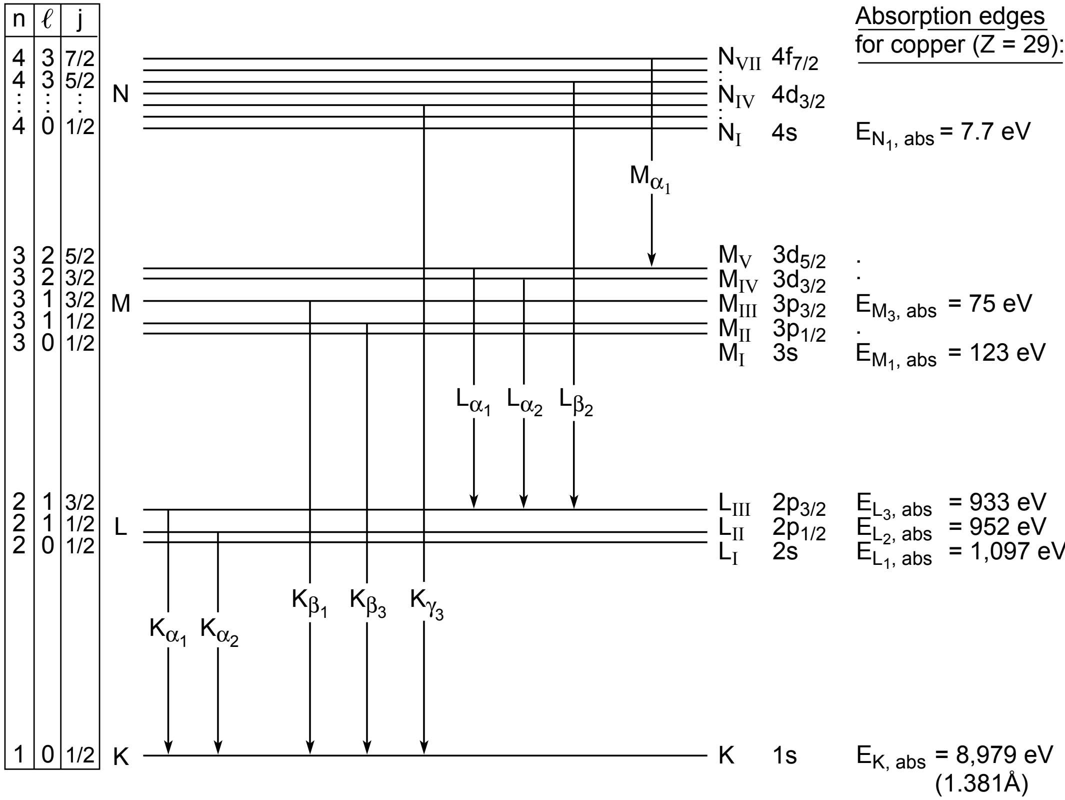
The atom is most stable in its lowest energy state or ground state, so it is likely that N an electron from one of the higher shells will change its state and fill the inner-shell vacancy at lower energy, emitting radiation as it changes its state.





The characteristic spectrum





Absorption edges for copper ($Z = 29$):

$E_{N_1,\text{ abs}}$ = 7.7 eV

$E_{M_3,\text{ abs}}$ = 75 eV

$E_{M_1,\text{ abs}}$ = 123 eV

$E_{L_3,\text{ abs}}$ = 933 eV

$E_{L_2,\text{ abs}}$ = 952 eV

$E_{L_1,\text{ abs}}$ = 1,097 eV

$E_{K,\text{ abs}}$ = 8,979 eV (1.381Å)

Cu K_{α_1} = 8,048 eV (1.541Å)

Cu K_{α_2} = 8,028 eV (1.544Å)

Cu K_{β_1} = 8,905 eV

Cu L_{α_1} = 930 eV

Cu L_{α_2} = 930 eV

Cu L_{β_1} = 950 eV

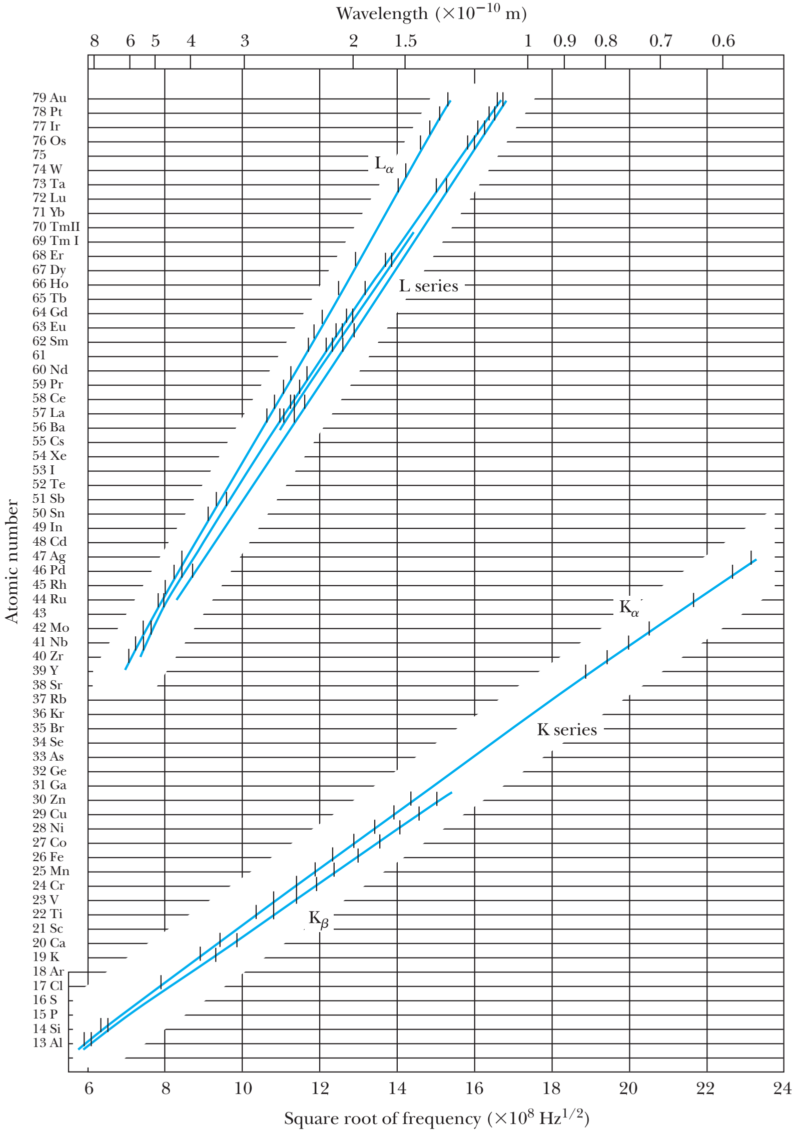
6.2 X ray production

Moseley formula

$$\frac{1}{\lambda_{K_\alpha}} = R(Z-1)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = \frac{3}{4}R(Z-1)^2$$

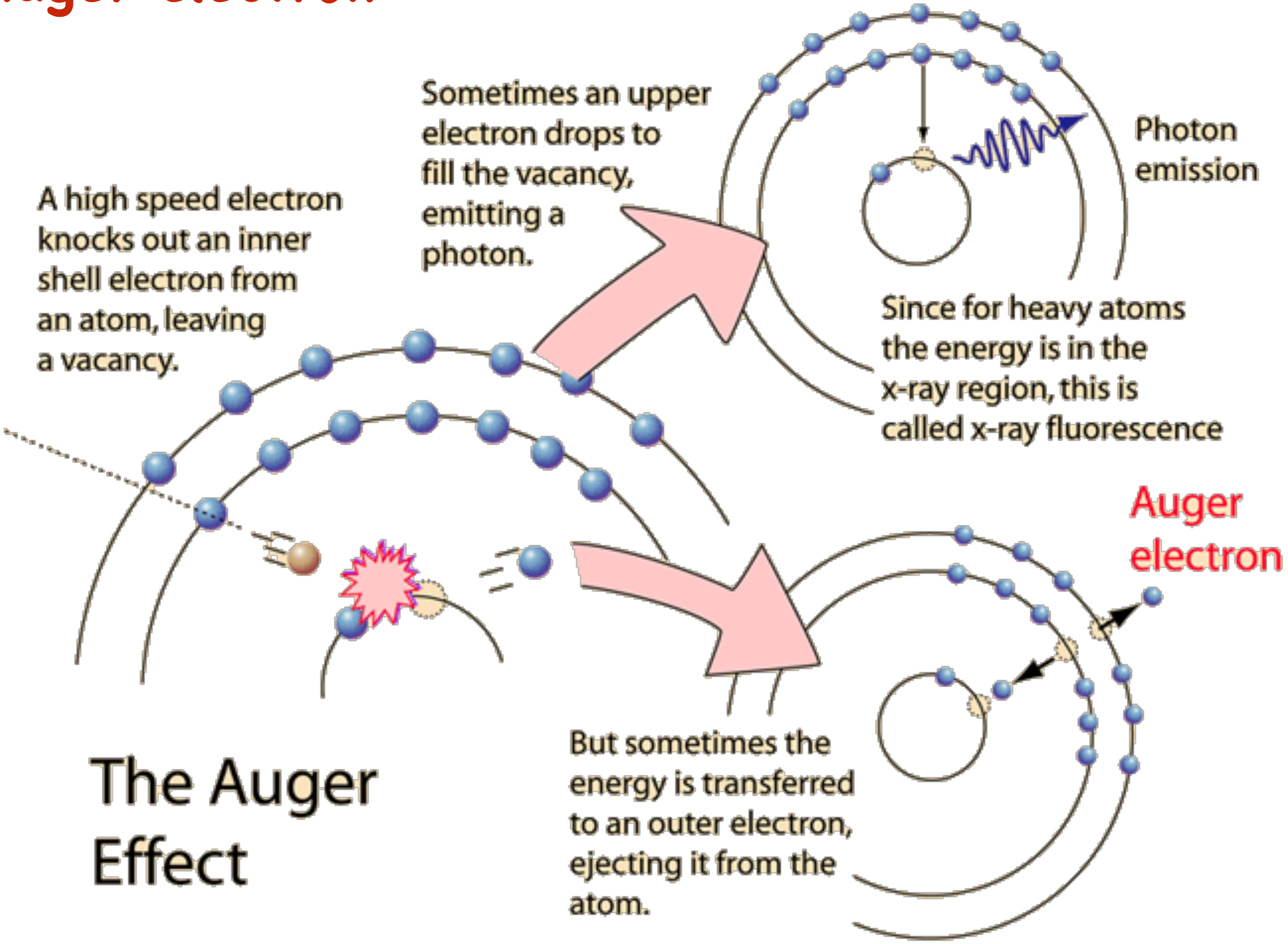
$$f_{K_\alpha} = \frac{c}{\lambda_{K_\alpha}} = \frac{3cR}{4}(Z-1)^2$$

$$\frac{1}{\lambda_K} = R(Z-1)^2\left(\frac{1}{1^2}-\frac{1}{n^2}\right) = R(Z-1)^2\left(1-\frac{1}{n^2}\right)$$



6.2 X ray production

Auger electron



The Auger Effect

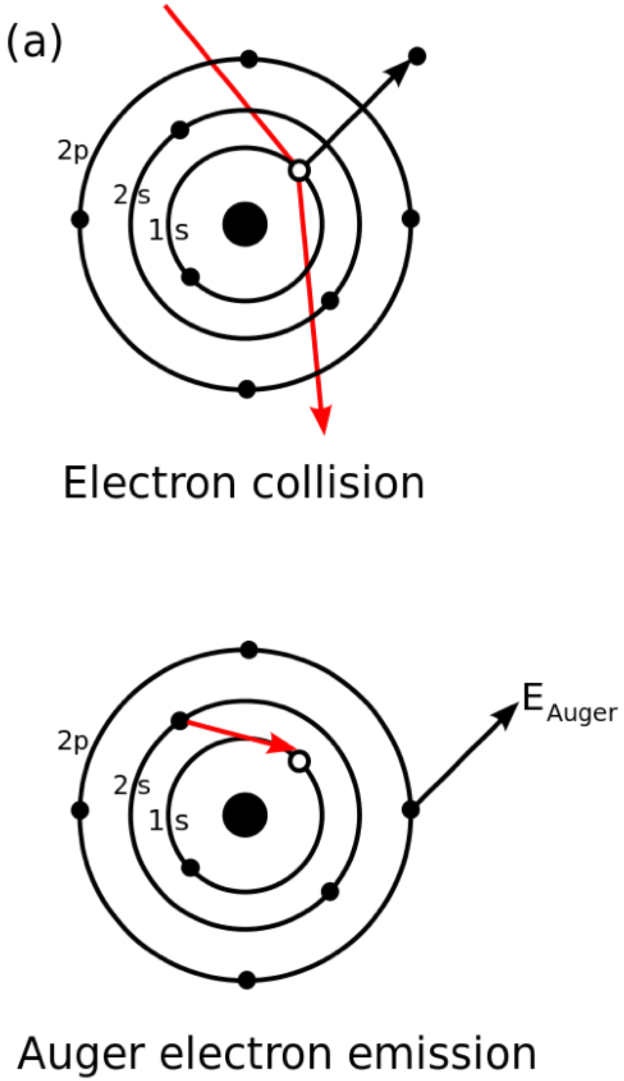
A high speed electron knocks out an inner shell electron from an atom, leaving a vacancy.

Sometimes an upper electron drops to fill the vacancy, emitting a photon.

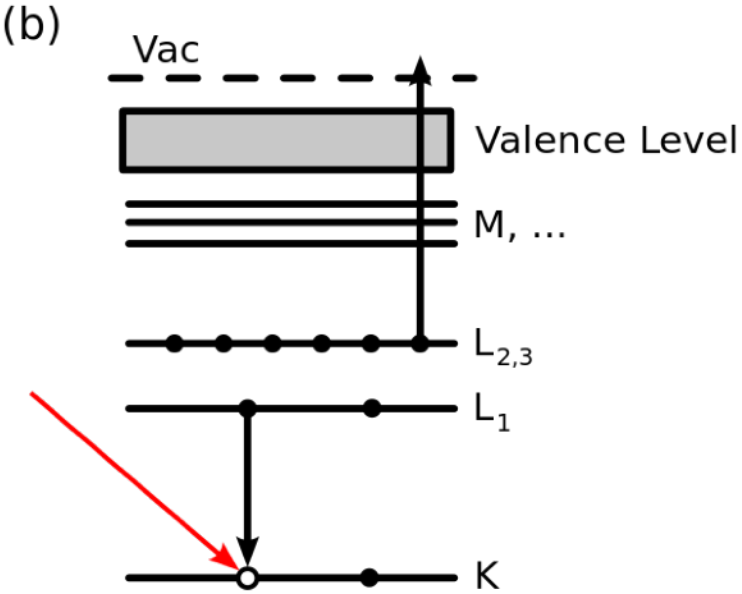
Since for heavy atoms the energy is in the x-ray region, this is called x-ray fluorescence

But sometimes the energy is transferred to an outer electron, ejecting it from the atom.

Auger electron



Auger electron emission

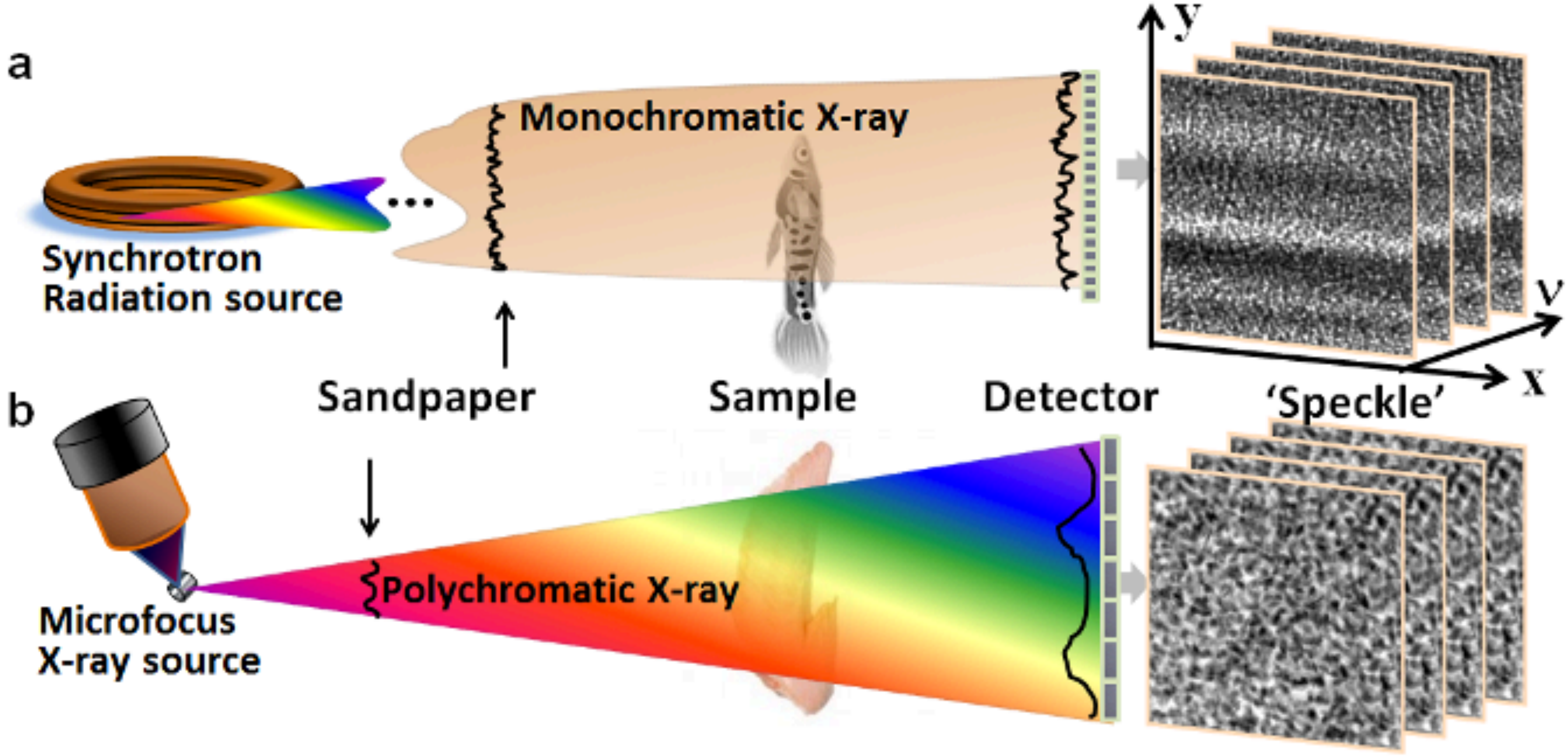


The kinetic energy of Auger electron

$$E_{ae} = E_K - E_{L_1} - E_{L_{2,3}}$$

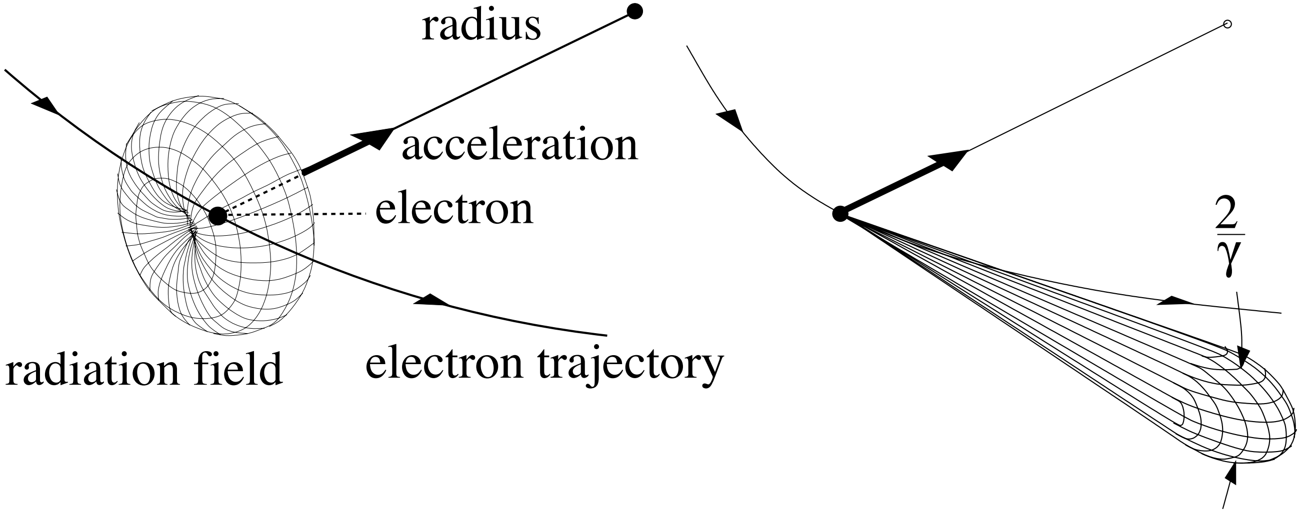
6.2 X ray production

Synchrotron Radiation: In cyclic accelerators, when charged particles are accelerated, they radiate electromagnetic energy called synchrotron radiation.



Synchrotron Radiation

Rest frame $\xrightarrow{\text{Lorentz transformation}}$ Laboratory frame



$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{E}{m_0c^2}$$

Opening angle

Synchrotron Radiation

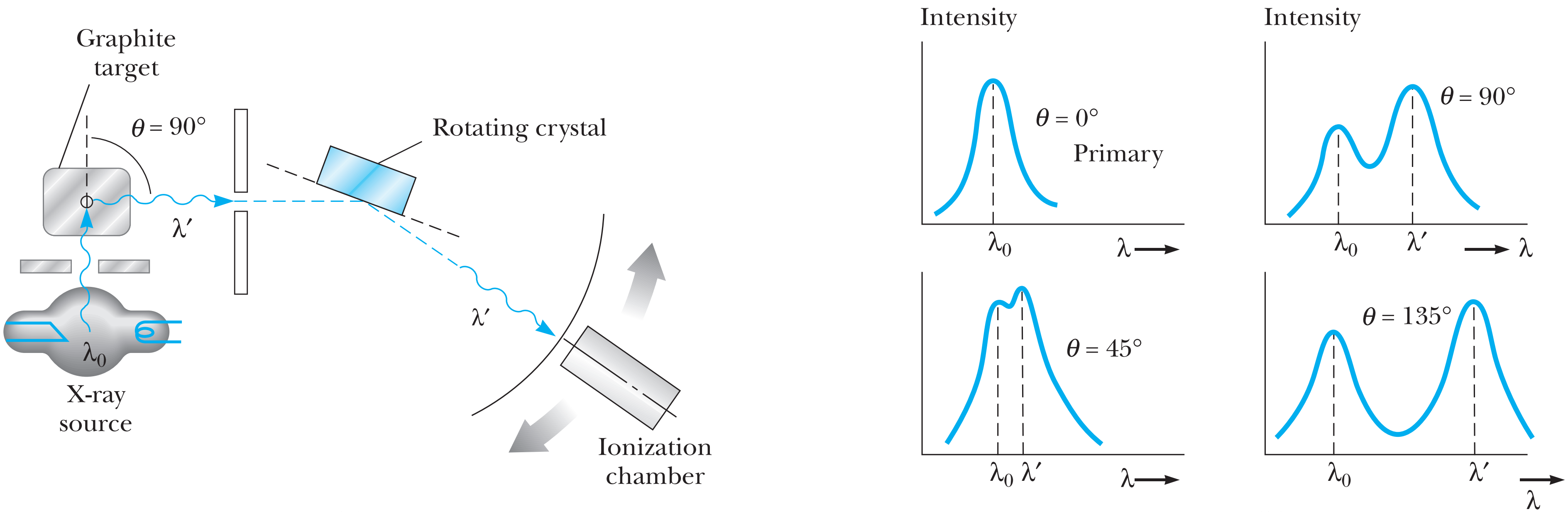
The very high intensity of the source yields images with a high signal-to-noise ratio on short time-scales, which enables fast radiographic investigations.

The beam can be easily monochromated. This allows correlations between attenuation values and the chemical constituents of the sample

The option to vary the energy of the radiation enables the investigation of objects with very different absorption coefficients within the same measuring environment.

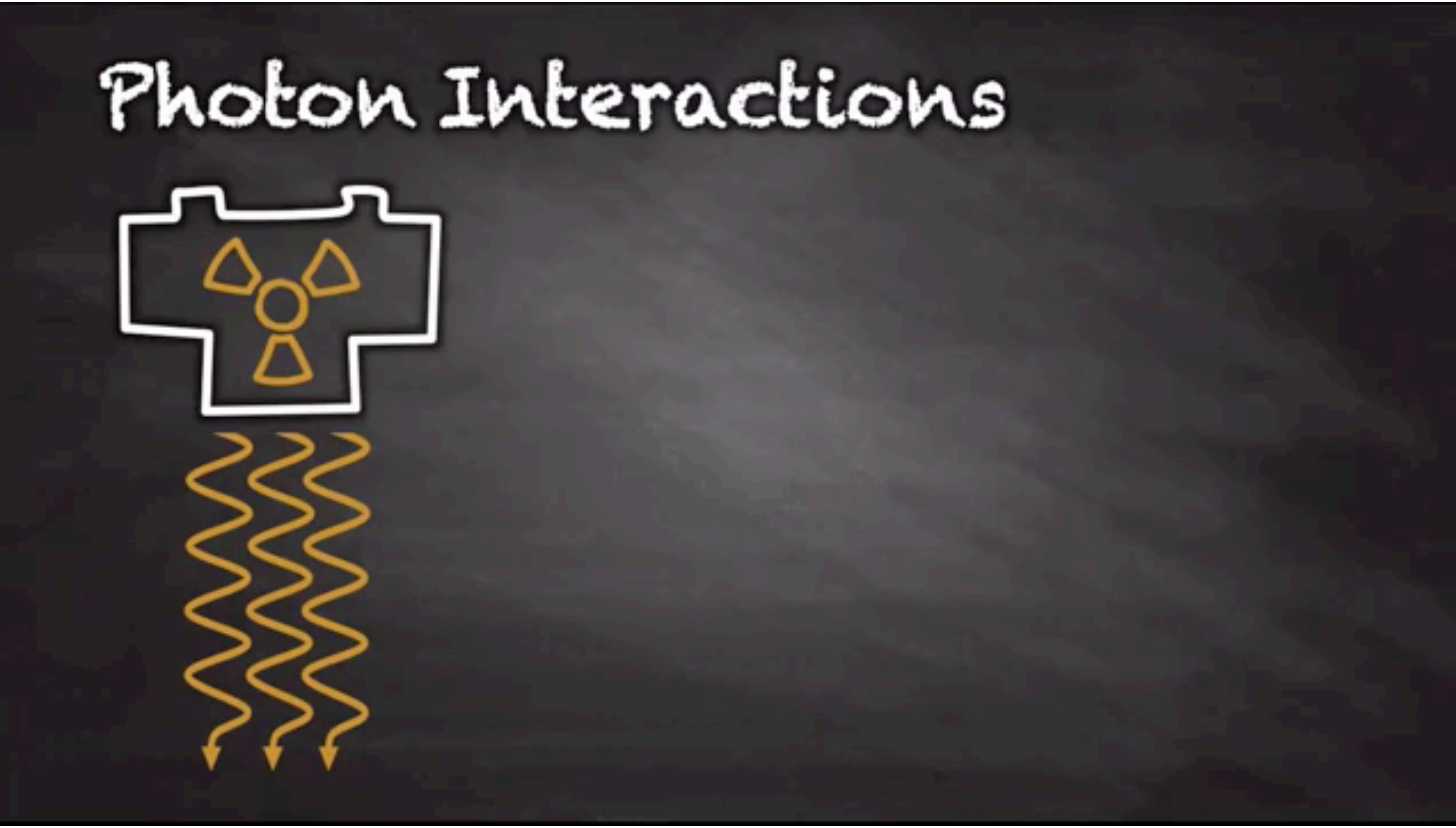
6.3 Compton scattering

At backward-scattering angles, there appeared to be a component of the emitted radiation (called a modified wave) that had a longer wavelength than the original primary (unmodified) wave.



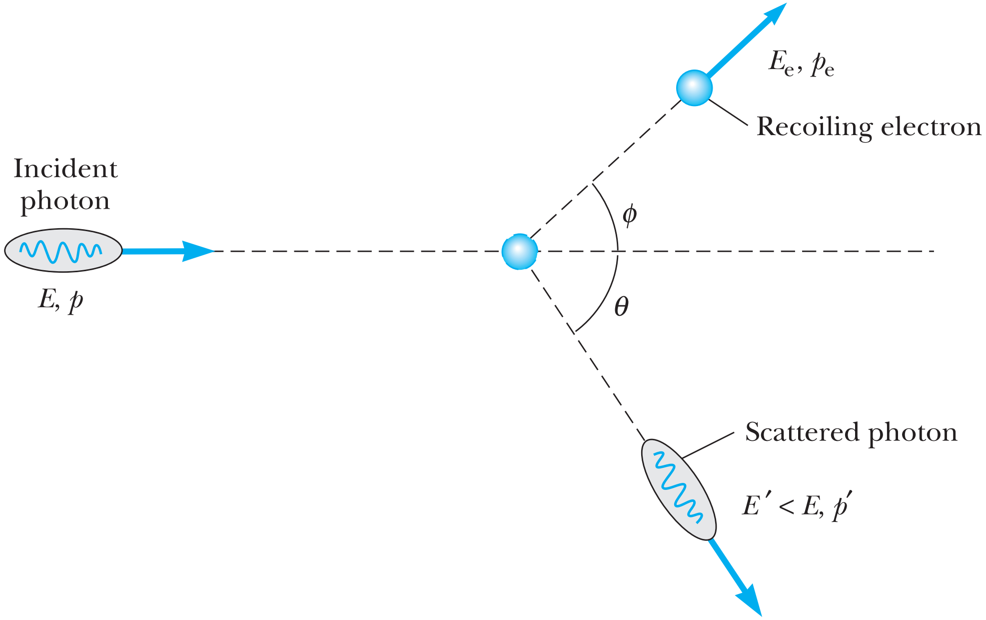
(a) (b)

6.3 Compton scattering



6.3 Compton scattering

Diagram representing Compton scattering of a photon by an electron.



Conservation of energy

$$E + m_e c^2 = E' + E_e$$

Conservation of momentum

$$p = p' \cos\theta + p_e \cos\phi$$

$$p' \sin\theta = p_e \sin\phi$$

6.3 Compton scattering

Therefore

$$p_e^2 = (p')^2 + p^2 - 2pp' \cos\theta$$

With De Broglie relation

$$p_{\text{photon}} = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

We have

$$E_e = hf - hf' + m_e c^2 \qquad E_e^2 = p_e^2 c^2 + m_e^2 c^4$$

$$p_e^2 = \left(\frac{hf'}{c}\right)^2 + \left(\frac{hf}{c}\right)^2 - \frac{2h^2 ff'}{c^2} \cos\theta$$

Finally

$$\lambda' - \lambda_0 = \boxed{\frac{h}{m_e c}}(1 - \cos\theta)$$

Compton wavelength

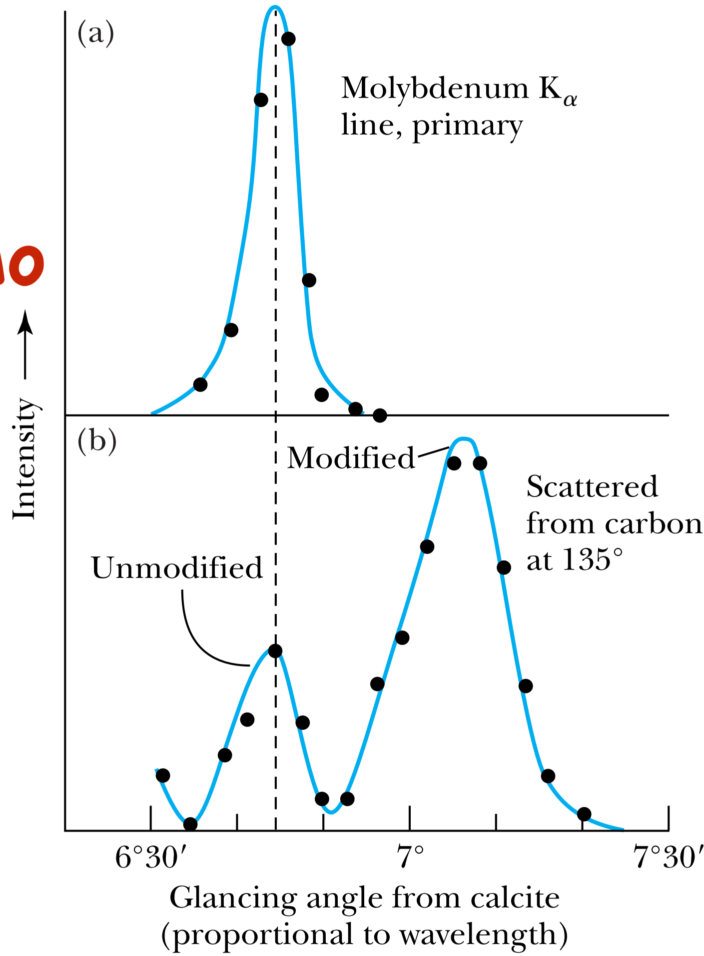
$\lambda_C = 2.426 \times 10^{-3}$ nm

6.3 Compton scattering

Compton's original data showing

1.the primary x-ray beam from Mo unscattered

2. the scattered spectrum from carbon at 135° showing both the modified and unmodified wave.



Pair Production

In the photoelectric effect, bremsstrahlung, and the Compton effect, we have studied exchanges of energy between photons and electrons. Have we covered all possible mechanisms?

For example, can the kinetic energy of a photon be converted into particle mass and vice versa?

It would appear that if none of the conservation laws are violated, then such a process should be possible.

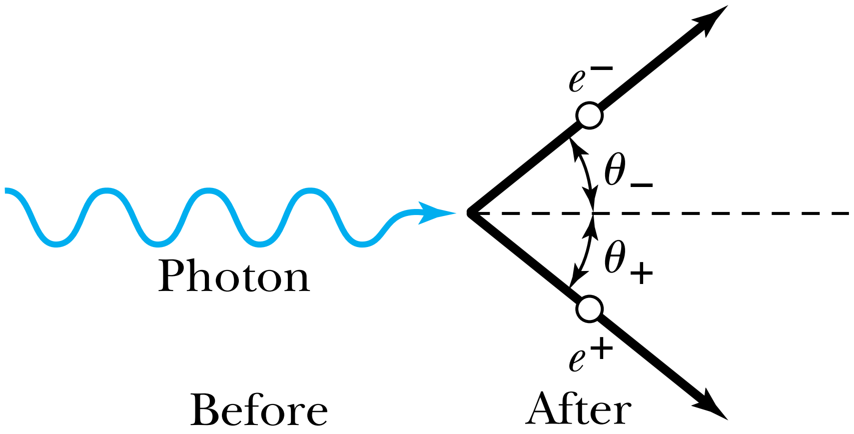
Pair Production

Experiments show that a photon's energy can be converted entirely into an electron and a positron in a process called pair production. The reaction is

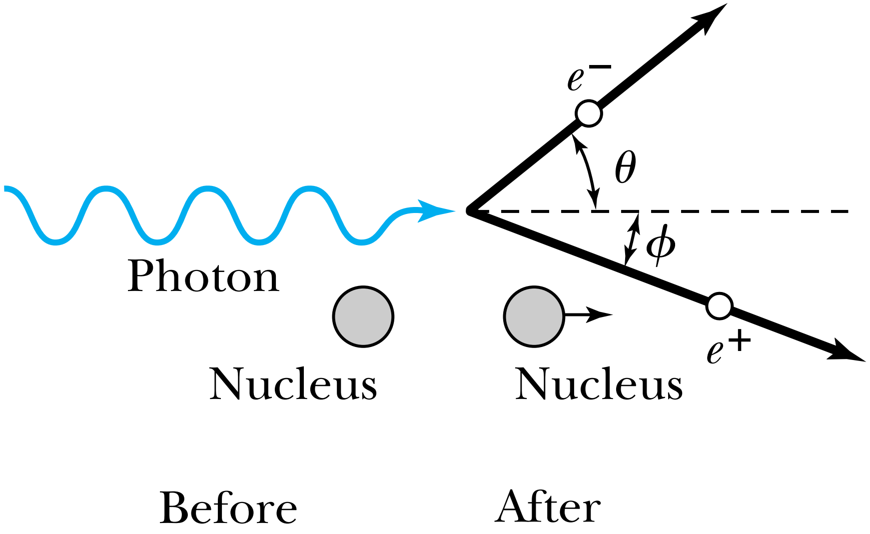
$$\gamma \rightarrow e^{+} + e^{-}$$

However, this process occurs only when the photon passes through matter, because energy and momentum would not be conserved if the reaction took place in isolation. The missing momentum must be supplied by interaction with a nearby massive object such as a nucleus.

Pair Production



(a) Free space (**cannot occur**)



(b) Beside nucleus

Pair Production

Consider the conversion of a photon into an electron and a positron that takes place inside an atom where the electric field of a nucleus is large. The nucleus recoils and takes away a negligible amount of energy but a considerable amount of momentum. The conservation of energy will now be

$$hf = E_+ + E_- + \text{K.E. (nucleus)}$$

The photon energy must be at least equal to $2m_ec^2$ in order to create the masses of the electron and positron.

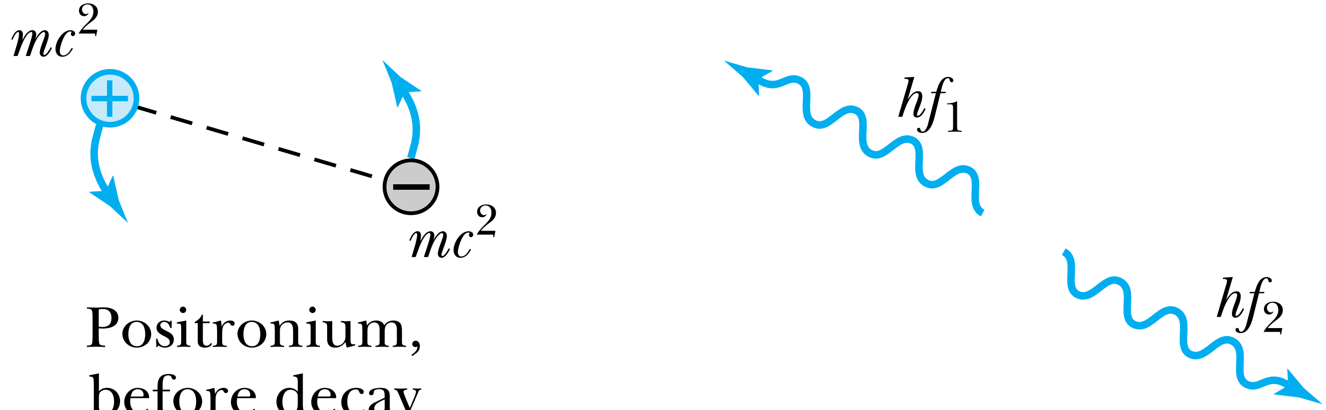
$$hf > 2m_ec^2 = 1.022\ \text{MeV} \qquad \text{(for pair production)}$$

Pair Annihilation

Eventually the electron and positron annihilate each other (typically in 10^{-10} s), producing electromagnetic radiation (photons). The process

$$e^{+} + e^{-} \rightarrow \gamma + \gamma$$

is called **pair annihilation.**



(a) Positronium, before decay (schematic only)

(b) After annihilation

Pair Annihilation

The conservation laws for the process will be

$$2m_e c^2 \approx hf_1 + hf_2$$

$$0 = \frac{hf_1}{c} - \frac{hf_2}{c}$$

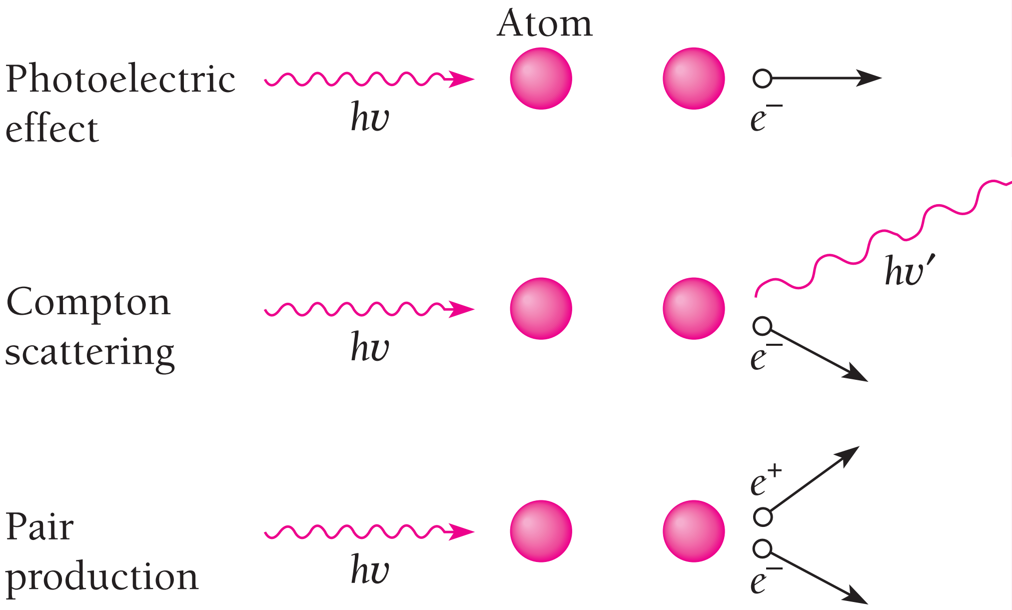
The frequencies are identical

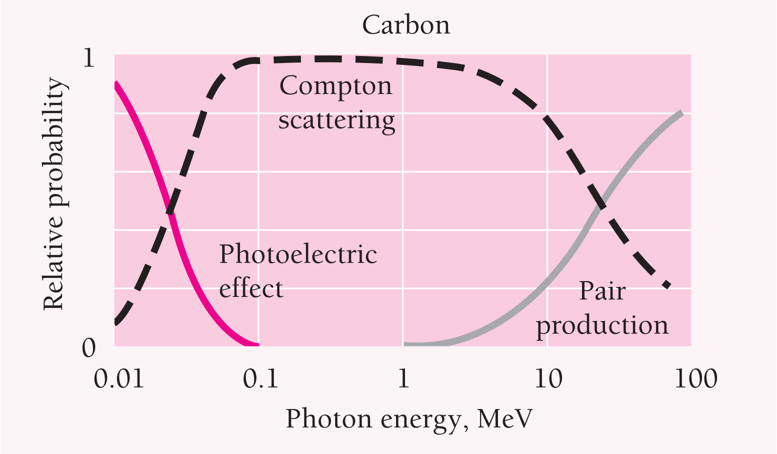
$$hf = m_e c^2 = 0.511 \text{ MeV}$$

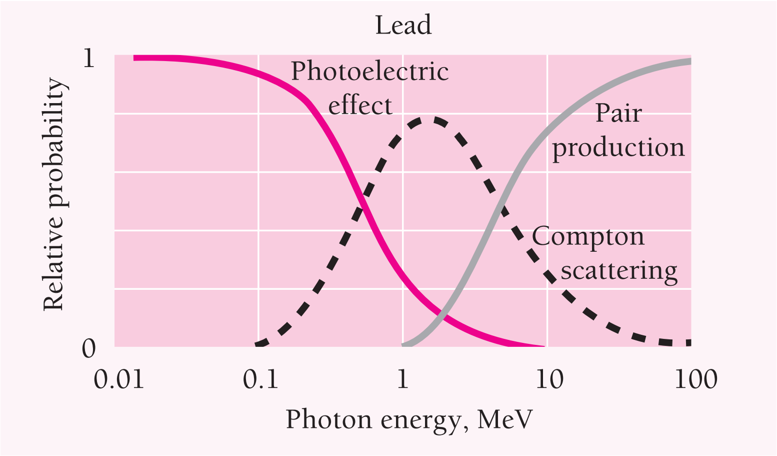
In other words, the two photons from positronium annihilation will move in opposite directions, each with energy 0.511 MeV. This is exactly what is observed experimentally.

6.4 Photon absorption

The three chief ways in which photons of light, x-rays, and gamma rays interact with matter







6.4 Photon absorption

The intensity I of an x- or gamma-ray beam is equal to the rate at which it transports energy per unit cross-sectional area of the beam.

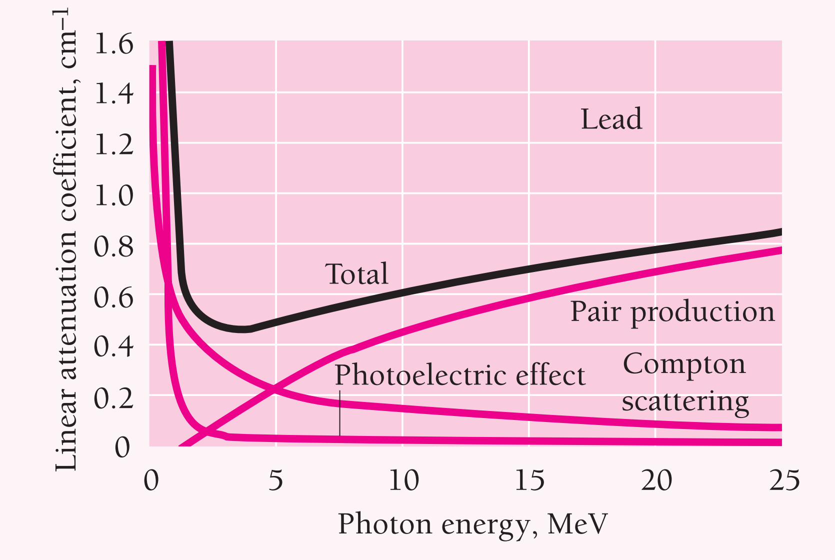
$$-\frac{dI}{I} = \mu\, dx$$

Radiation intensity I

$$I = I_0 e^{-\mu x}$$

Absorber thickness

$$x = \frac{\ln (I_0/I)}{\mu}$$



6.4 linear absorption coefficient

	Linear Absorption Coefficient (cm^{-1})				
λ (pm)	Air	Water	Aluminum	Copper	Lead
10		0.16	0.43	3.2	43
20		0.18	0.76	13	55
30		0.29	1.3	38	158
40		0.44	3.0	87	350
50	8.6×10^{-4}	0.66	5.4	170	610
60	1.3×10^{-3}	1.0	9.2	286	1000
70	1.95×10^{-3}	1.5	14	430	1600
80	2.73×10^{-3}	2.1	20	625	
90	3.64×10^{-3}	2.8	30	875	
100	4.94×10^{-3}	3.8	41	1200	
150	1.56×10^{-2}	12	124		
200	3.64×10^{-2}	28	275		
250	6.63×10^{-2}	51	524		

Exercise

1. Which element has a K_α x-ray line whose wavelength is 0.180 nm?

Exercise

1. Which element has a K_α x-ray line whose wavelength is 0.180 nm?

Solution:

The frequency corresponding to a wavelength of 0.180 nm is

$$\nu = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{1.80 \times 10^{-10} \text{ m}} = 1.67 \times 10^{18} \text{ Hz}$$

Therefore

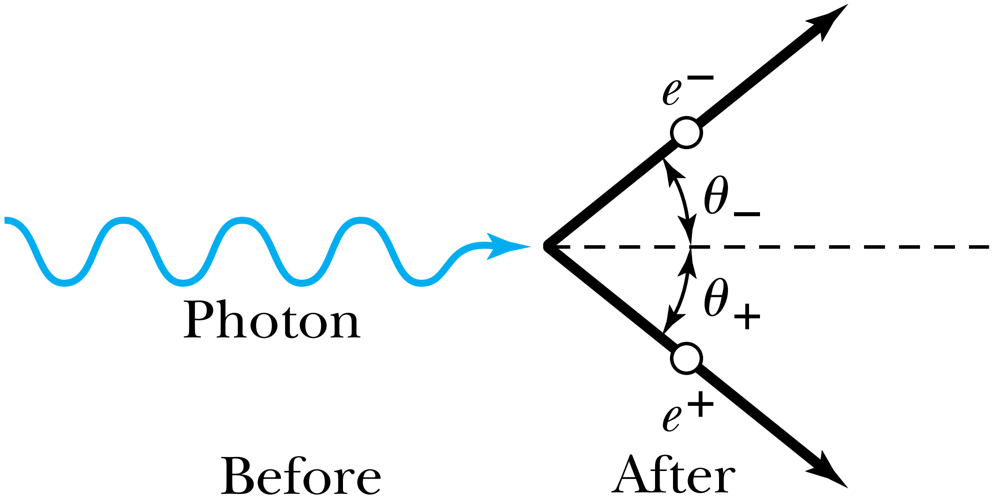
$$Z - 1 = \sqrt{\frac{4}{3cR}} = \sqrt{\frac{(4)(1.67 \times 10^{18} \text{ Hz})}{(3)(3.00 \times 10^8 \text{ m/s})(1.097 \times 10^7 \text{ m}^{-1})}} = 26$$

$$Z = 27$$

The element with atomic number 27 is cobalt.

Exercise

2. Show that a photon cannot produce an electron-positron pair in free space as following figure



Exercise

2. Show that a photon cannot produce an electron-positron pair in free space as following figure

Solution:

Let the total energy and momentum of the electron and thepositron be E_-, p_-, and E^+,p^+, respectively. The conservation laws are then

$$hf = E_+ + E_-$$

$$\frac{hf}{c} = p_- \cos\theta_- + p_+ \cos\theta_+$$

$$0 = p_- \sin\theta_- - p_+ \sin\theta_+$$

From the second equation, we have

$$hf_{\max} = p_{-}c + p_{+}c$$

When we inserted the mass-energy relation to first equation

$$hf = \sqrt{{p_{+}}^{2}c^{2} + m^{2}c^{4}} + \sqrt{{p_{-}}^{2}c^{2} + m^{2}c^{4}}$$

Therefore

$$hf > p_{-}c + p_{+}c$$

The two frequency equations are inconsistent and cannot simultaneously be valid.

Homework

The Physics of Atoms and Quanta

18.1 18.2 18.3 18.4 18.8