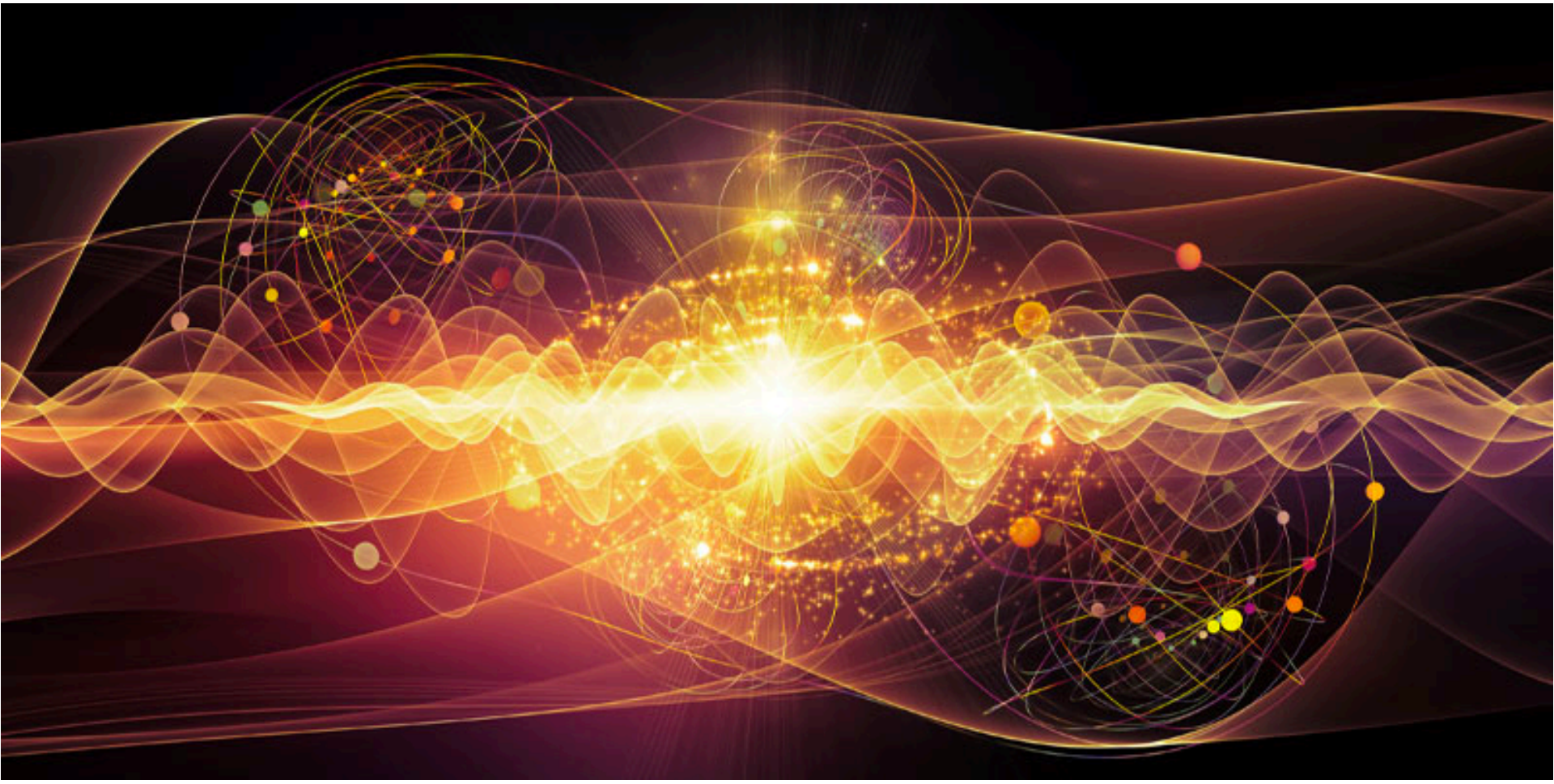
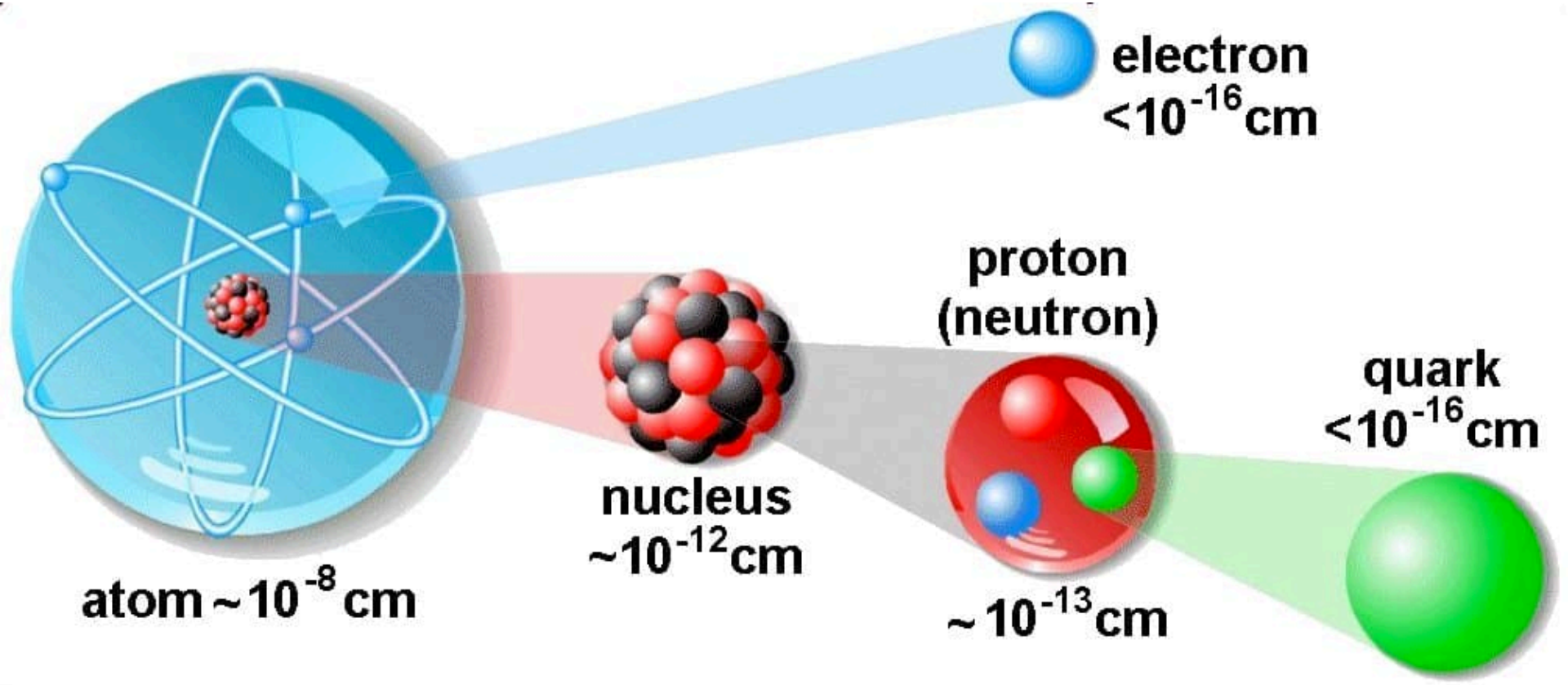
Atomic Physics

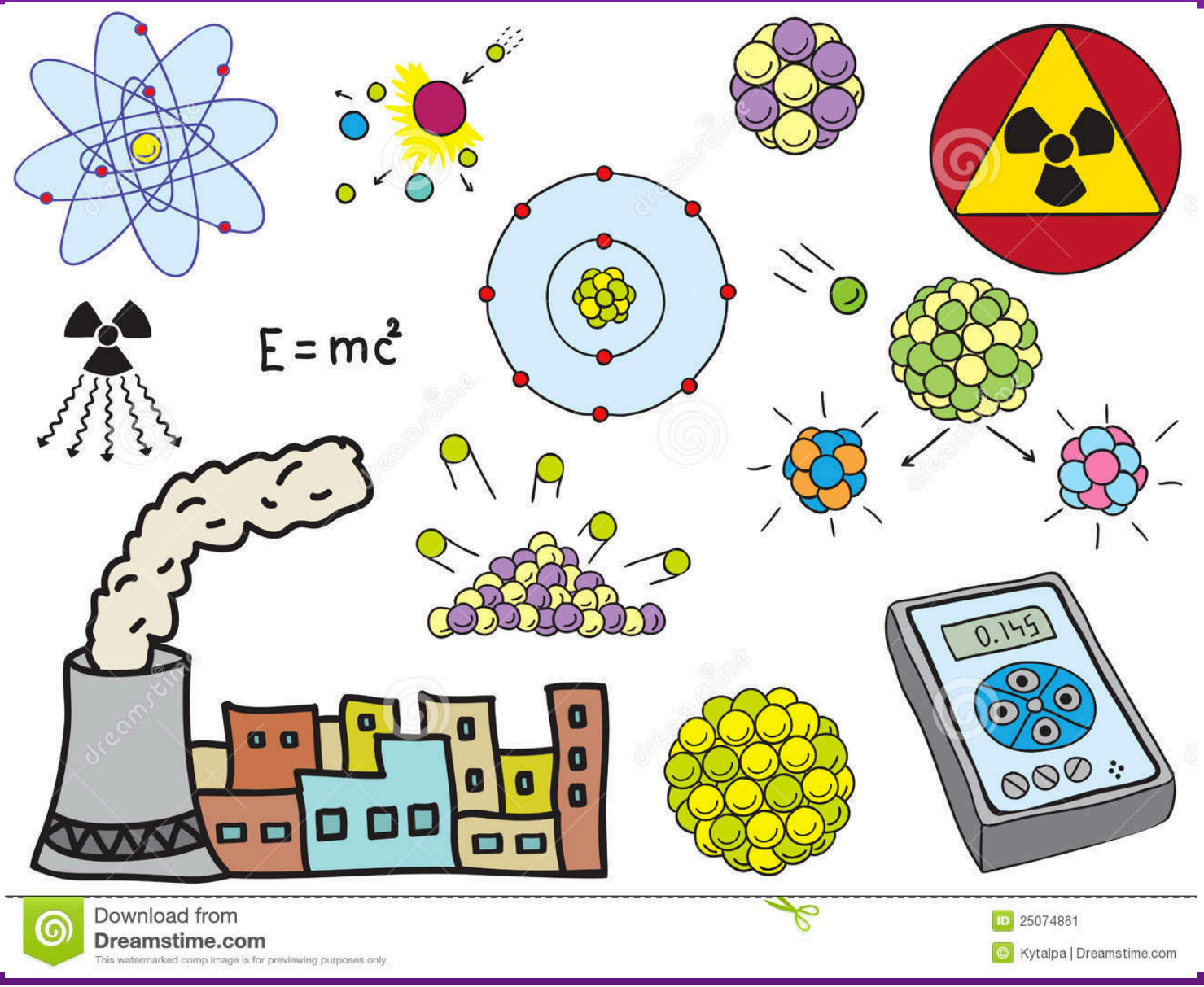
Chapter 7 Introduction to Nuclear Physics



Nuclear structure

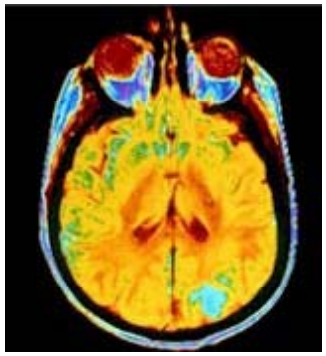


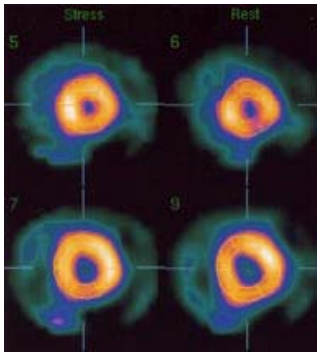
Nuclear physics application



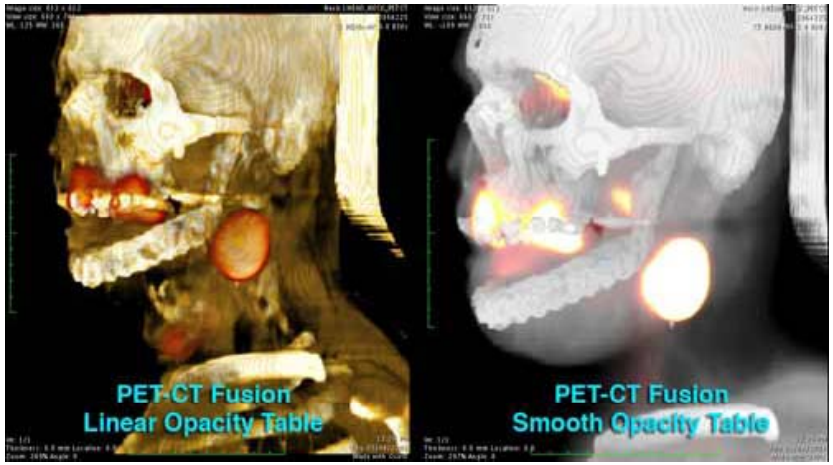
Nuclear physics application

Nuclear Imaging

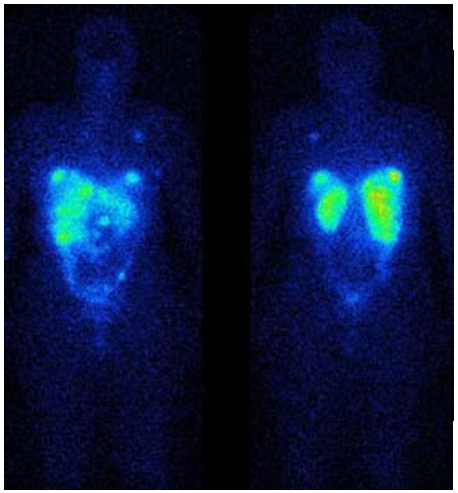




Blood flow with radiopharmaceuticals



Imaging software and analysis



- Gamma Camera
- SPEC & PEP
- Isotopes & Isomer
- Parmaceuticals

Tumor mapping & visualization by radioactive isotope accumulation.

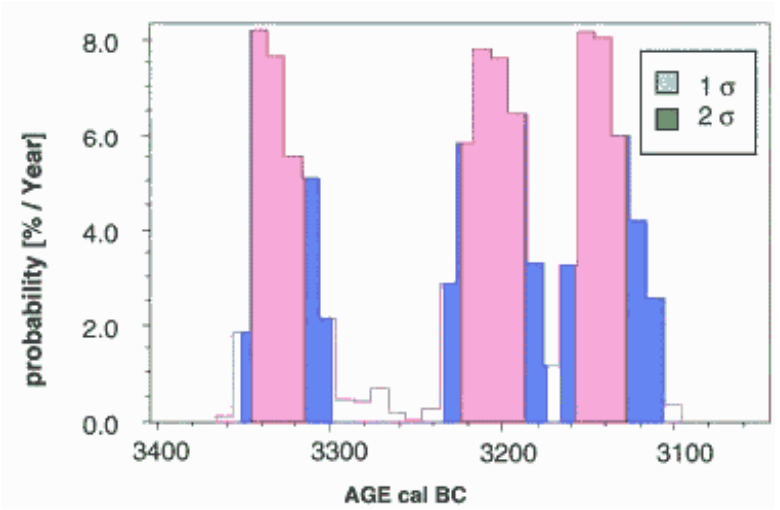


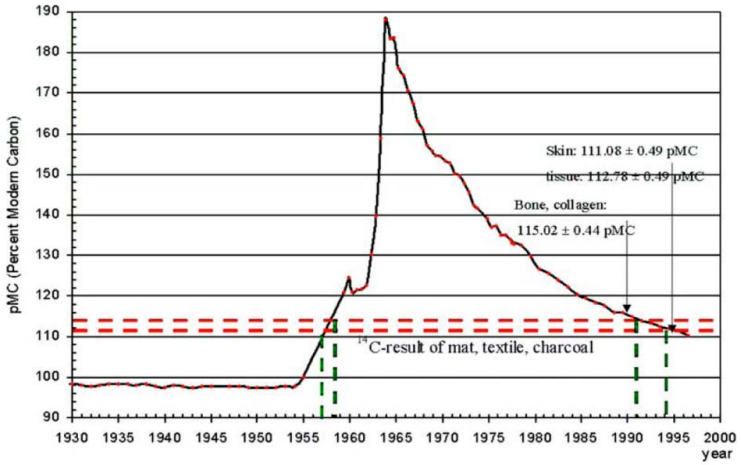
Imaging system development

Dating real and false mummies









The history of nuclear physics



✓ 1896: Henri Becquerel–discovery of radioactivity

Photographic plates blackened when placed near certain minerals (uranium salts). Radioactivity could not be explained by e-m (or gravity), and was one of the unsolved problems.



✓ 1898: Maria and Pierre Curie– discovery of Polonium and Radium
(much more radioactive than uranium)

Becquerel and the Curies shared the Nobel Prize in Physics in 1903.

Later, Marie Curie isolates metallic radium and receives the Nobel Price in Chemistry in 1911.

Radioactivity Unit: Bacquerel and Curie

The history of nuclear physics

Ernest Rutherford – "the father of nuclear physics"

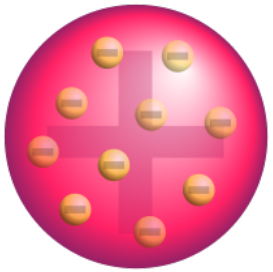
- ✓ 1899: Rutherford shows 2 types of radiation exits and calls them named α and β.
- ✓ 1900: Villard gives evidence for a 3rd type of radiation coming from radium and calls it γ
- ✓ 1902: Curies show that β radiation is electrons
- ✓ 1904: Rutherford shows α particles are helium

Ernest Rutherford was awarded the Nobel Prize in Chemistry in 1908 "for his investigations into the disintegration of the elements, and the chemistry of radioactive substances".

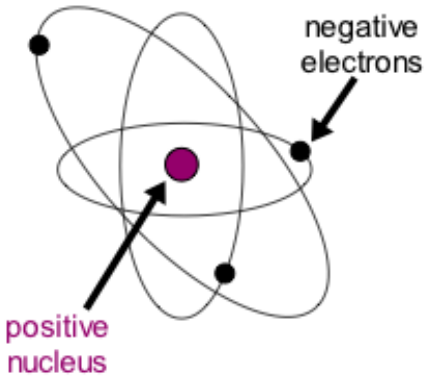
"I have dealt with many different transformations with various periods of time, but the quickest that I have met was my own transformation in one moment from a physicist to a chemist." E. Rutherford (Nobel banquet 1908).

The history of nuclear physics

✓ 1911: Rutherford proposed the existence of a massive nucleus as a small central part of an atom



J.J. Thompson's
Plum Pudding Model (1904)

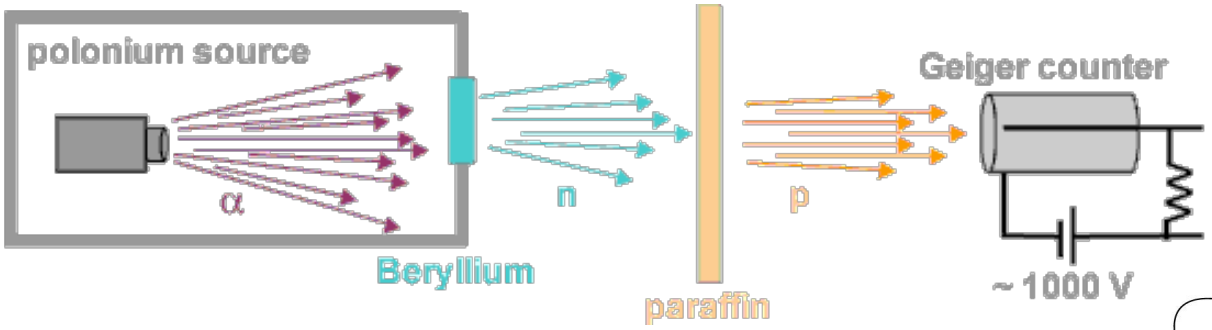


Rutherford's
Planetary Model (1911)

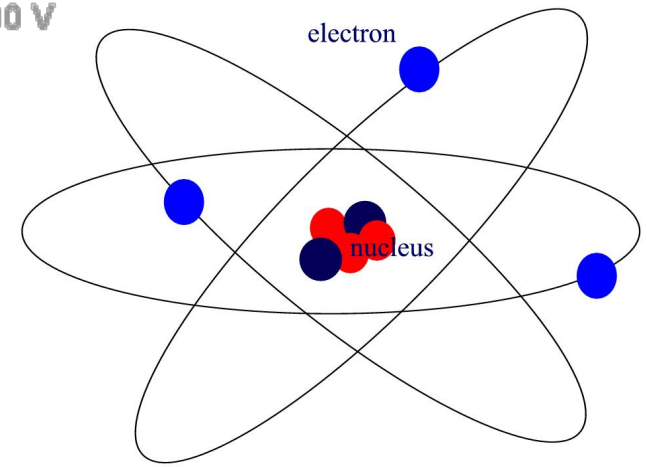
The history of nuclear physics

✓1932: Nobel Prize in Physics for James Chadwick for a discovery of the neutron





Nucleus = protons + neutrons
(short range nuclear force)

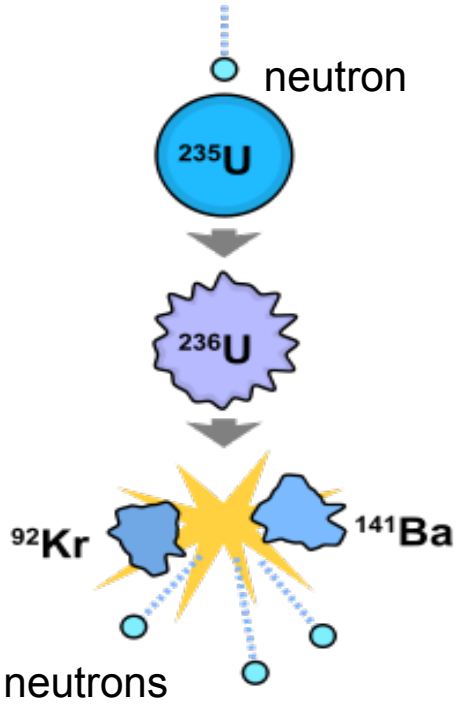


The history of nuclear physics

南開大學

- ✓ Chadwick's discovery made it possible to create elements heavier than Uranium in the lab. Later, Enrico Fermi discovered nuclear reactions (slow neutrons) which led to a revolutionary discovery of nuclear fission (Otto Hanh and Fritz Strassmann).

- ✓ Chadwick's discovery was crucial for the fission of uranium 235. Unlike α particles, neutrons do not need to overcome Coulomb barrier and thus can penetrate and split the nuclei of the heaviest elements. The release of neutrons sustains the fission reaction.



Isotope	Natural Abundance (atomic percentage)	Half-Life (years)
U^{234}	0.0054	2.440×10^5
U^{235}	0.7200	7.040×10^8
U^{238}	99.2746	4.488×10^9

Uranium-235 has the distinction of being the only naturally occurring fissile isotope.

Uranium-238 cannot fission with low energy neutrons (stable nuclear shell structure)

The history of nuclear physics

✓1914: J. Chadwick shows spectrum of β radiation is continuous, contrary to the fundamental principle of energy conservation

✓1930:W.Pauli proposed a neutrino to explain the continuous spectrum of β decay.

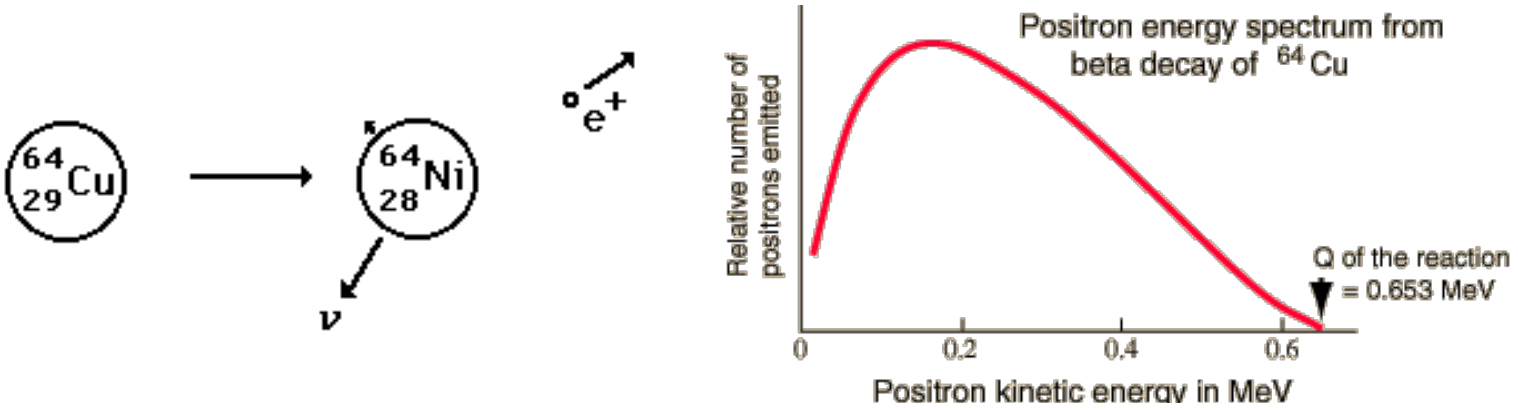
"I have done a terrible thing, I have postulated a particle that cannot be detected."
W. Pauli

Wolfgang Pauli was awarded the Nobel Prize in Physics in 1945 "for the discovery of the Exclusion Principle, also called the Pauli Principle".

The history of nuclear physics

✓ 1933: E. Fermi used the neutrino to explain neutron β decay (model of weak interactions)



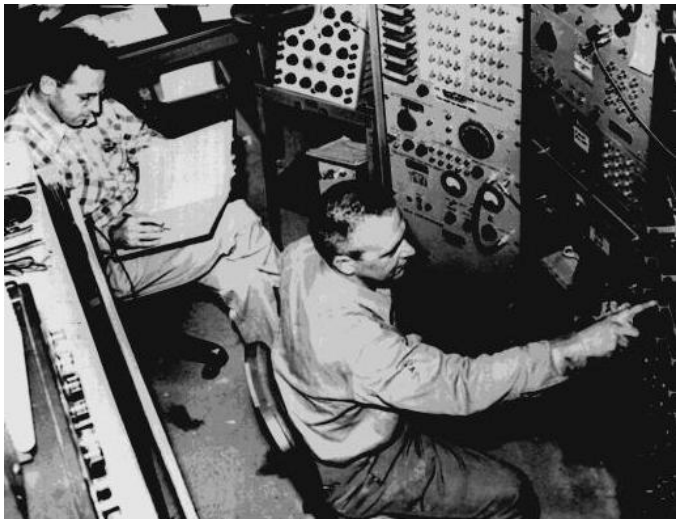


Enrico Fermi was awarded the Nobel Prize in Physics in 1938 "for his demonstrations of the existence of new radioactive elements produced by neutron irradiation, and for his related discovery of nuclear reactions brought about by slow neutrons".

The history of nuclear physics

✓ 1956: F. Reines and C. Cowan detection of a neutrino via inverse beta decay reaction

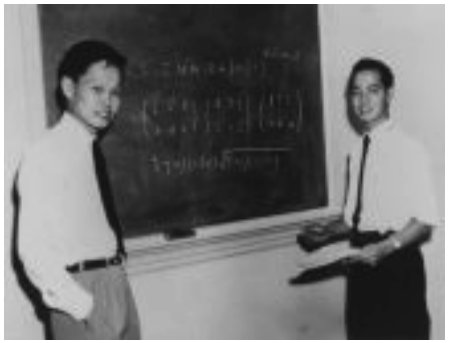
$$\bar{\nu}_e + p \rightarrow n + e^+$$



From then on Reines dedicated his career to the study of the neutrino's properties and interactions, including the discovery of neutrinos emitted from SN1987A by the Irvine-Michigan-Brookhaven Collaboration. This discovery helped to inaugurate the field of neutrino astronomy.

F. Reines was awarded the Nobel Prize in Physics in 1995 **for his co-detection of the neutrino with Clyde Cowan in the neutrino experiment"**

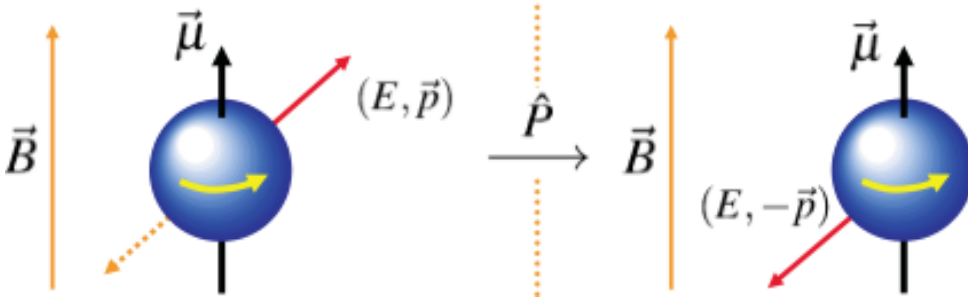
The history of nuclear physics



✓ 1957: Lee and Yang – proposed the concept of parity violation in weak interactions (Nobel Prize in Physics)

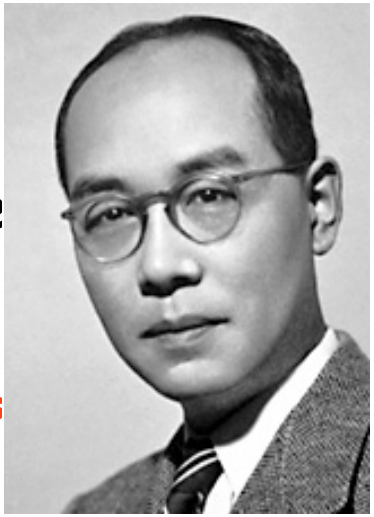
Implications: if parity is not conserved in weak interactions, it means that the Universe sometimes distinguishes between left and right

✓ 1958: C.S. Wu experimentally confirmed parity violation in weak interactions (β decay of polarized cobalt-60 nuclei)



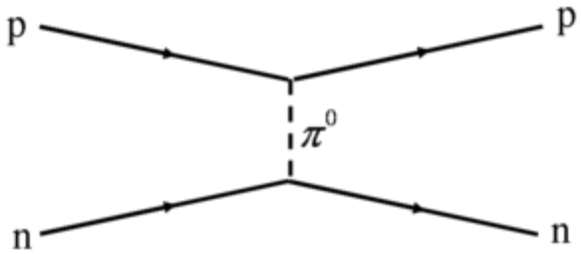
Observed electrons emitted preferentially in direction opposite to to applied field: If parity were conserved, expect equal rate of electrons in directions along and opposite to the nuclear spin.

The history of nuclear physics



✓1935: Hideki Yukawa proposed the force between nucleons arises from meson exchange

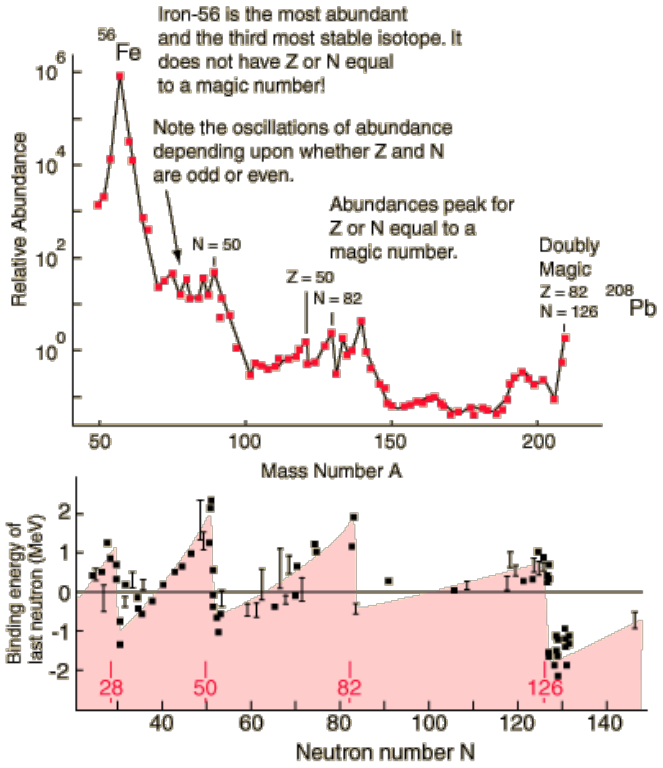
Awarded the Nobel Prize in Physics in 1947 ***"for his prediction of the existence of mesons on the basis of theoretical work on nuclear forces***

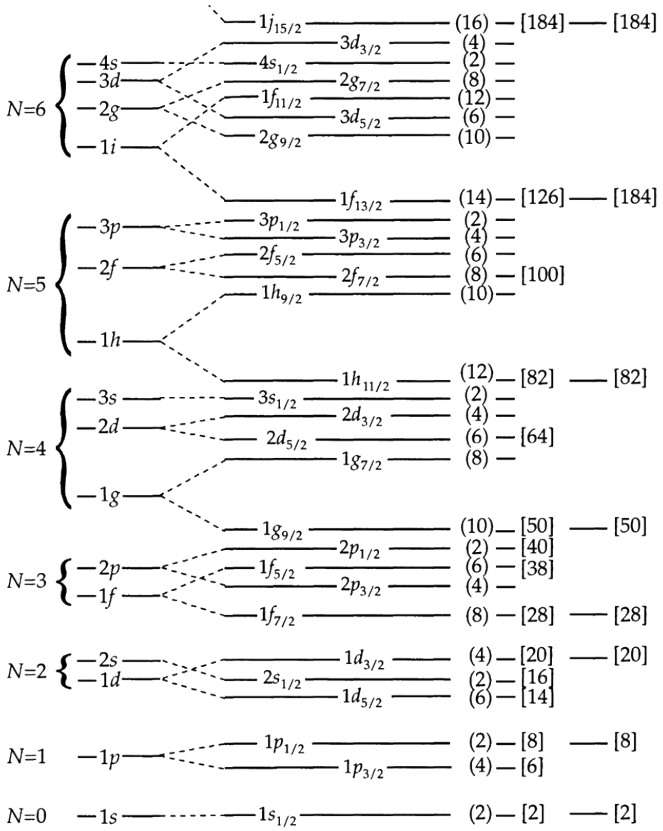


The meson-exchange concept (where hadrons are treated as elementary particles) continues to represent the best working model for a quantitative NN potential.

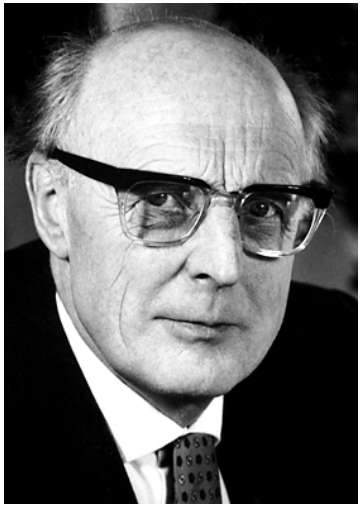
The history of nuclear physics

✓ 1949: M. Meyer and J. Jensen used shell model with spin-orbit interaction to explain magic number





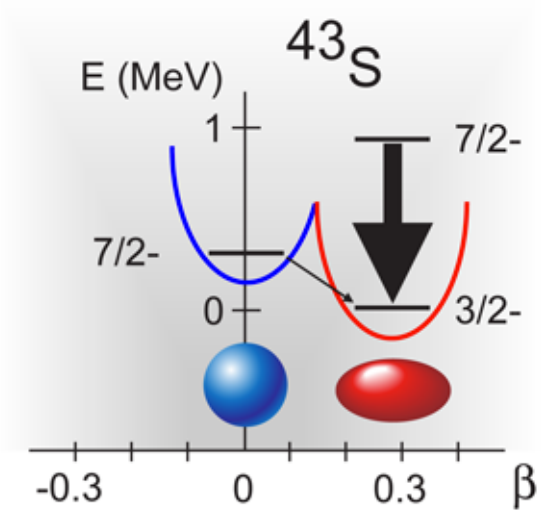


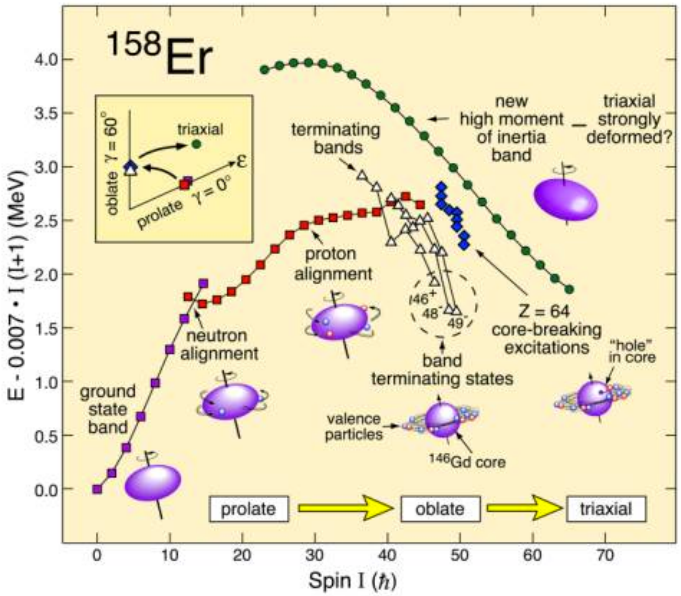


1963 Mayer and Jensen are awarded Nobel prize in physics "for their discoveries concerning nuclear shell structure".

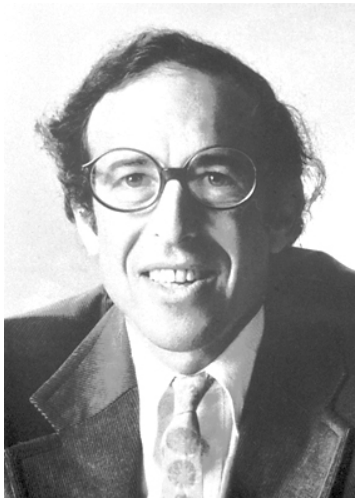
The history of nuclear physics

✓ 1951: Collective model
(Bohr, Mottelson, Rainwater)



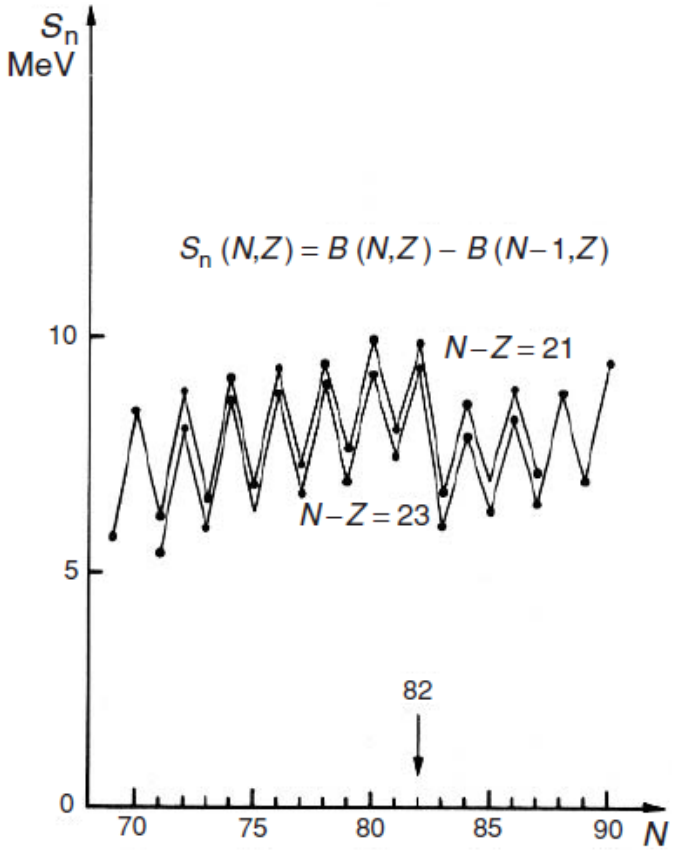


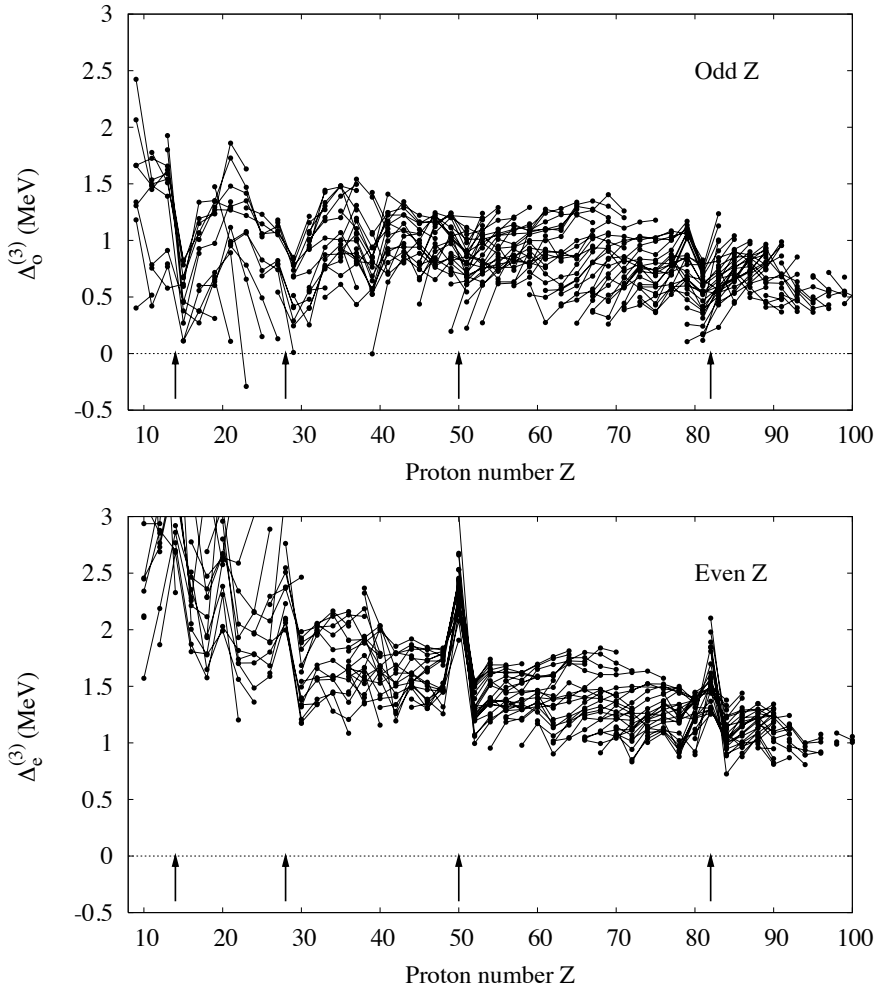




The Nobel Prize in Physics 1975 was awarded jointly to Aage Niels Bohr, Ben Roy Mottelson and Leo James Rainwater "for the discovery of the connection between collective motion and particle motion in atomic nuclei and the development of the theory of the structure of the atomic nucleus based on this connection".

✓ 1957: Nuclear superfluidity (Bohr, Mottelson)





✓ Nuclear reaction

decay, fusion, fission, heavy ion collision, ...

✓ Nuclear structure

nuclear basic properties:

nuclear size, nuclear binding, nuclear shape

✓ Models of nuclear structure theory

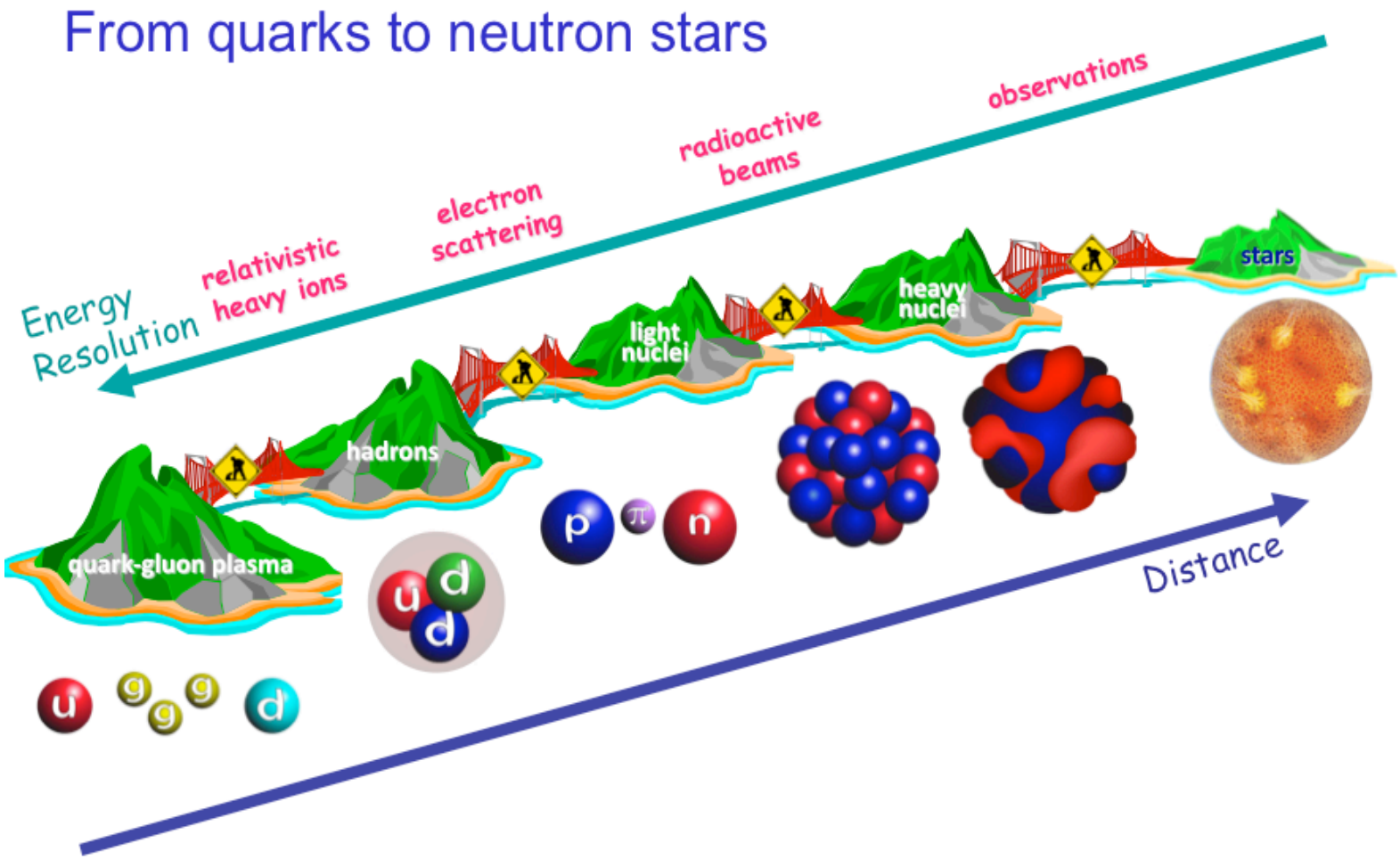
Collective models

The degrees of freedom are some bulk property of nucleus as a whole

Microscopic models

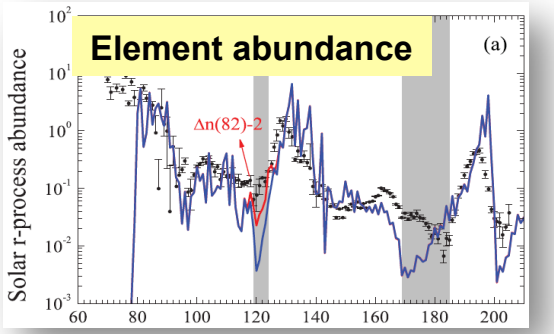
The degrees of freedom are those of the constituent particles of the nucleus

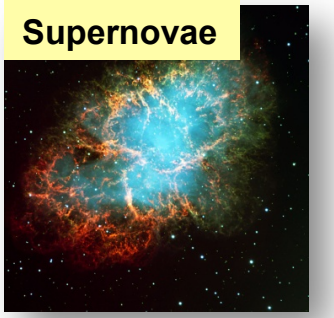
Present nuclear physics

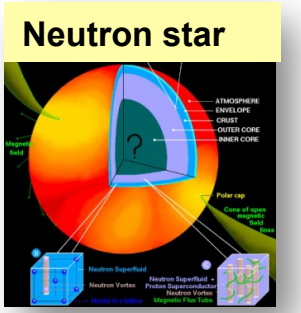


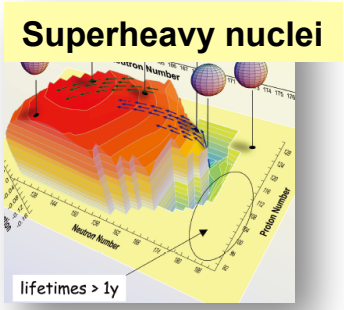
Hot topics in nuclear structure theory

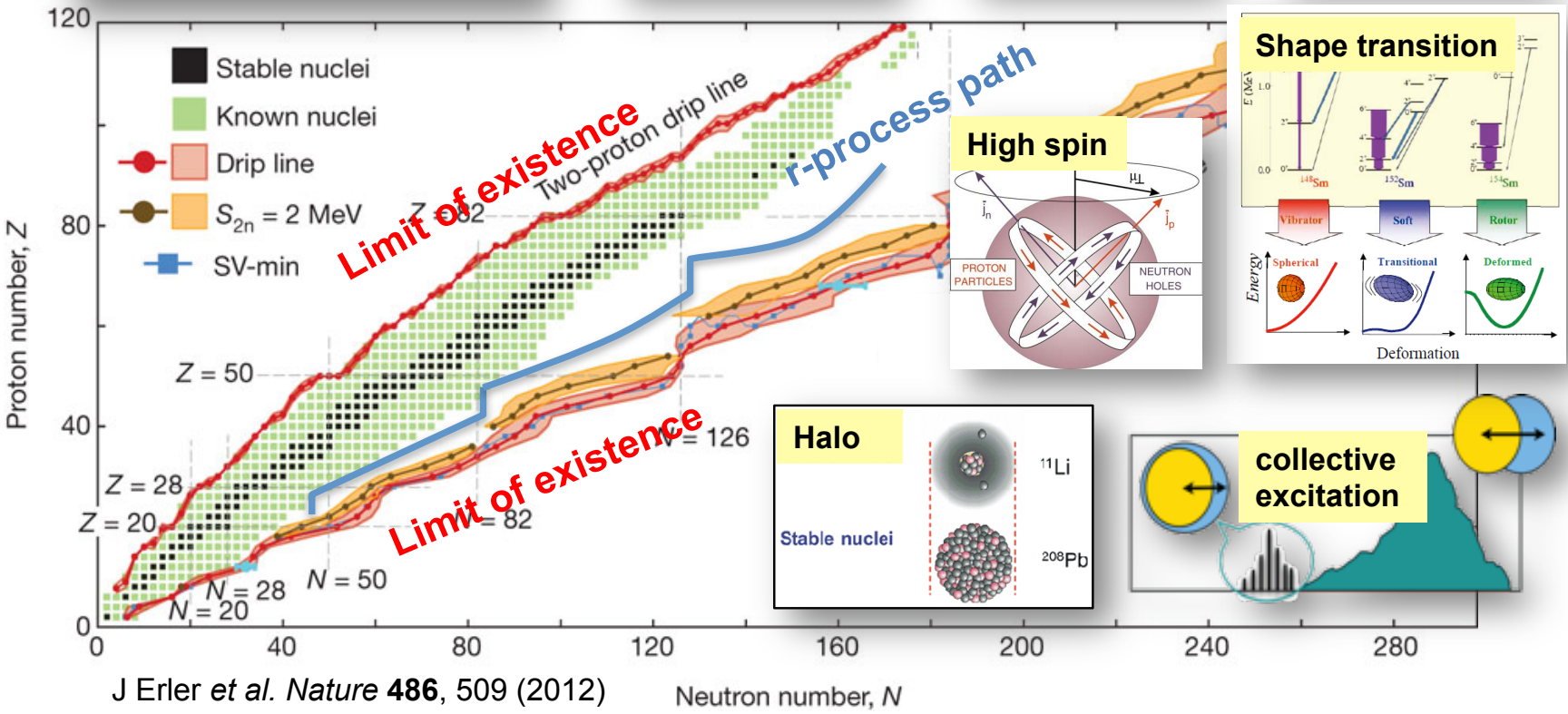
南開大學











J Erler *et al. Nature* **486**, 509 (2012)

Some nomenclature

${}^{A}_{Z}\mathrm{X}_{N}$	Isotope notation
X	Chemical symbol, *e.g.* Ca, Pb
A	Atomic mass number (sum of n's and p's in the nucleus)
Z	Atomic number (or, proton number), the number of p's in the nucleus
N	Neutron number, the number of n's in the nucleus
Examples	${}^{3}_{2}\mathrm{He}_{1}$, ${}^{40}_{20}\mathrm{Ca}_{20}$, ${}^{208}_{82}\mathrm{Pb}_{126}$
Variants	${}^{40}\mathrm{Ca}$, Calcium-40, Ca-40

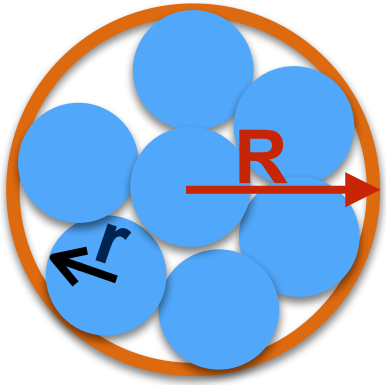
Note that, once X (which encodes Z) and A are given, the rest of the information is redundant, since $A = Z + N$. The full form is usually used only for emphasis.

isotope	Same Z, different N *e.g.* ${}^{40}\mathrm{Ca}$ and ${}^{41}\mathrm{Ca}$ Mnemonic: From Greek *isos* (same) *topos* (place) (coined by F. Soddy 1913) *i.e.* same place in the periodic table
isotone	Different Z, same N, *e.g.* ${}^{13}\mathrm{C}$ and ${}^{12}\mathrm{B}$ Mnemonic: isoto**P**e and isoto**N**e (coined by K. Guggenheimer 1934)
isobar	Different Z, and N, but same A, *e.g.* ${}^{12}\mathrm{C}$ and ${}^{12}\mathrm{B}$ Mnemonic: From Greek *isos* (same) *baros* (weight)

Nuclear Radii

- A nucleons with hard spheres of radius r

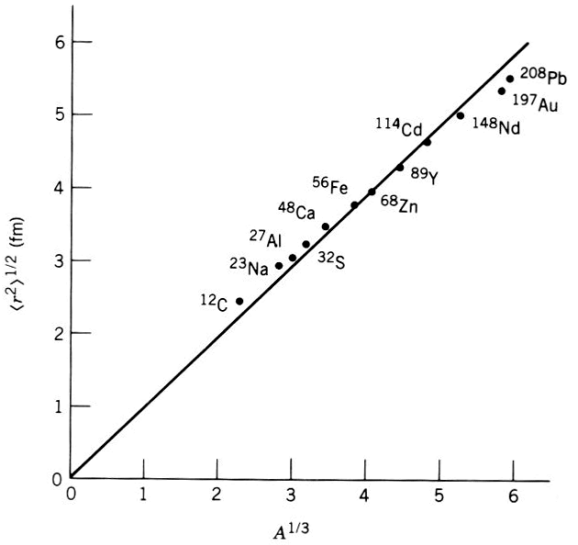
so,

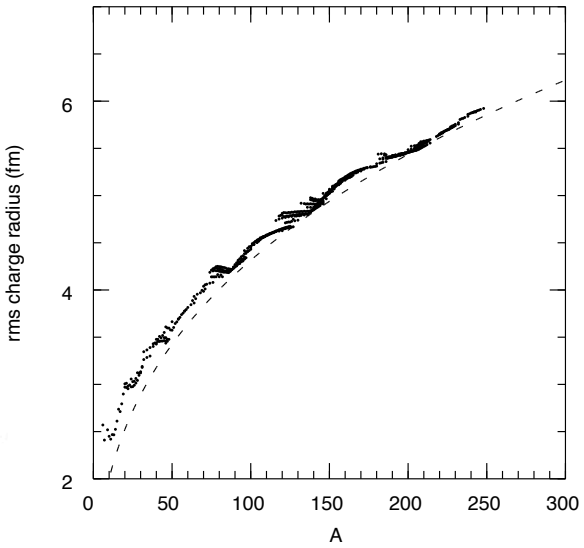


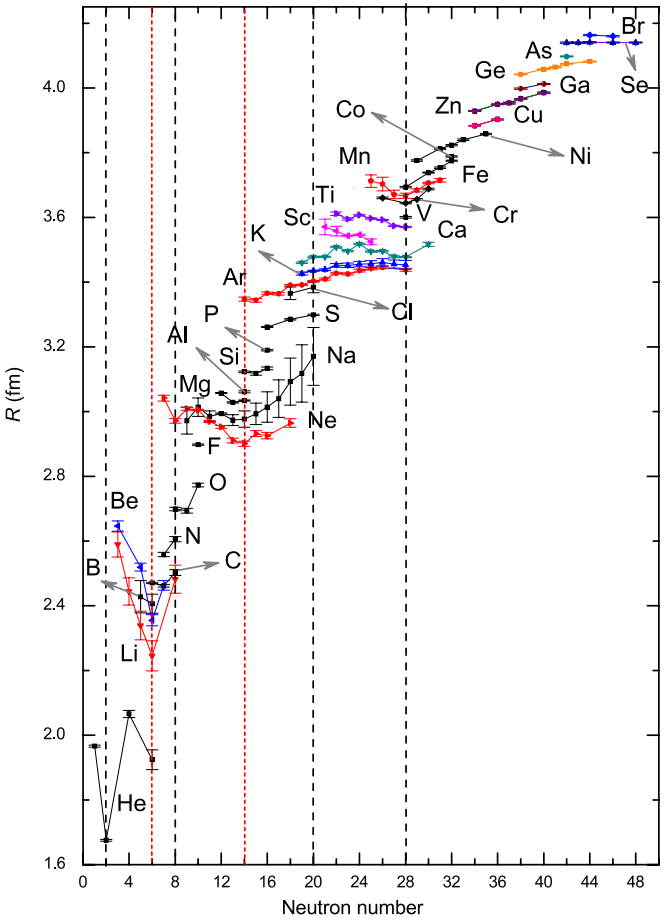
$$\frac{4}{3}\pi R^3 \approx A\frac{4}{3}\pi r^3$$

$$R \approx r_0 A^{1/3}$$

$$r_0 \sim 1.20 - 1.25 \text{ fm}$$







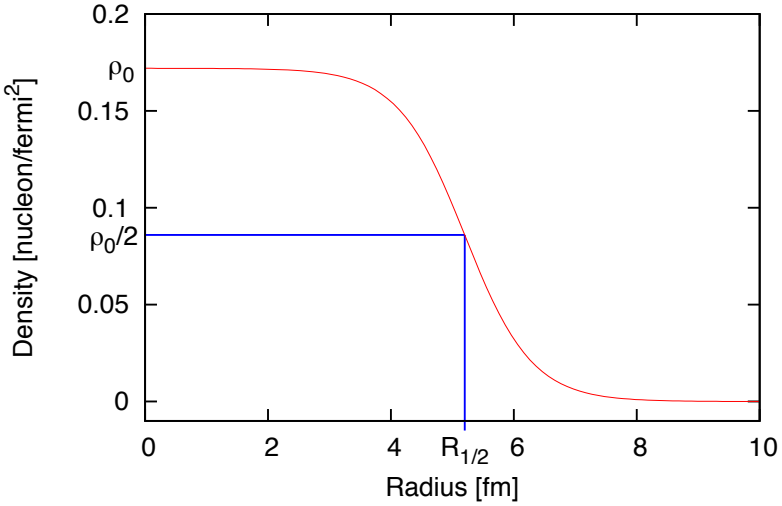
Nuclear density

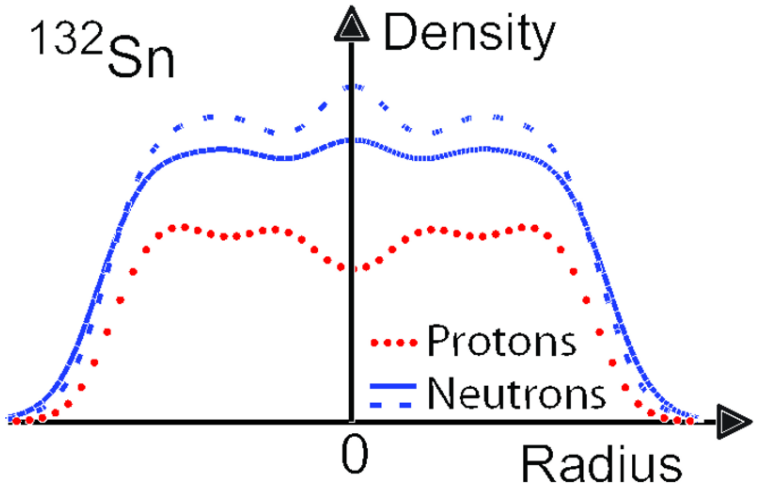
- **Woods-Saxon distribution**

$$\rho(r) = \rho_0 \frac{1}{1 + \exp\left(\frac{r - R_{1/2}}{a}\right)}$$

- **nuclear radii**

$$\rho(R_{1/2}) = \rho_0 \frac{1}{1 + \exp\left(\frac{R_{1/2} - R_{1/2}}{a}\right)} = \rho_0 \frac{1}{1+1} = \frac{1}{2}\rho_0$$





➢ A uniform charge distribution

In spherical case

$$
\begin{aligned}
F(q) &= \int_0^{2\pi} \mathrm{d}\phi \int_0^{\infty} r^2 \mathrm{d}r\ \rho_p(r) \int_0^{\pi} \sin\theta\ \mathrm{d}\theta\ e^{iqr\cos\theta} \quad [\text{note } F(\vec{q}) \longrightarrow F(q)] \\
&= 2\pi \int_0^{\infty} r^2 \mathrm{d}r\ \rho_p(r) \int_0^{\pi} \sin\theta\ \mathrm{d}\theta\ e^{iqr\cos\theta} \quad [\text{did the integral over } \phi] \\
&= 2\pi \int_0^{\infty} r^2 \mathrm{d}r\ \rho_p(r) \int_{-1}^{1} \mathrm{d}\mu\ e^{iqr\mu} \quad [\text{change of variable } \mu = \cos\theta] \\
&= 2\pi \int_0^{\infty} r \mathrm{d}r\ \rho_p(r) \left(\frac{2}{q}\right) \sin qr \quad [\text{did the integral over } \mu] \\
&= \frac{4\pi}{q} \int_0^{\infty} r \mathrm{d}r\ \rho_p(r) \sin qr \quad [\text{in final form}]
\end{aligned}
$$

Form factor

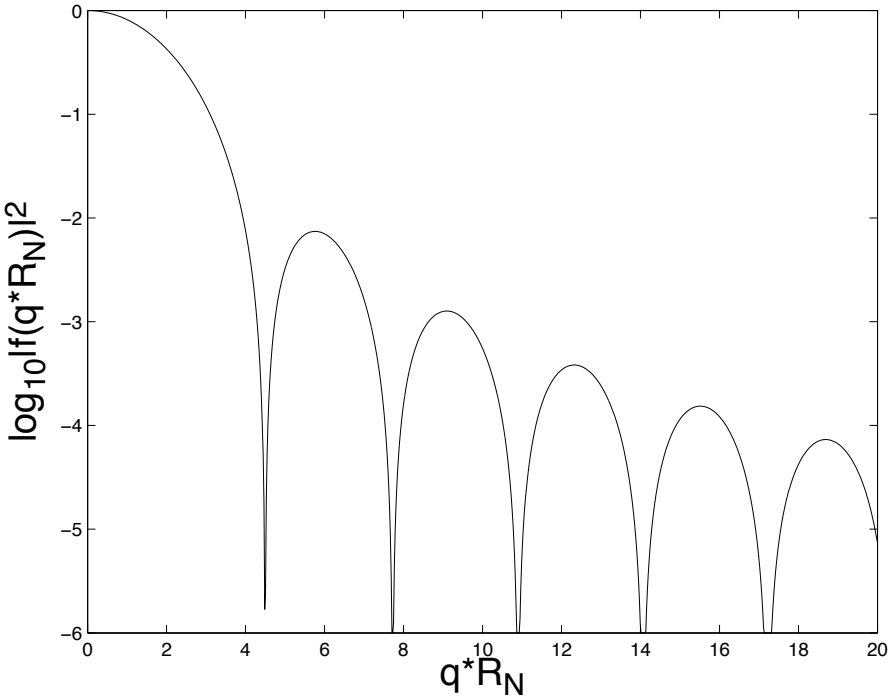
- **A uniform charge distribution**

$$\rho_p(r) = \frac{3}{4\pi R_N^3}\Theta(R_N - r)$$

- **Form factor from in uniform charge distribution**

$$F(q) = \frac{3}{(qR_N)^3}\int_0^{(qR_N)} \mathrm{d}z\ z\sin z$$

$$= \frac{3[\sin(qR_N) - qR_N\cos(qR_N)}{(qR_N)^3}$$



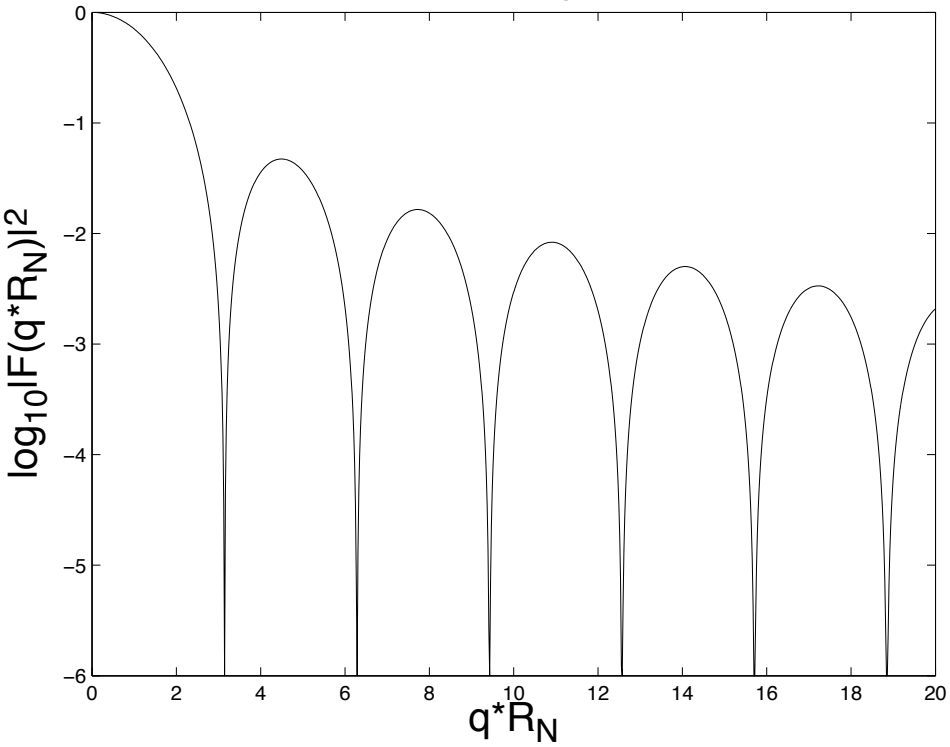
Form factor

- **A delta charge distribution**

$$\rho_p(r) = \delta(R_N - r)/4\pi R_N^2$$

- **Form factor from in delta charge distribution**

$$F(q) = \sin(qR_N)/qR_N$$



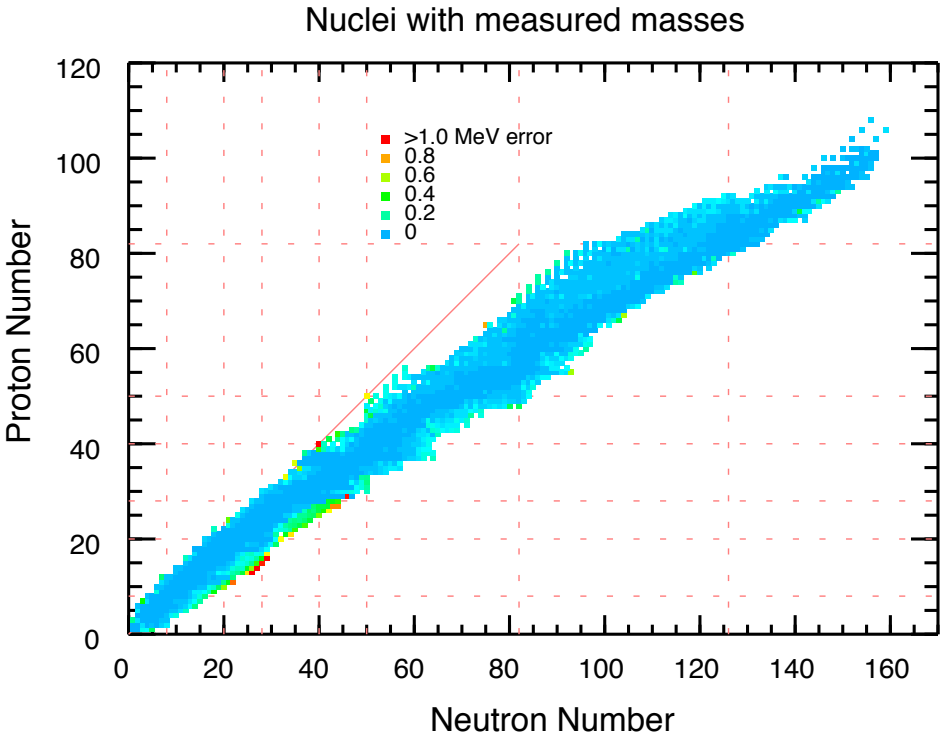
Nuclear mass

- **Mass excess**

$$\Delta(N,Z) \equiv M(N,Z) - uA,$$

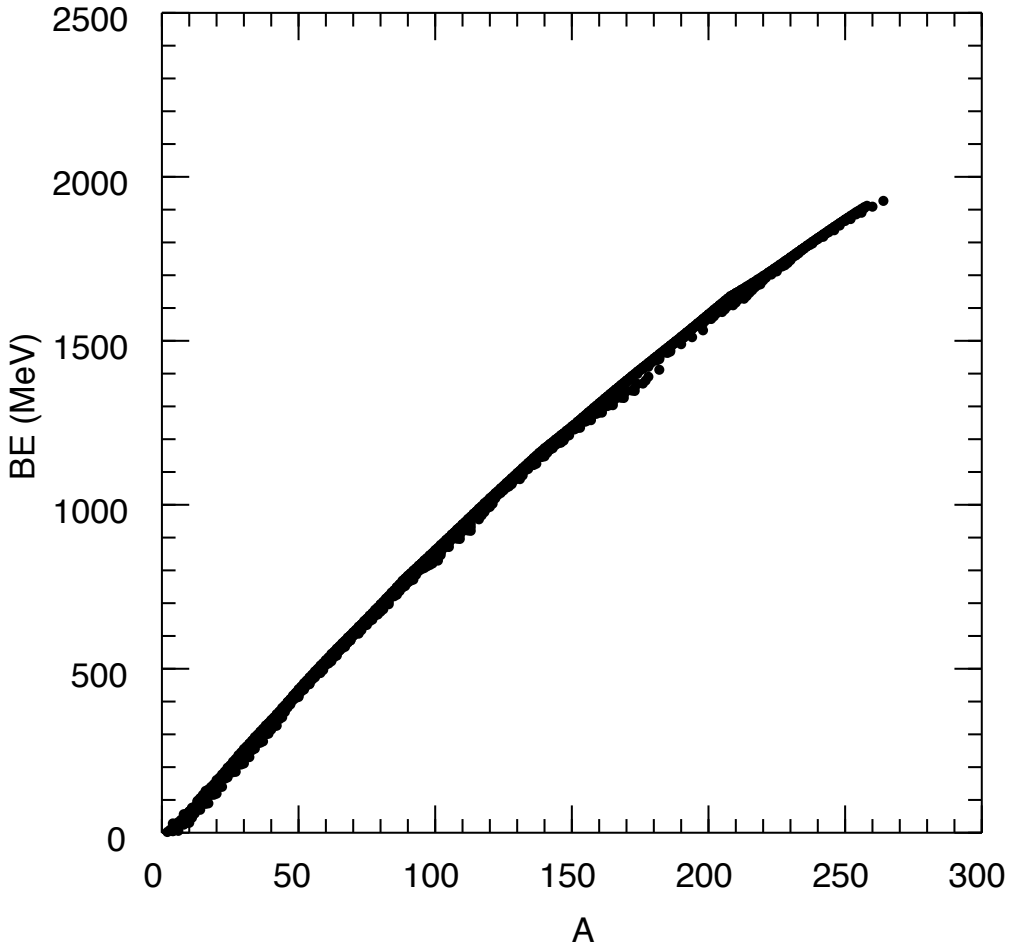
- **Atomic Mass Unit**

$$u = M(^{12}\mathrm{C})/12 = 931.49386\,\mathrm{MeV}/c^2$$



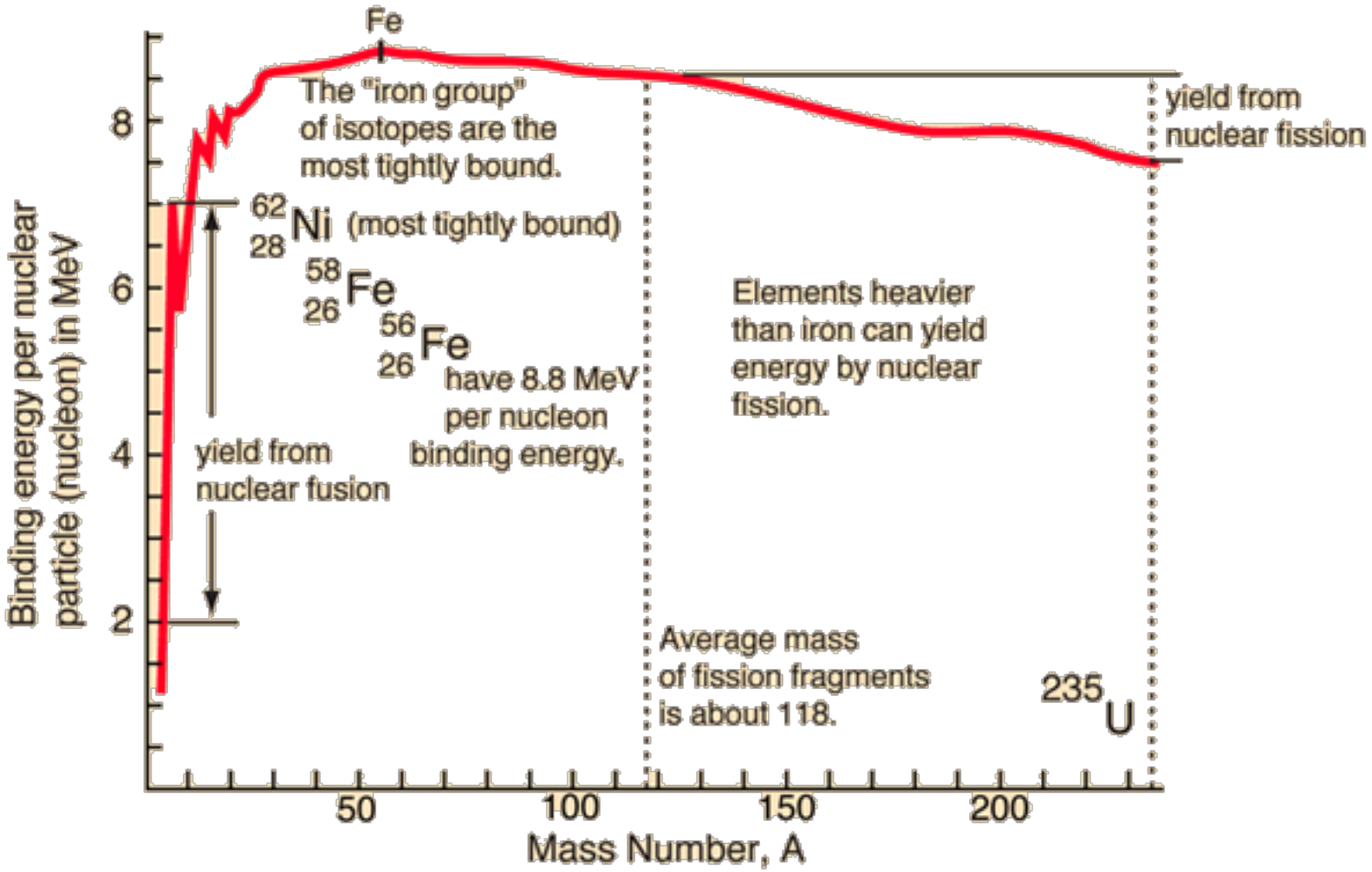
➢ Nuclear binding energy

$$B_N(Z,A) = \left\{ Zm_p + Nm_n - \left[m(^A X) - Zm_e \right] \right\} c^2$$



Nuclear binding energy

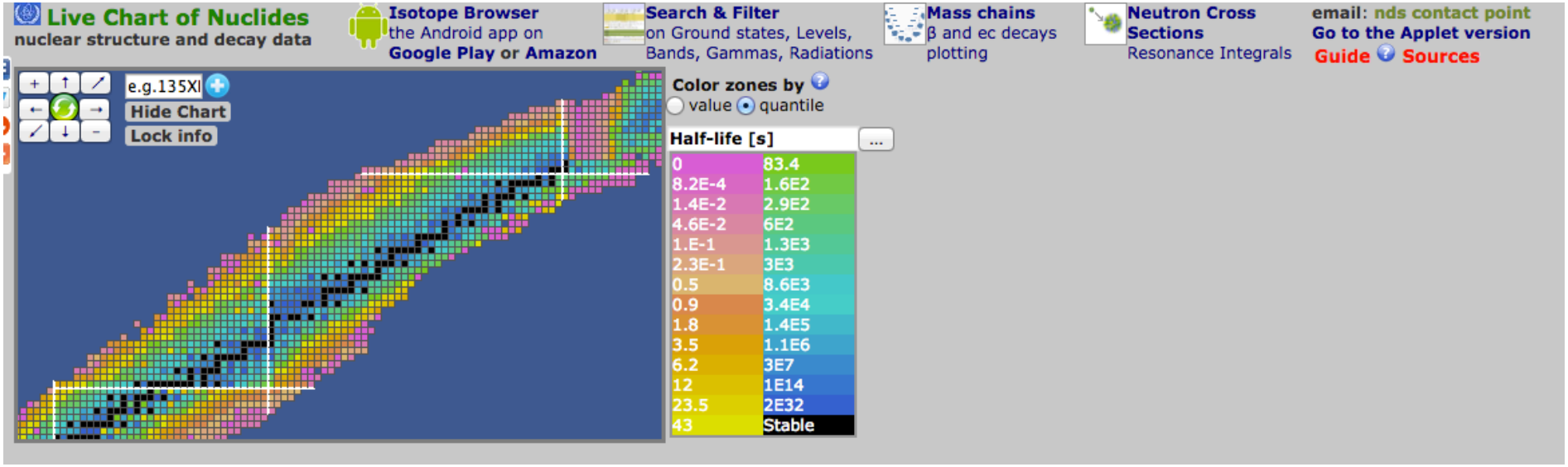
- Binding energy per nucleon



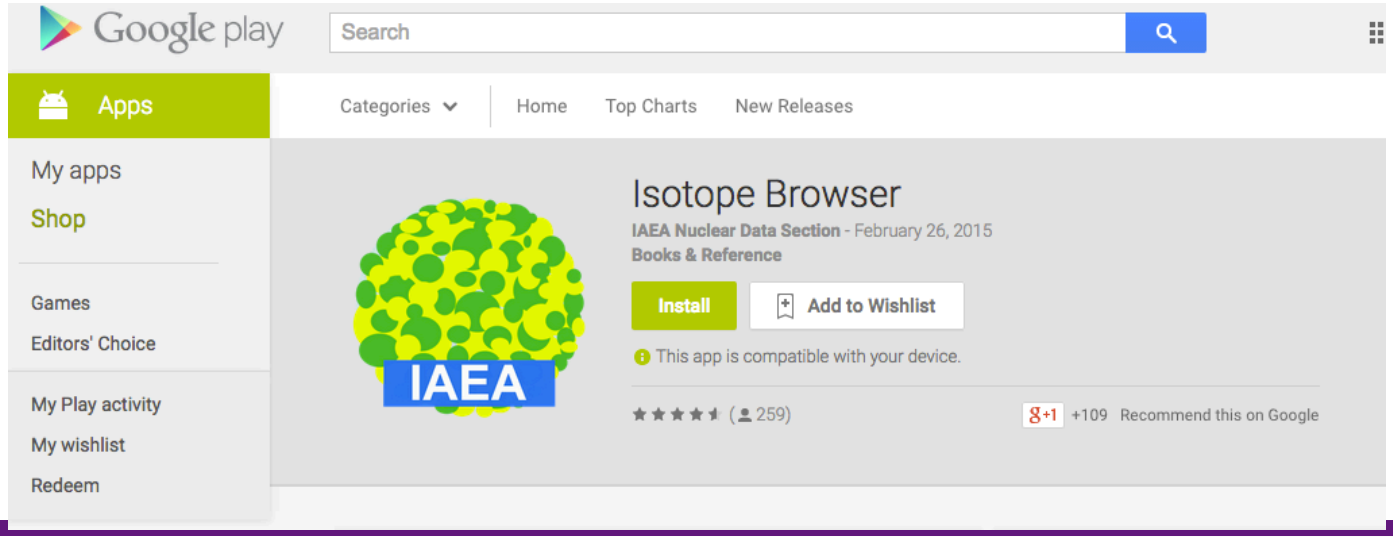
Nuclear mass table

Element	Z	N	A	Atomic mass M_A (u)	Mass excess $M_A - A$ (μu)	Mass defect ΔM_A (μu)	Binding energy E_B (MeV)	E_B/A (MeV/A)
n	0	1	1	1.008665	8665	0	—	—
H	1	0	1	1.007825	7825	0	—	—
D	1	1	2	2.014102	14102	−2388	2.22	1.11
T	1	2	3	3.016049	16049	−9106	8.48	2.83
He	2	1	3	3.016029	16029	−8286	7.72	2.57
He	2	2	4	4.002603	2603	−30377	28.30	7.07
He	2	4	6	6.018886	18886	−31424	29.27	4.88
Li	3	3	6	6.015121	15121	−34348	32.00	5.33
Li	3	4	7	7.016003	16003	−42132	39.25	5.61
Be	4	3	7	7.016928	16928	−40367	37.60	5.37
Be	4	5	9	9.012182	12182	−62442	58.16	6.46
Be	4	6	10	10.013534	13534	−69755	64.98	6.50
B	5	5	10	10.012937	12937	−69513	64.75	6.48
B	5	6	11	11.009305	9305	−81809	76.20	6.93
C	6	6	12	12.000000	0	−98940	92.16	7.68
N	7	7	14	14.003074	3074	−112356	104.7	7.48
O	8	8	16	15.994915	−5085	−137005	127.6	7.98
F	9	10	19	18.998403	−1597	−158671	147.8	7.78
Ne	10	10	20	19.992436	−7564	−172464	160.6	8.03
Na	11	12	23	22.989768	−10232	−200287	186.6	8.11
Mg	12	12	24	23.985042	−14958	−212837	198.3	8.26
Al	13	14	27	26.981539	−18461	−241495	225.0	8.33
Si	14	14	28	27.976927	−23073	−253932	236.5	8.45
P	15	16	31	30.973762	−26238	−282252	262.9	8.48
K	19	20	39	38.963707	−36293	−358266	333.7	8.56
Co	27	32	59	58.933198	−66802	−555355	517.3	8.77
Zr	40	54	94	93.906315	−93685	−874591	814.7	8.67
Ce	58	82	140	139.905433	−94567	−1258941	1172.7	8.38
Ta	73	108	181	180.947993	−52007	−1559045	1452.2	8.02
Hg	80	119	199	198.968254	−31746	−1688872	1573.2	7.91
Th	90	142	232	232.038051	38051	−1896619	1766.7	7.62
U	92	143	235	235.043924	43924	−1915060	1783.9	7.59
U	92	144	236	236.045563	45563	−1922087	1790.4	7.59
U	92	146	238	238.050785	50785	−1934195	1801.7	7.57
Pu	94	146	240	240.053808	53808	−1946821	1813.5	7.56

Nuclear mass table



https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html



- **Nuclear combinations**

$$\sum_i [N,Z]_i \rightarrow \sum_f [N,Z]_f$$

- **Particle conserved**

$$\sum_i N_i = \sum_f N_f \text{ and } \sum_i Z_i = \sum_f Z_f$$

- **Q values**

$$Q = \sum_i M(N_i,Z_i)c^2 - \sum_f M(N_f,Z_f)c^2 = \sum_f B(N_f,Z_f) - \sum_i B(N_i,Z_i)$$

- **One neutron separation energy**

$$S_n = -Q_n = B(N,Z) - B(N-1,Z)$$

- **One proton separation energy**

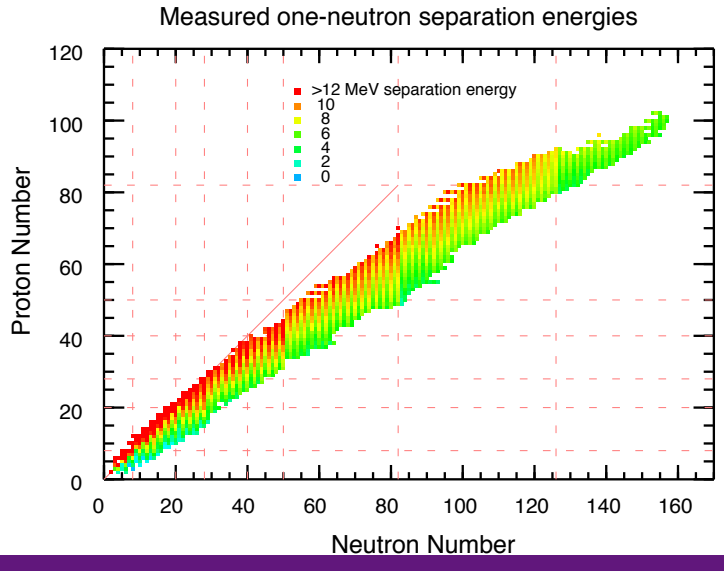
$$S_p = -Q_p = B(N,Z) - B(N,Z-1)$$

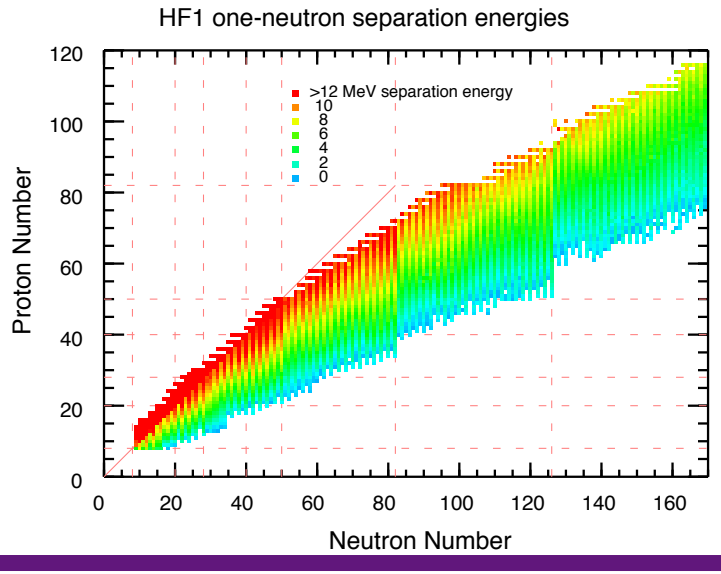
- **Two neutron separation energy**

$$S_{2n} = -Q_{2n} = B(N,Z) - B(N-2,Z)$$

- **Two proton separation energy**

$$S_{2p} = -Q_{2p} = B(N,Z) - B(N,Z-2)$$



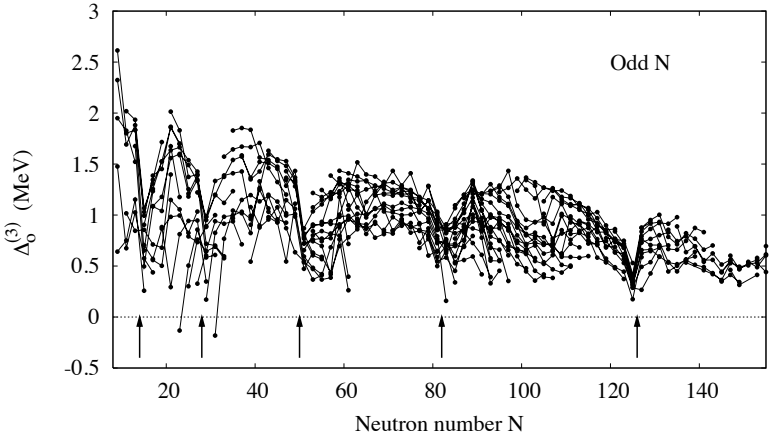


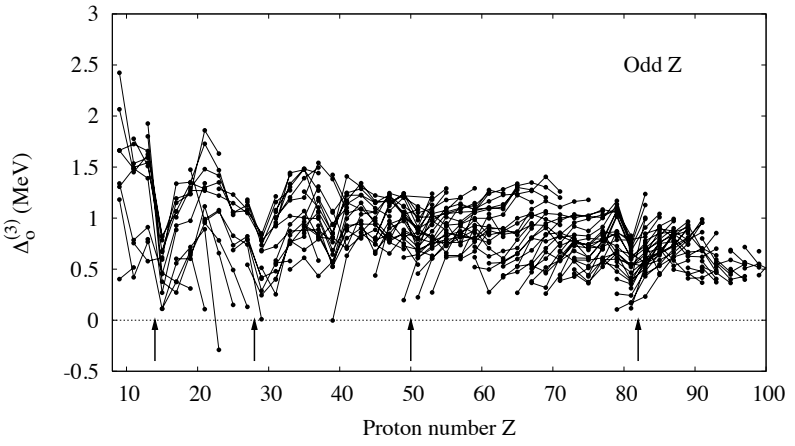
南開大學

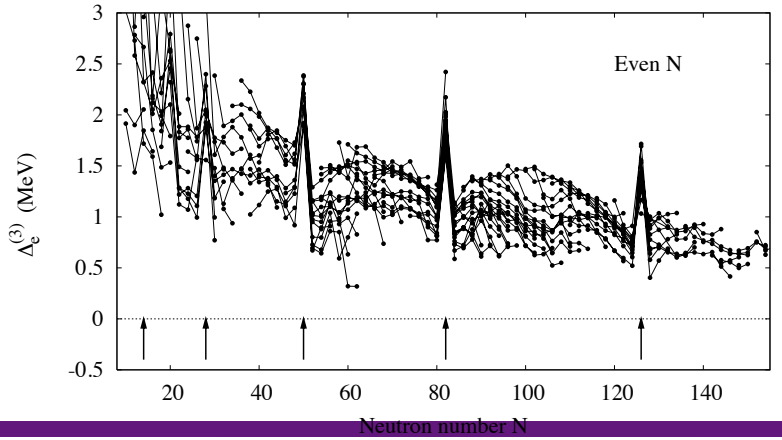
➢ Pairing energy

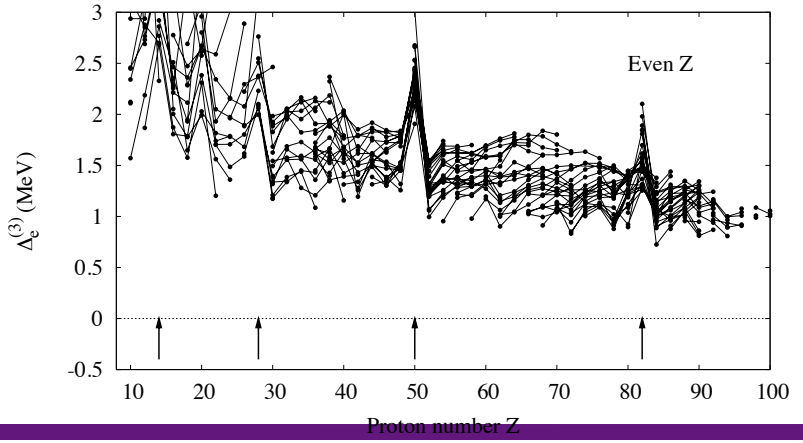
$$\Delta S_n = B(N,Z) - B(N-1,Z) - [B(N+1,Z) - B(N,Z)]$$

$$= 2B(N,Z) - B(N-1,Z) - B(N+1,Z).$$







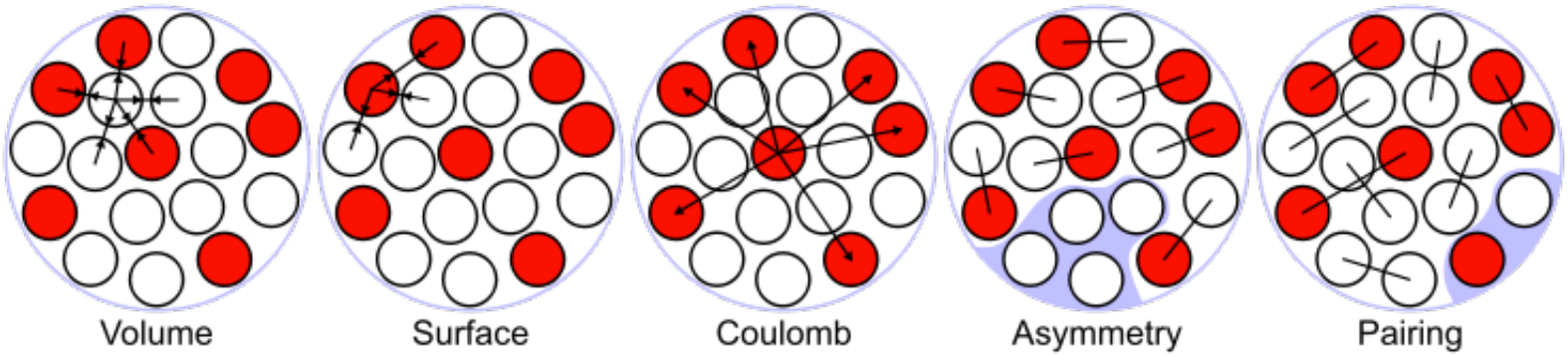


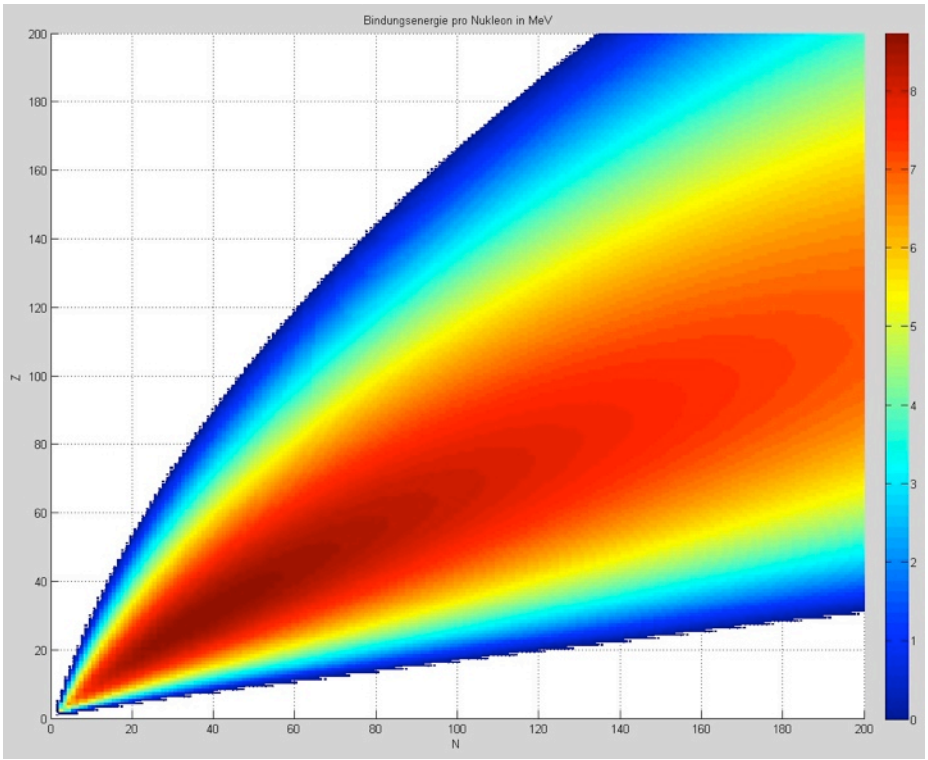
➢ Semi-empirical mass formula

$$
\begin{aligned}
B(Z,A) = \quad & a_{\mathrm{V}} A && (\text{“volume” term}) \\
& -a_{\mathrm{S}} A^{2/3} && (\text{“surface” term}) \\
& -a_{\mathrm{C}} Z(Z-1) A^{-1/3} && (\text{“Coulomb repulsion” term}) \\
& -a_{\mathrm{sym}} \frac{(A-2Z)^2}{A} && (\text{“symmetry” term}) \\
& +a_{\mathrm{p}} \frac{(-1)^Z [1+(-1)^A]}{2} A^{-3/4} && (\text{“pairing” term})
\end{aligned}
$$

a_i	[MeV]	Description	Source
a_{V}	15.5	Volume attraction	Liquid Drop Model
a_{S}	16.8	Surface repulsion	Liquid Drop Model
a_{C}	0.72	Coulomb repulsion	Liquid Drop Model + Electrostatics
a_{sym}	23	n/p symmetry	Shell model
a_{p}	34	n/n, p/p pairing	Shell model

Semi-empirical mass formula

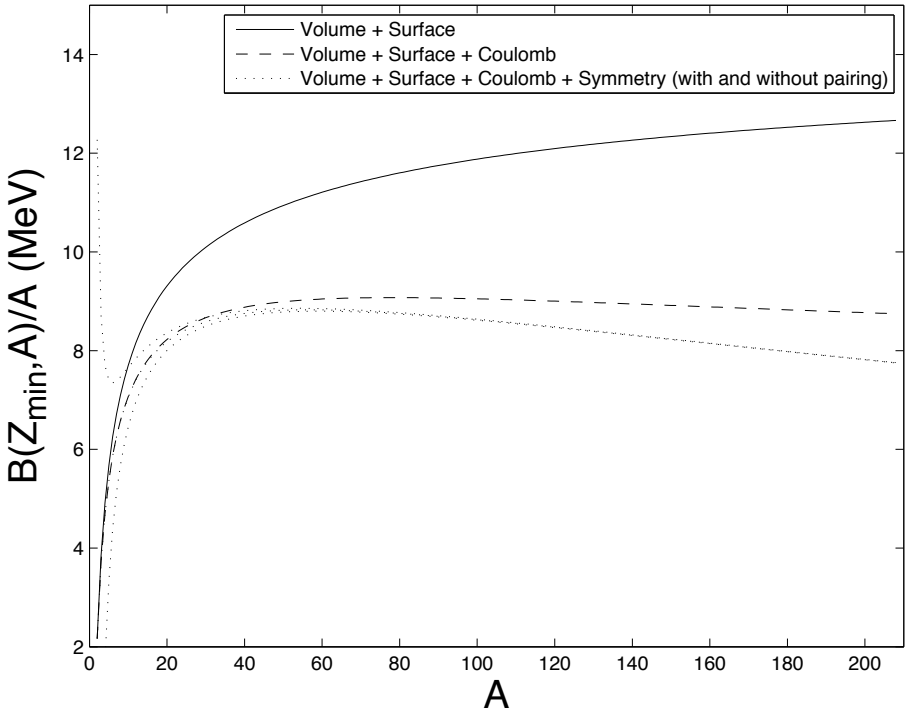




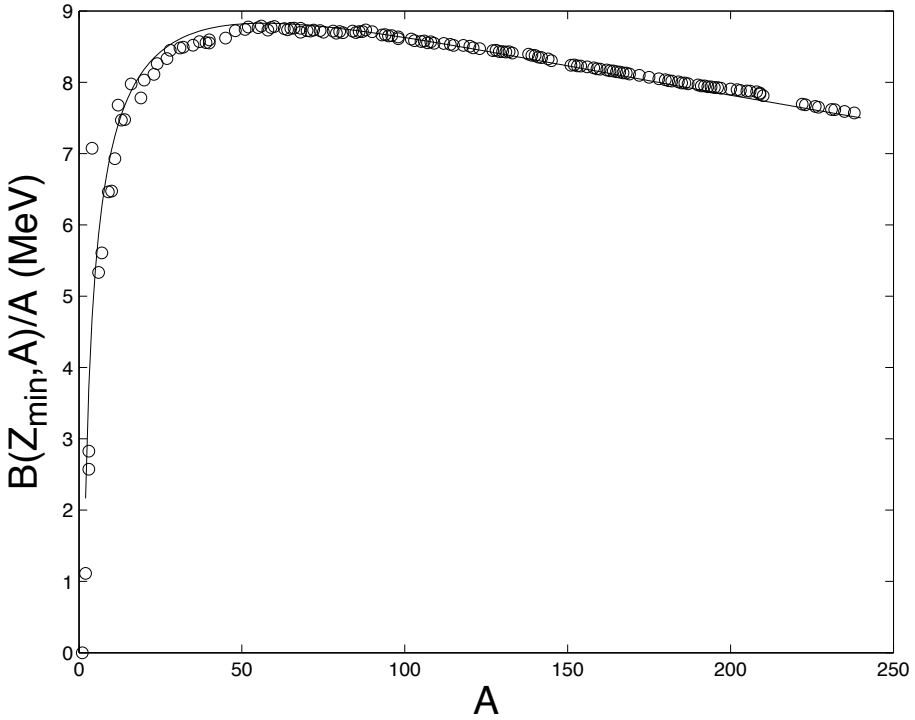
Semi-empirical mass formula

$$B(Z,A) = a_{\rm V}A - a_{\rm S}A^{2/3} - a_{\rm C}Z(Z-1)A^{-1/3} - a_{\rm sym}\frac{(A-2Z)^2}{A} + a_{\rm p}\frac{(-1)^Z[1+(-1)^A]}{2}A^{-3/4}$$

Binding energy per nucleon *vs.* A



Binding energy per nucleon *vs.* data



Nuclear stability

- α decay

$$^{A}Z \rightarrow {}^{(A-4)}(Z-2) + {}^{4}\mathrm{He}$$

- Electron capture

$$e^{-} + {}^{A}Z \rightarrow {}^{A}(Z-1) + \nu_e$$

- β- decay

$$^{A}Z \rightarrow {}^{A}(Z+1) + e^{-} + \bar{\nu}_e$$

- β+ decay

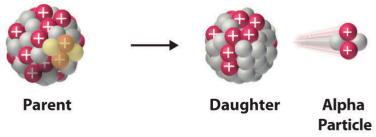
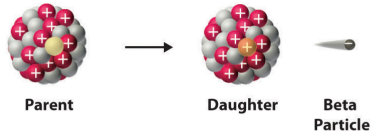
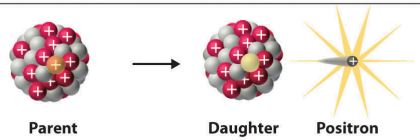
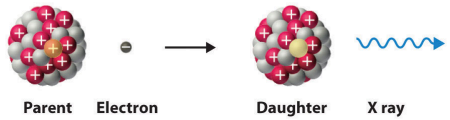
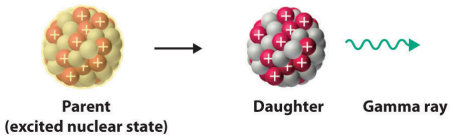
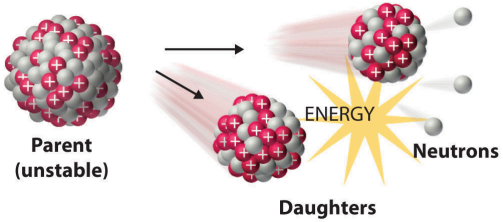
$$^{A}Z \rightarrow {}^{A}(Z-1) + e^{+} + \nu_e$$

- Light fragment emission

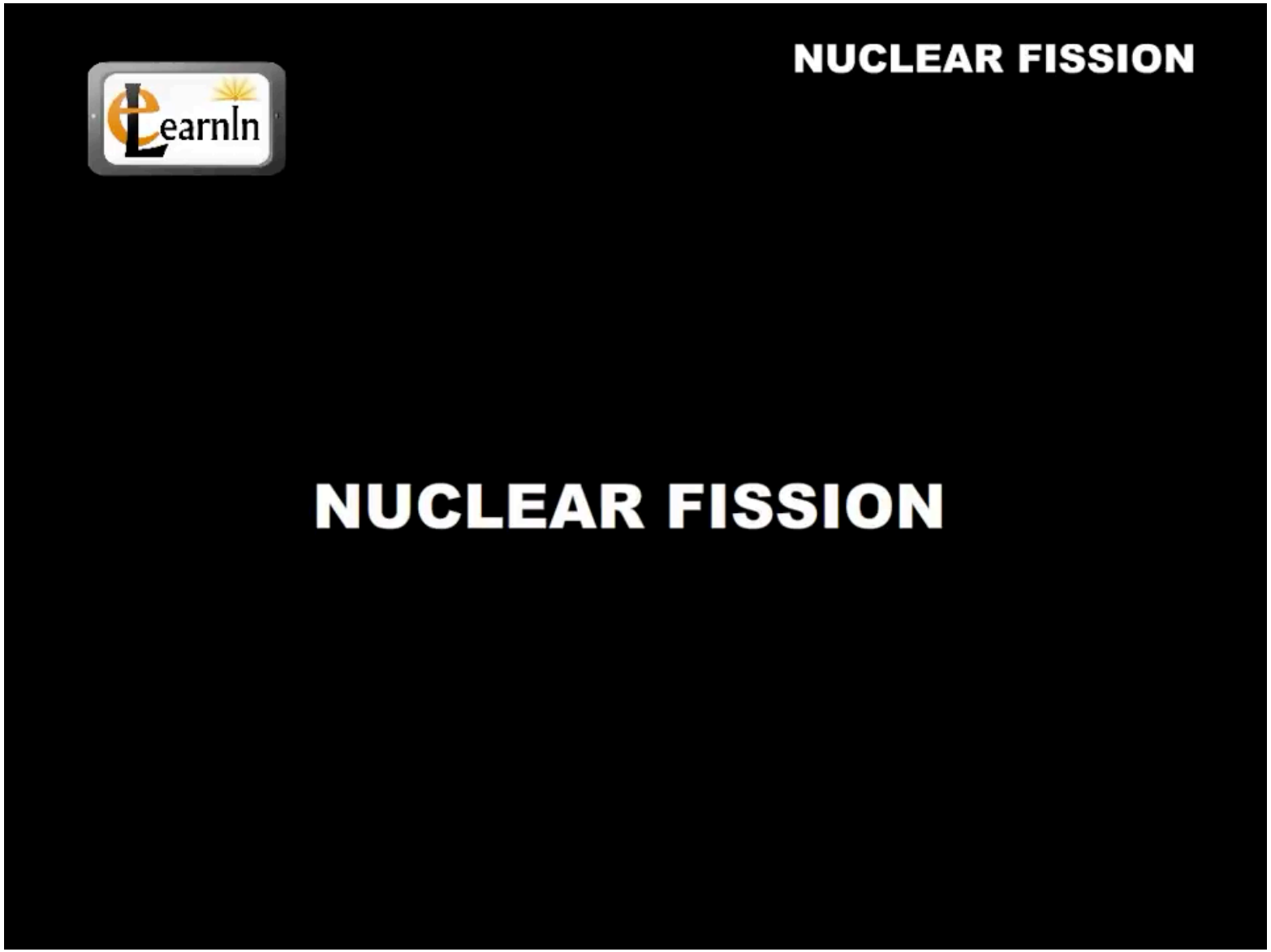
- Fission

Nuclear stability

Nuclear stability

Decay Type	Radiation Emitted	Generic Equation	Model
Alpha decay	${}^{4}_{2}\alpha$	${}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}X' + {}^{4}_{2}\alpha$	
Beta decay	${}^{0}_{-1}\beta$	${}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}X' + {}^{0}_{-1}\beta$	
Positron emission	${}^{0}_{+1}\beta$	${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}X' + {}^{0}_{+1}\beta$	
Electron capture	X rays	${}^{A}_{Z}X + {}^{0}_{-1}e \longrightarrow {}^{A}_{Z-1}X' + \text{X ray}$	
Gamma emission	${}^{0}_{0}\gamma$	${}^{A}_{Z}X^{*} \xrightarrow{\text{Relaxation}} {}^{A}_{Z}X' + {}^{0}_{0}\gamma$	
Spontaneous fission	Neutrons	${}^{A+B+C}_{Z+Y}X \longrightarrow {}^{A}_{Z}X' + {}^{B}_{Y}X' + C\,{}^{1}_{0}n$	

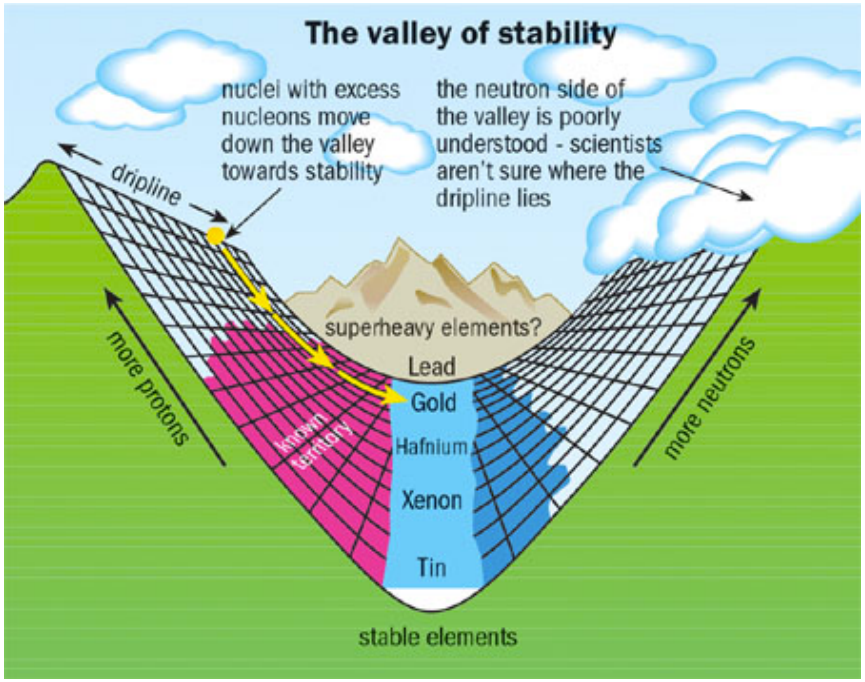
Nuclear stability

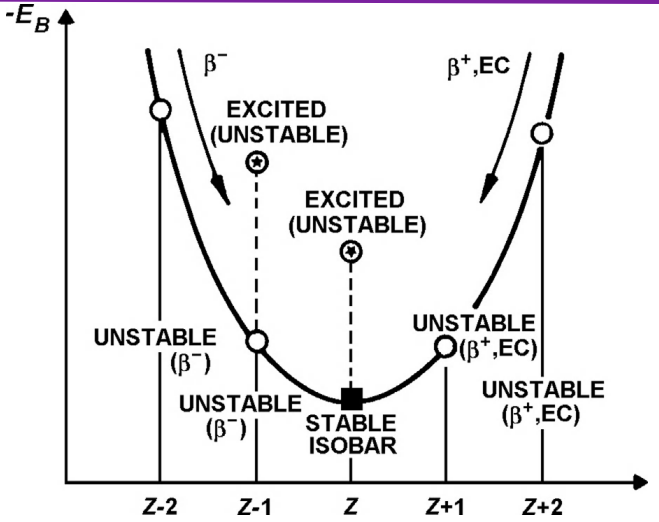


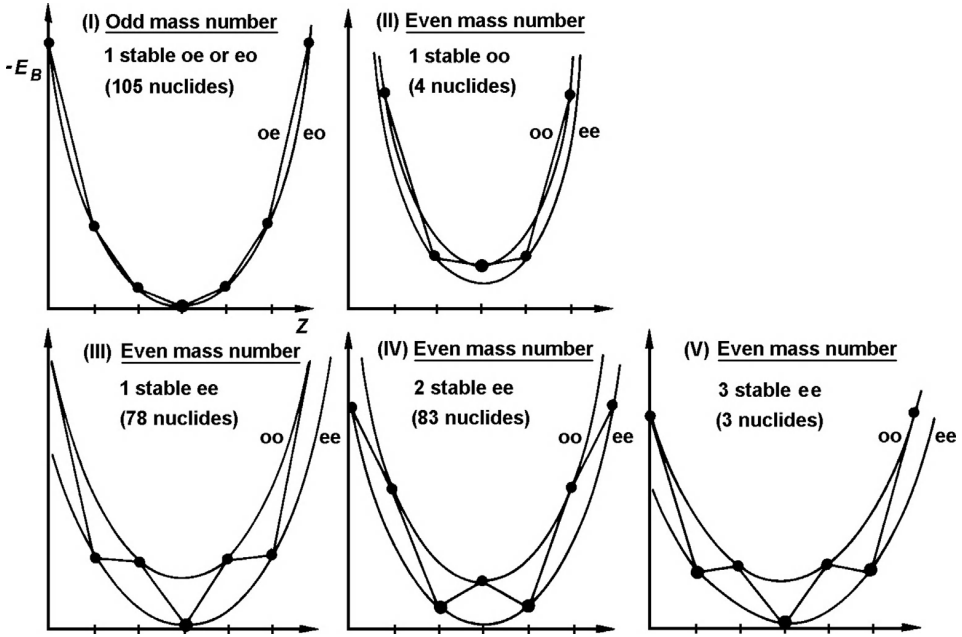
➢ Nuclear stability

$$Z_{\min} = \frac{[m_n - m(^1\mathrm{H})]c^2 + a_{\mathrm{C}}A^{-1/3} + 4a_{\mathrm{sym}}}{2a_{\mathrm{C}}A^{-1/3} + 8a_{\mathrm{sym}}A^{-1}}$$

$$Z_{\min} \approx \frac{A}{2}\frac{1}{1 + (1/4)(a_{\mathrm{C}}/a_{\mathrm{sym}})A^{2/3}}$$



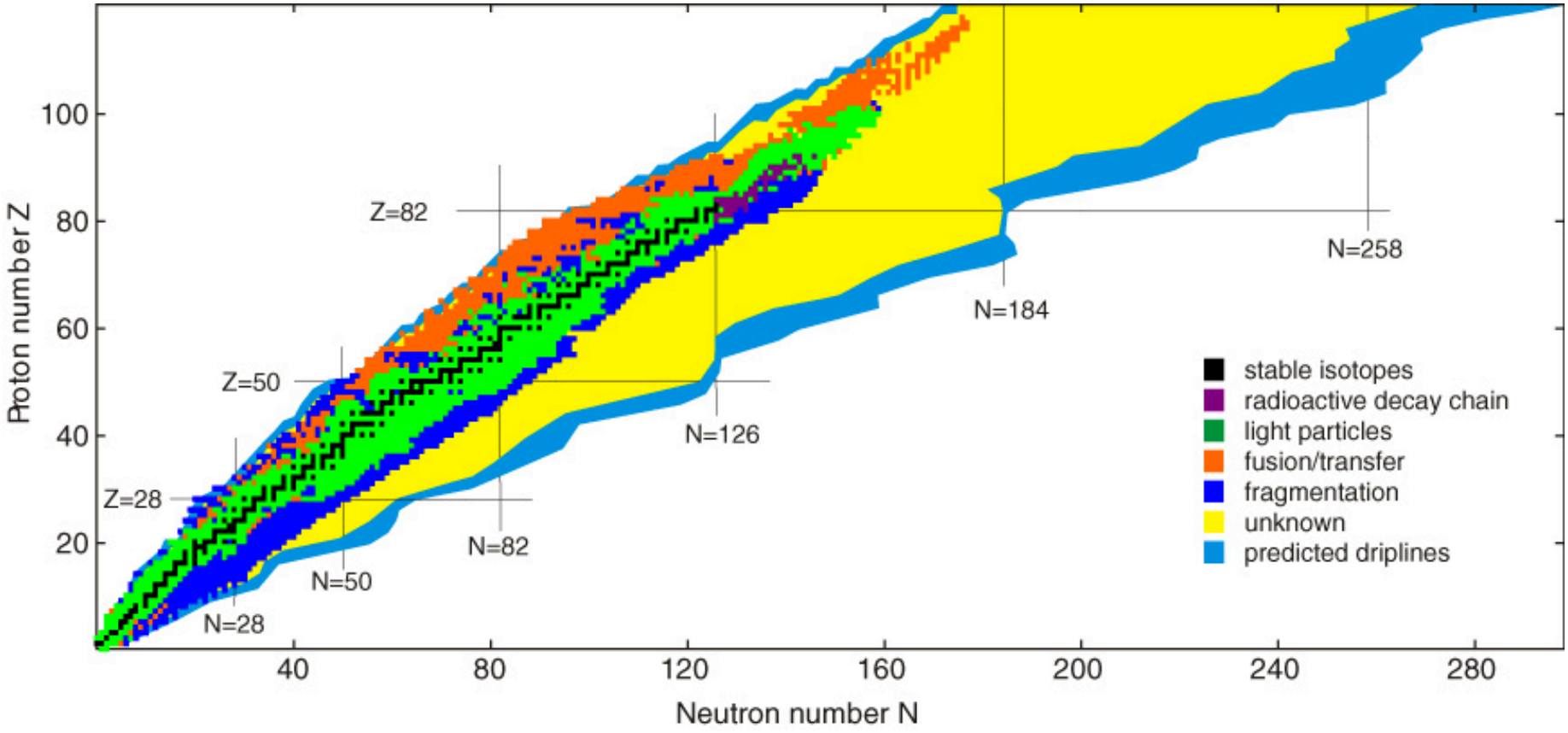




Stability Nuclei

Type of nuclide by stability class	Number of nuclides in class	Running total of nuclides in all classes to this point	Notes
Theoretically stable to according to the Standard Model	90	90	Includes first 40 elements. If protons decay, then there are no stable nuclides.
Theoretically stable to alpha decay, beta decay, isomeric transition, and double beta decay but not spontaneous fission, which is possible for "stable" nuclides ⩾ niobium–93	56	146	Note that spontaneous fission has never been observed for nuclides with mass number < 230.
Energetically unstable to one or more known decay modes, but no decay yet seen. Considered stable until radioactivity confirmed.	106 [*citation needed*]	252	Total is the observationally stable nuclides.
Radioactive primordial nuclides.	34	286	Includes Bi, Th, U
Radioactive nonprimordial, but naturally occurring on Earth.	~61 significant	~347 significant	Cosmogenic nuclides from cosmic rays; daughters of radioactive primordials such as francium, etc.

Nuclear stability



Angular momentum and parity

南開大學

Symbol	meaning
l_i	orbital angular momentum of the i'th nucleon
s_i	intrinsic spin angular momentum of the i'th nucleon
j_i	total angular momentum of the i'th nucleon, *i.e.* $\vec{j_i} = \vec{l_i} + \vec{s_i}$
$\vec{L}$	sum of all orbital angular momenta in a nucleus *i.e.* $\vec{L} = \sum_{i=1}^{A} \vec{l_i}$
$\vec{S}$	sum of all intrinsic spin angular momenta in a nucleus *i.e.* $\vec{S} = \sum_{i=1}^{A} \vec{s_i}$

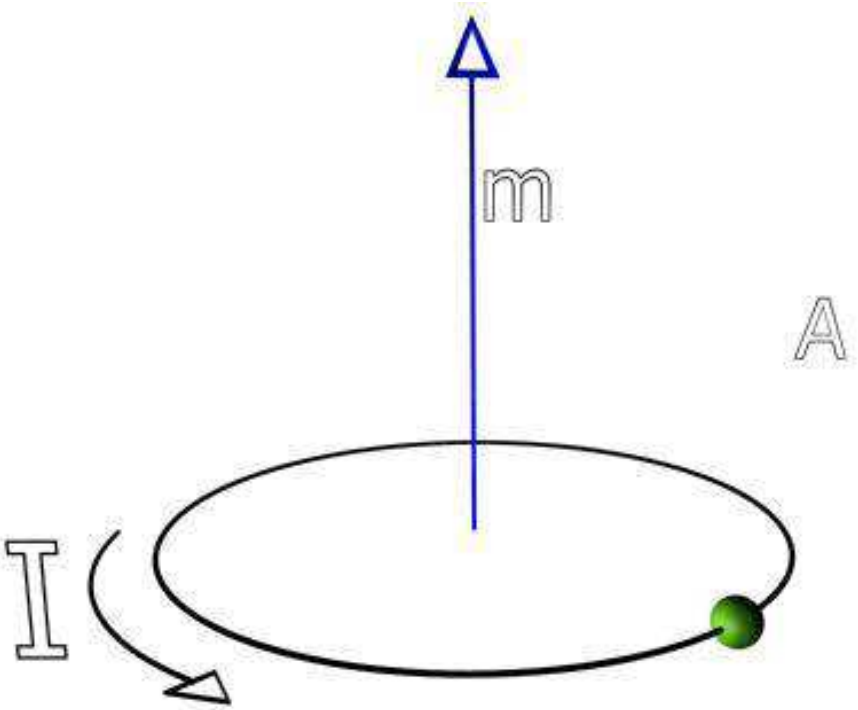
➢ Nuclear angular momentum

$$\vec{I} = \sum_{i=1}^{A} (\vec{l_i} + \vec{s_i}) = \vec{L} + \vec{S}$$

$$\begin{aligned} \langle \vec{I}^2 \rangle &= \hbar^2 I(I+1) \\ I &= 0, \frac{1}{2}, 1, \frac{3}{2} \cdots \\ \langle I_z \rangle &= \hbar m_I \\ m_I &: -I \leq m_I \leq I \\ \Delta m_I &: \text{integral jumps} \end{aligned}$$

$$\begin{aligned} I &= n\hbar \quad \text{for odd-odd nuclei} \\ I &= (n + \frac{1}{2})\hbar \quad \text{for odd-A nuclei} \\ I &= 0 \quad \text{for even-even nuclei} \end{aligned}$$

Nuclear magnetic moment



➢ **Nuclear magnetic moment**

$$\vec{\mu} = \frac{1}{2}\int \mathrm{d}\vec{x}\ \vec{x} \times J(\vec{x})$$

➢ Nuclear magnetic moment

$$\mu_z = g^l l_z + g^s s_z$$

so

$$\mu = g^l j + (g^s - g^l)\langle s_z \rangle$$

finally

$$\mu = g^l \left(j - \frac{1}{2} \right) + \frac{1}{2} g^s, \quad j = l + \frac{1}{2}$$

$$\mu = g^l \frac{j(j+\frac{3}{2})}{j+1} - \frac{j g^s}{2(j+1)}, \quad j = l - \frac{1}{2}$$

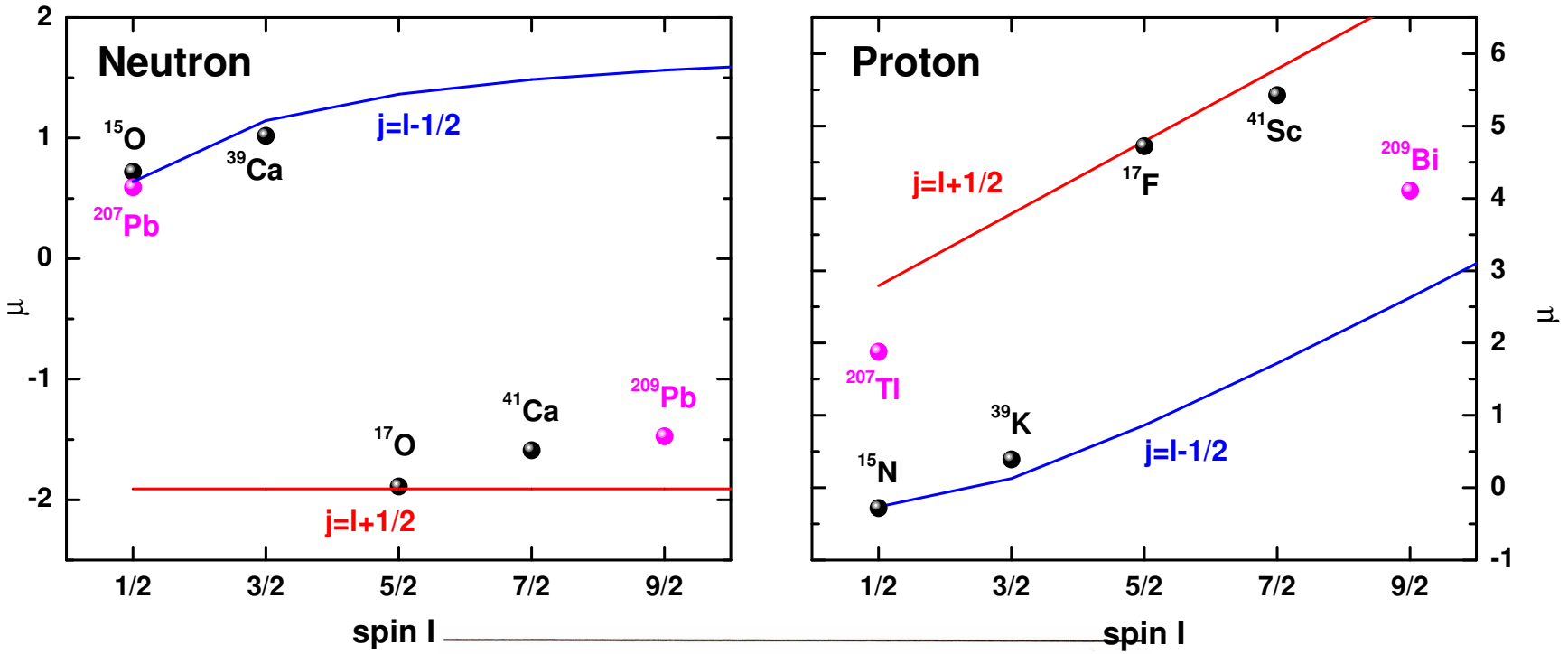
where, we used

$$\langle s_z \rangle = \left\langle \frac{(\vec{j} \cdot \vec{s}) j_z}{j^2} \right\rangle$$

$$= \frac{j}{2j(j+1)} [j(j+1) - l(l+1) + s(s+1)]$$

$$|ljm> = \sum_{m_l m_s} | < l m_l \frac{1}{2} m_s | jm > | l m_l > | \frac{1}{2} m_s >$$

Nuclear magnetic moment



Nuclide	$\mu(\mu_N)$
n	−1.9130418
p	+2.7928456
^{2}H (D)	+0.8574376
^{17}O	−1.89379
^{57}Fe	+0.09062293
^{57}Co	+4.733
^{93}Nb	+6.1705

Nuclear quadrupole moment

南開大學

- **Electrostatic potential of nuclei**

$$V(\vec{x}) = \frac{Ze}{4\pi\epsilon_0}\int \mathrm{d}\vec{x}'\ \frac{\rho_p(\vec{x}')}{|\vec{x}-\vec{x}'|}$$

- **Taylor expansion**

$$V(\vec{x}) = \frac{Ze}{4\pi\epsilon_0}\left[\frac{1}{|\vec{x}|}\int \mathrm{d}\vec{x}'\ \rho_p(\vec{x}') + \frac{\vec{x}}{|\vec{x}|^3}\cdot\int \mathrm{d}\vec{x}'\ \vec{x}'\rho_p(\vec{x}') + \frac{1}{2|\vec{x}|^5}\int \mathrm{d}\vec{x}'\ \left(3(\vec{x}\cdot\vec{x}')^2 - |\vec{x}|^2|\vec{x}'|^2\right)\rho_p(\vec{x}')\cdots\right]$$

It can be simplified

$$V(\vec{x}) = \frac{Ze}{4\pi\epsilon_0}\left[\frac{1}{|\vec{x}|} + \frac{Q}{2|\vec{x}|^3}\cdots\right]$$

where

$$Q = \int \mathrm{d}\vec{x}\ \left(3z^2 - r^2\right)\rho_p(\vec{x})$$

➢ Nuclear quadrupole moment

$$Q = \int \mathrm{d}\vec{x}\ \psi_N^*(\vec{x})(3z^2 - r^2)\psi_N(\vec{x})$$

➢ For j=l+1/2 case

$$\psi = \frac{u_l(r)}{r} Y_{ll}(\theta, \phi)\chi_{\mathrm{spin}}\chi_{\mathrm{isospin}}$$

so

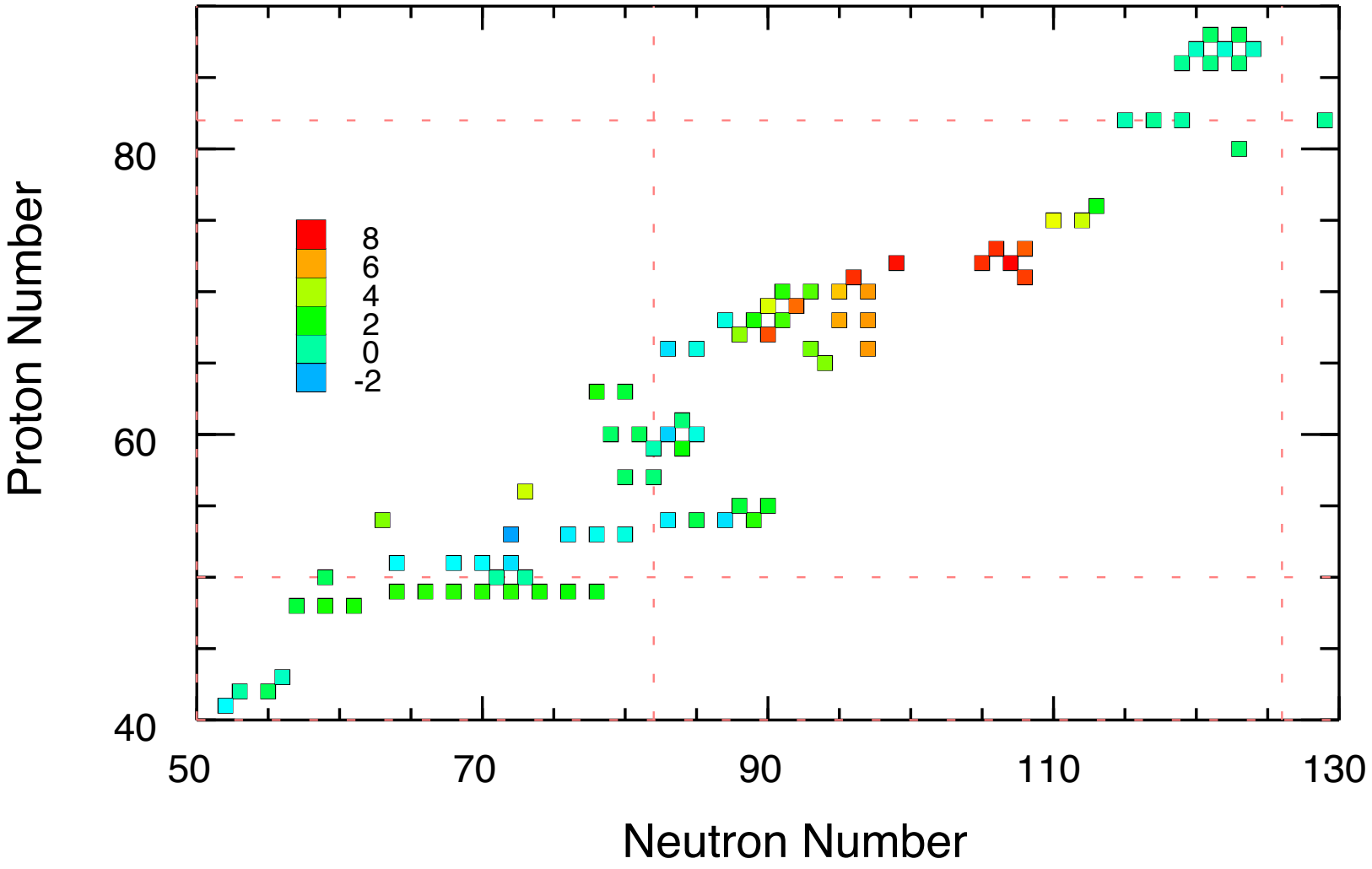
$$Q = \int u_l^2(r)|Y_{ll}(\theta, \phi)|^2 r^2(3\cos^2\theta - 1)\sin\theta dr d\theta d\phi$$

finally

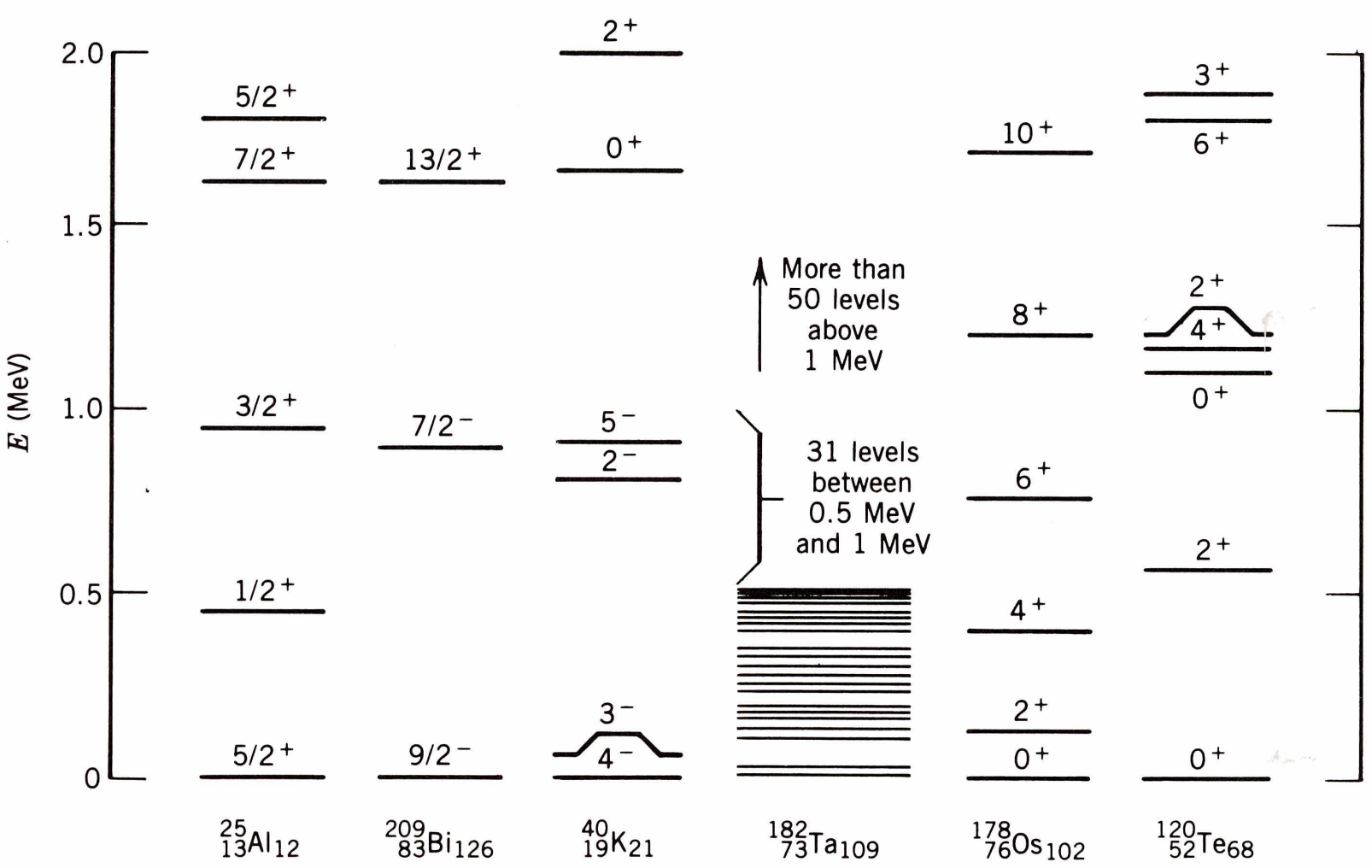
$$Q = -\langle r^2\rangle \frac{2j-1}{2j+2}$$

Nuclear quadrupole moment

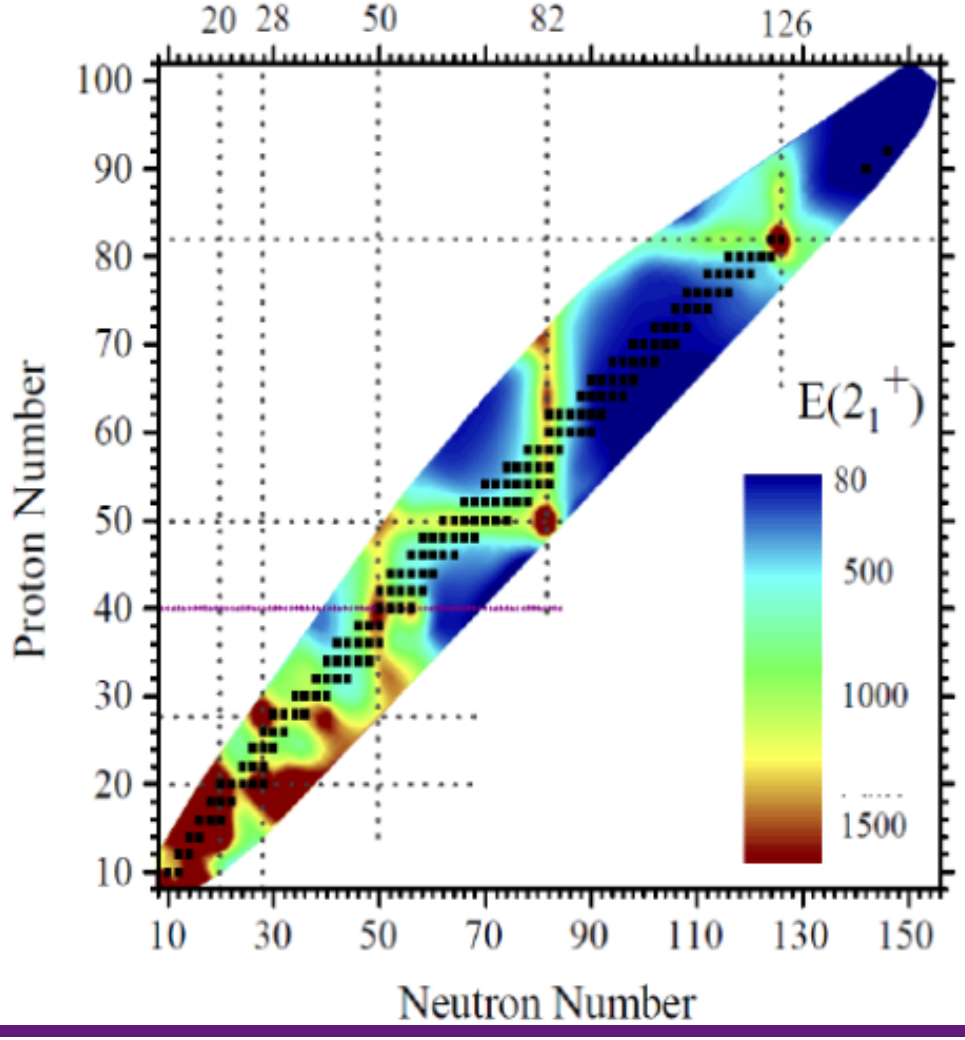
南開大學

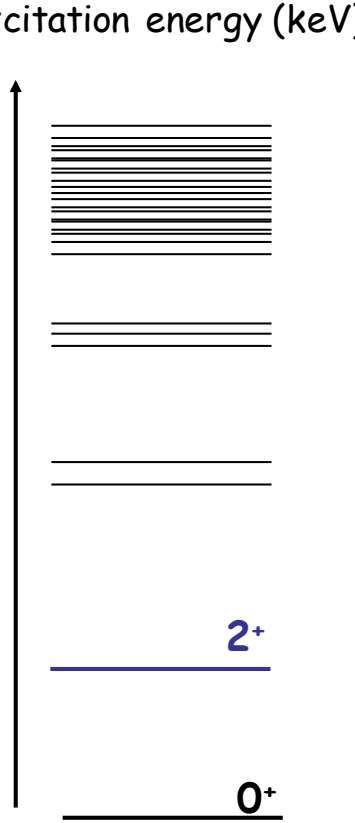


Excited state of nuclei



First excited state of even-even nuclei



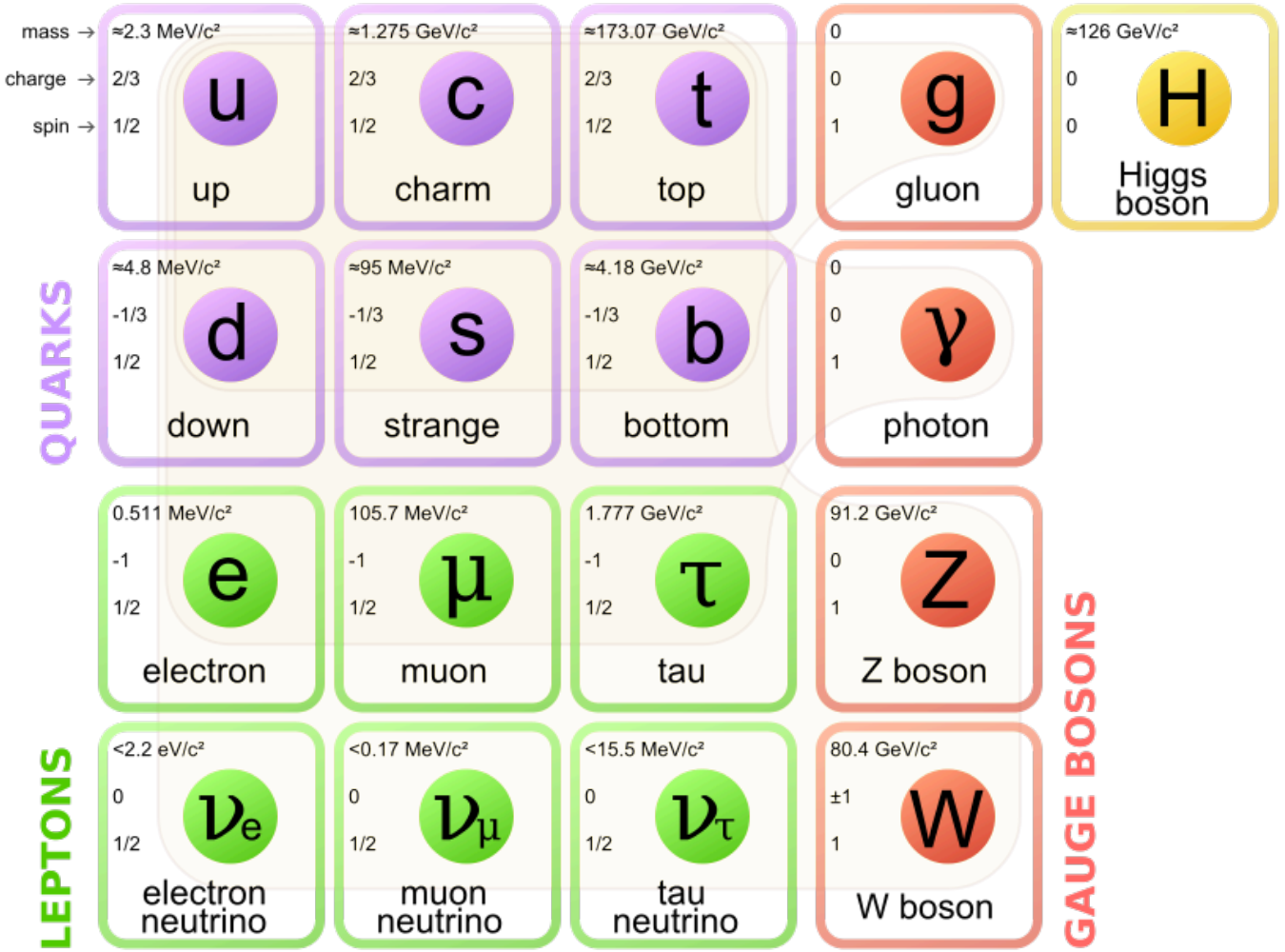


Ground state Configuration.
Spin/parity $I^{\pi}=0^{+}$;
E_x = 0 keV

Nucleon properties

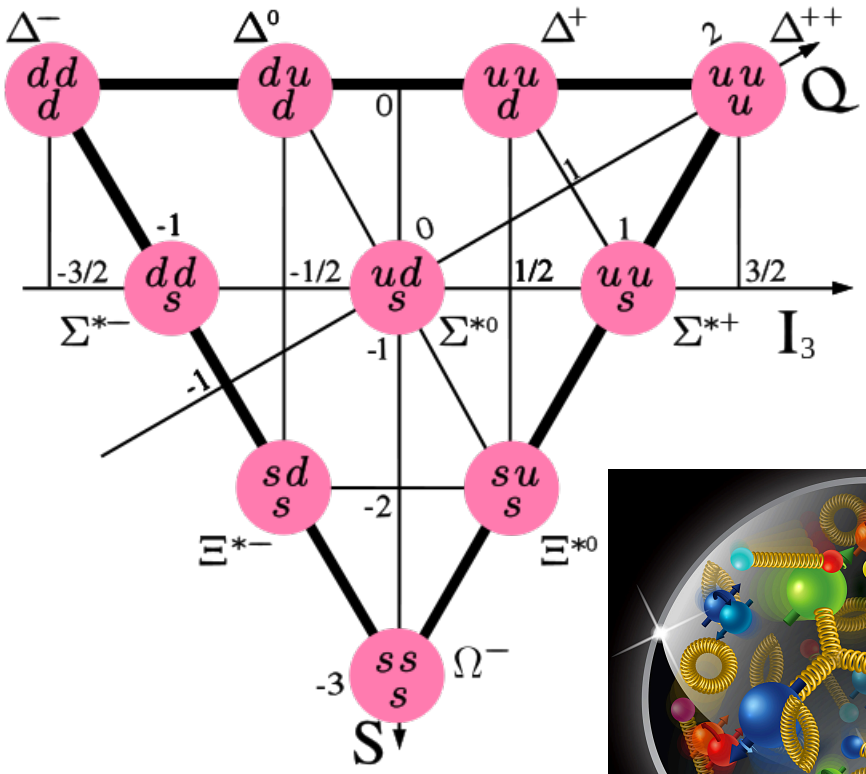
南開大學

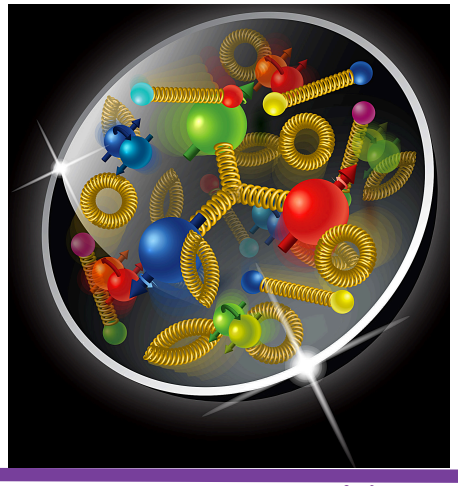
➢ Standard model of particle physics



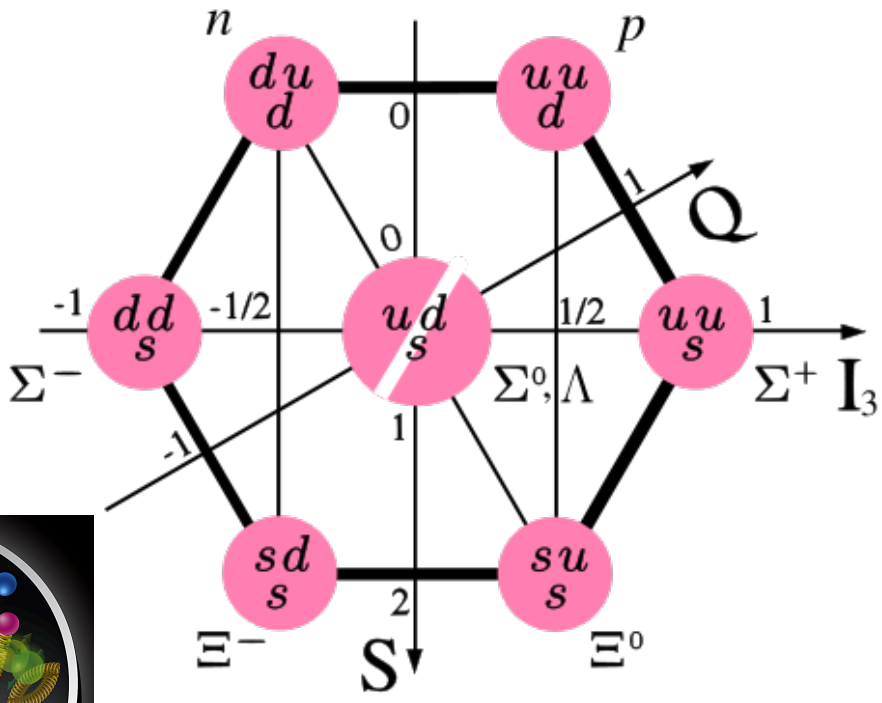
Nucleon properties

➢ Baryon decuplet states





➢ Baryon octet states



Nucleon properties

南開大學

- **Isospin operator**

$$[t_x, t_y] = i\hbar t_z, \quad [t_y, t_z] = i\hbar t_x, \quad [t_z, t_x] = i\hbar t_y$$

$$\vec{t} = \frac{1}{2}\vec{\tau}$$

- **Proton and neutron**

$$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- **Charge operator**

$$Q = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \frac{1}{2}(1 + t_x)$$

$$Qp = 1p, \quad Qn = 0$$

Nucleon properties

南開大學

- **Baryonic number**

$$B = \begin{cases} 1 & \text{Baryon} \\ 0 & \text{Meson} \\ -1 & \text{Antibaryon} \end{cases}$$

- **Strangeness number**

$$S = (n_s - n_{\bar{s}})$$

- **Gell-Mann–Nishijima formula**

$$\begin{aligned} Q &= I_3 + \frac{1}{2}(B+S) \\ &= I_3 + \frac{1}{2}Y \end{aligned}$$

Nucleon properties

南開大學

- Parity operator

$$P(\vec{r}) = -\vec{r}$$

- Parity operator on spherical function

$$PY_{lm}(\vec{r}) = Y_{lm}(-\vec{r}) = (-1)^l Y_{lm}(\vec{r})$$

- Parities of meson and baryons

$$P = \begin{cases} (-1)^l & \text{Baryon} \\ (-1)^{l+1} & \text{Meson} \end{cases}$$

The parity of π is -1

The parities of proton and neutron are 1

Basic interactions

Interactions	Theory	Relative strength	Range(m)
Strong	QCD	10^{38}	10^{-15}
Electromagnetic	QED	10^{36}	infinity
Weak	Electroweak	10^{25}	10^{-18}
Gravity	General Relativity	1	infinity

Characters of Strong interaction

- Strength Strong
- Short range repulsive
- Gauge field
- Non perturbative at low energy

Nucleon-nucleon interaction

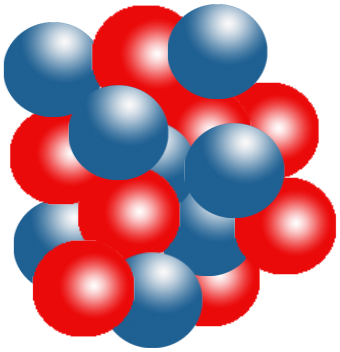
- Realistic nucleon-nucleon (NN) interaction:

  NN interaction in free space



- Effective nucleon-nucleon interaction

  NN interaction in nuclear medium





Effective NN interaction

- **Effective nucleon-nucleon interaction**

  Density functional theory from the properties of nuclear matter or finite nuclei

- **Skyrme interaction (zero range)**

  J. W. Negele and D. Vautherin, Phys. Rev. C 5(1972)1472

- **Gogny interaction (Finite range)**

  J. Decharge and D. Gogny, Phys. Rev. C 21(1980)1568

- **Relativistic mean field (RMF) (meson exchange force)**

  H. P. Duerr, Phys. Rev. 103(1956)469, PK1

- **Relativistic Hartree Fock (RHF)**

  A. Bouyssy, J. -F Mathiot, and N. Van Giai, Phys. Rev. C 36(1987)380, PKO, PKA,..

- **Point coupling interaction (zero range)**

  B.A. Nikolaus, T. Hoch, and D.G. Madland, Phys. Rev. C 46(1992)1757, PC-PK1,

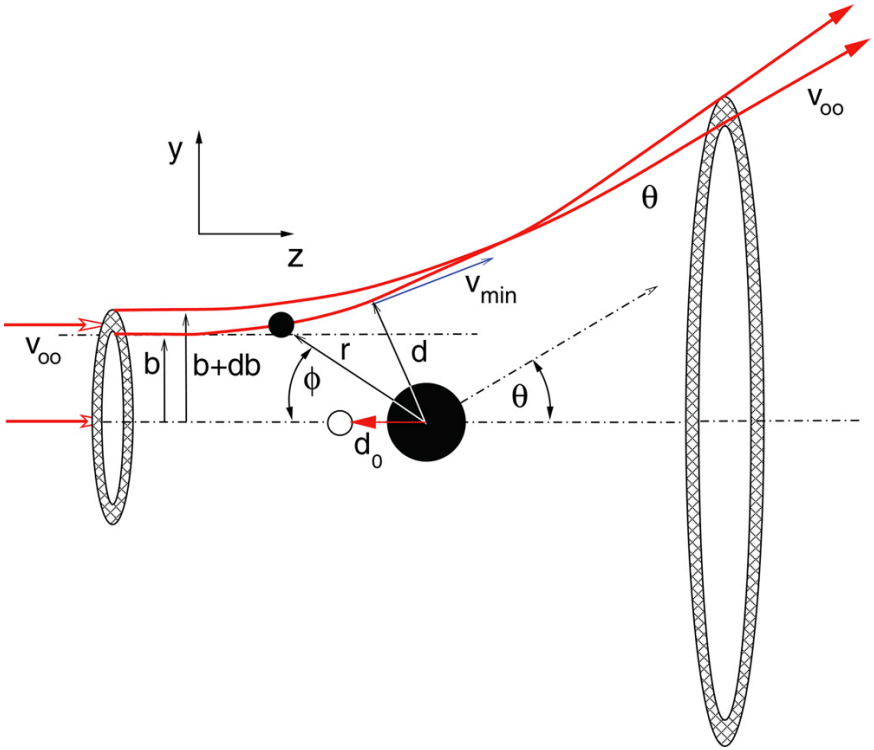
...

History

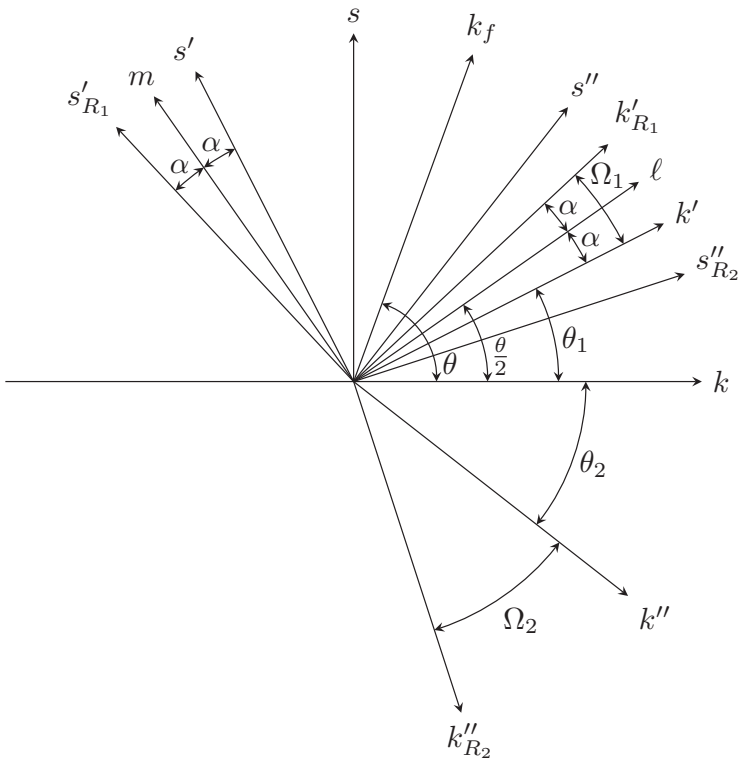
- One pion exchange — Yukawa (1935)
- Multi-pion — Taketani et al. (1951)
- Repulsive — Jastrow (1951)
- One Boson exchange — Bonn Group (1970~)
- EFT — Weinberg (1990)
- High precision potentials — (1990~)
- Lattice — (2000~)

Realistic NN interaction

Rutherford scattering



NN scattering



J. Bystricky, F. Lehar, and P. Winternitz, J. Phys. 39(1978)1.

南開大學

Differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{N_{\text{out}}(\theta,\phi)/\Delta\Omega}{N_{\text{in}}/S}$$

over a given time interval and detector of surface S_D corresponding to an angular resolution

$$\Delta\Omega = \sin\theta\Delta\theta\Delta\phi = S_{\text{D}}/R^2.$$

Total cross section

$$\sigma_T \equiv \int d\Omega\sigma(\Omega)$$

Compared to the classical geometrical value of a disk of radius a, one has

$$\sigma = \pi a^2$$

Realistic NN interaction

Spin observables

J. Bystricky, F. Lehar, and P. Winternitz, J. Phys. 39(1978)1.

$$\frac{d\sigma}{d\Omega}X_{pqik} = \frac{1}{4}\mathrm{Tr}\,\sigma_p^{(1)}\sigma_q^{(2)}M\sigma_i^{(1)}\sigma_k^{(2)}M^\dagger,$$

Scattering amplitude

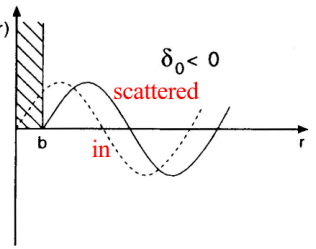
$$M^{s's}_{m'm}(\theta,\phi) = \frac{\sqrt{4\pi}}{2ip}\sum_{j,\ell,\ell'}(-1)^{s-s'}i^{\ell-\ell'}\hat{J}^2\hat{L}Y^{\ell'}_{m-m'}(\theta,\phi)$$

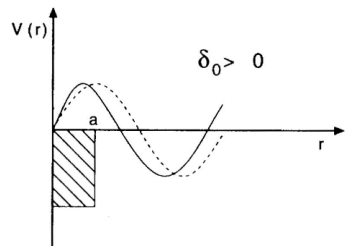
$$\times\begin{pmatrix}\ell' & s' & j\\ m-m' & m' & -m\end{pmatrix}\begin{pmatrix}\ell & s & j\\ 0 & m & -m\end{pmatrix}$$

$$\times\langle\ell's'|S^j-1|\ell s\rangle, \qquad \text{(B12)}$$

S-matrix

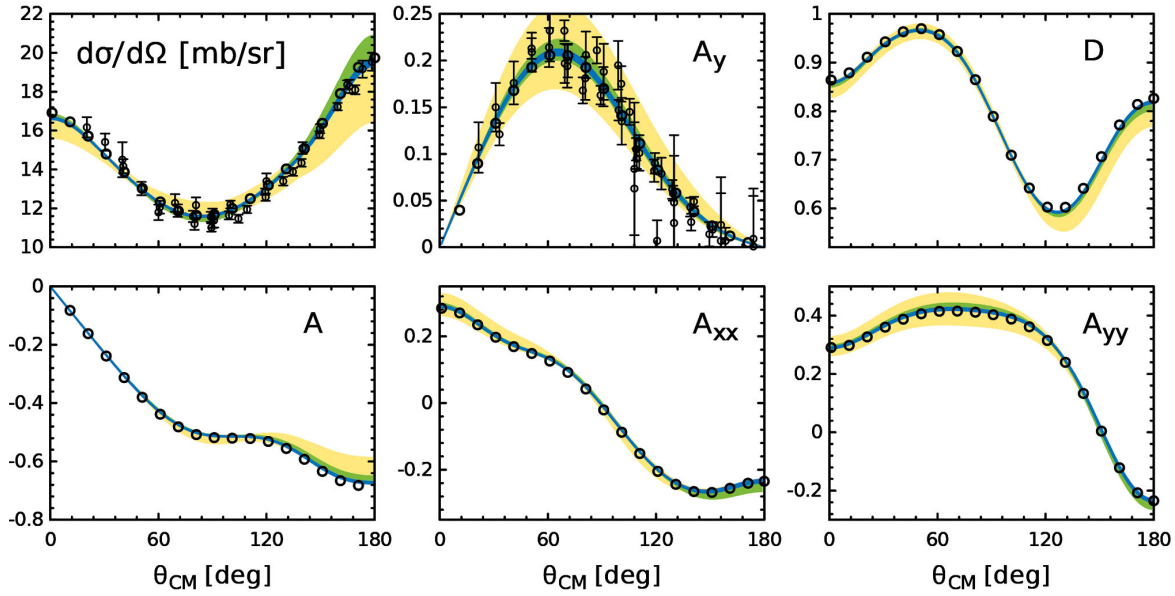
$$S^j_T = \begin{pmatrix}\cos 2\bar{\epsilon}_j e^{2i\bar{\delta}^{1j}_-} & i\sin 2\bar{\epsilon}_j e^{i(\bar{\delta}^{1j}_- + \bar{\delta}^{1j}_+)}\\ i\sin 2\bar{\epsilon}_j e^{i(\bar{\delta}^{1j}_- + \bar{\delta}^{1j}_+)} & \cos 2\bar{\epsilon}_j e^{2i\bar{\delta}^{1j}_+}\end{pmatrix}$$





NN scattering observables

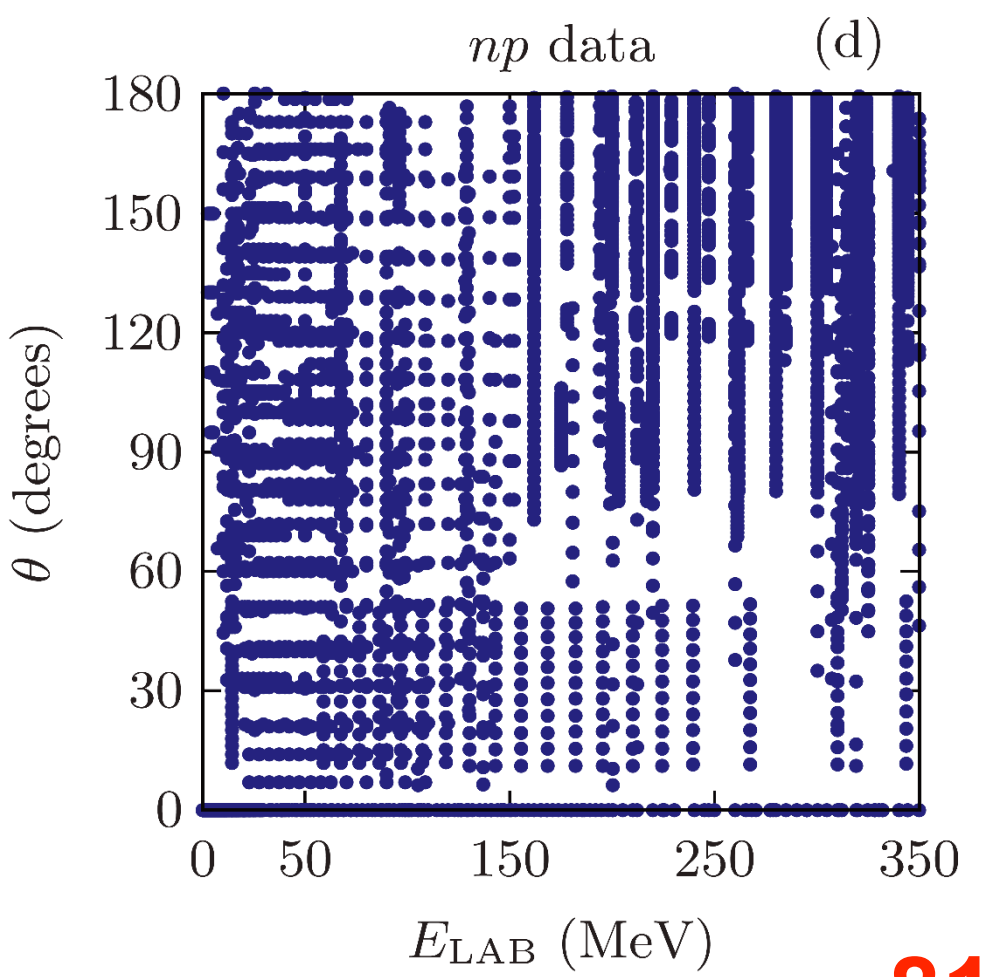
Name	X_{pqik}	Others
Differential cross section	I_{0000}	$\sigma, d\sigma/d\Omega$
Vector analyzing power	A_{00n0}	A_y, P_b
Polarization transfer	$D_{s'0k0}$	A
	D_{n0n0}	D
Spin correlation parameters	A_{00ss}	A_{xx}
	A_{00nn}	A_{yy}

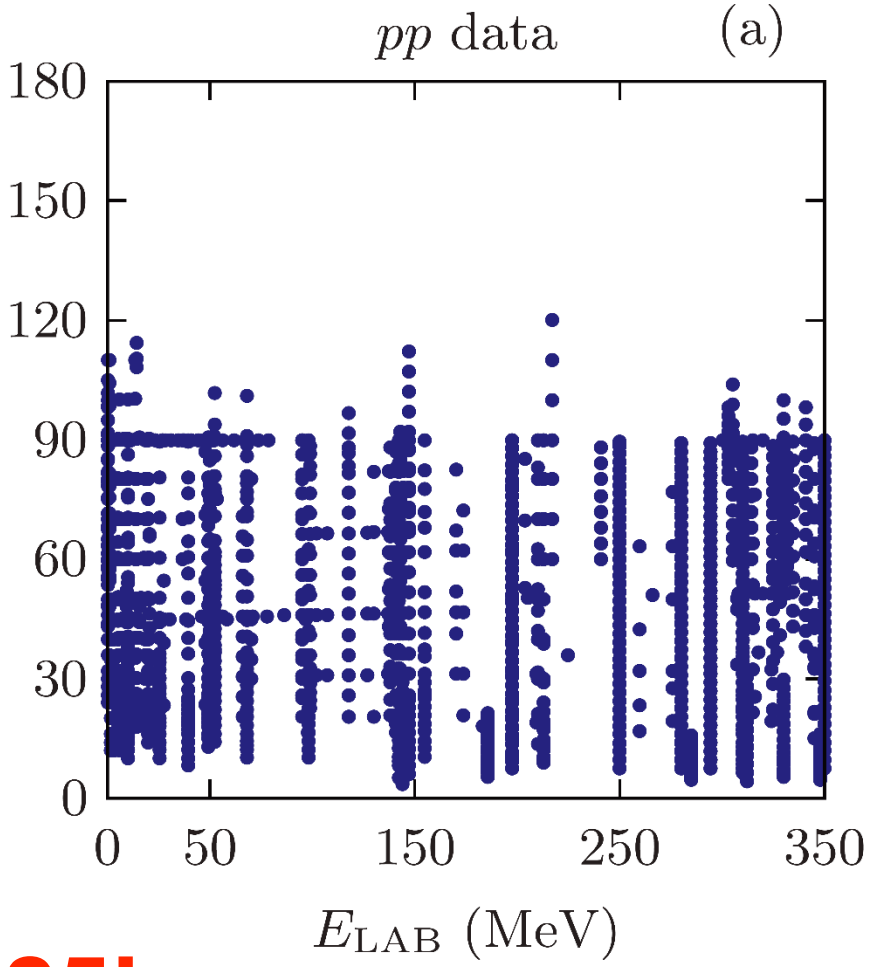


E. Epelbaum, H. Krebs, and U.-G. Meissner, Eur. Phys. J. A 51(2015)53

NN scattering data

R. Navarro Pérez, J. E. Amaro, and E. Ruiz Arriola, Phys. Rev. C 89(2014)064006





8125!

Features of NN interaction

- Short range repulsive and strong intermediate attractive
- Spin dependent
- Spin-orbit force
- Charge independent (almost)
- Non-central

Symmetries of NN interaction

- Translation invariance
- Galilean invariance
- Rotation invariance
- Space reflection invariance
- Time reversal invariance
- Interchange of particle 1 and 2 invariance
- Almost isospin symmetry

A function $f(\sigma_1,\sigma_2)$ represents a 4×4 matrix elements can be spanned by a linear combination of 16 matrices.

Type	Operators	Parity	Time reversal	Number
Scalar	1	+	+	1
Scalar	$\sigma_1 \cdot \sigma_2$	+	+	1
Vector	$\sigma_1 \times \sigma_2$	+	+	3
Vector	$\sigma_1 - \sigma_2$	+	-	3
Vector	$\sigma_1 + \sigma_2$	+	-	3
Tensor	$[\sigma_1 \times \sigma_2]^{[2]}$	+	+	5

Tensors build from r and p

Type	Operators	Parity	Time reversal
Scalar	r^2	+	+
Scalar	p^2	+	+
Scalar	$r \cdot p$	+	-
Vector	r	-	+
Vector	p	-	-
Vector	$r \times p$	+	-
Tensor	$[r \times r]^{[2]}$	+	+
Tensor	$[p \times p]^{[2]}$	+	+
Tensor	$[r \times p]^{[2]}$	+	-

南開大學

➢ **Vector-vector : spin-orbit operator**

$$\mathbf{L}\cdot\mathbf{S}=\frac{\mathbf{1}}{\mathbf{2}}(\mathbf{r}\times\mathbf{p})\cdot(\sigma_{\mathbf{1}}+\sigma_{\mathbf{2}})$$

➢ **Tensor-Tensor**

$$\left[\mathbf{r}^{[1]}\times\mathbf{r}^{[1]}\right]^{[2]}\cdot\left[\sigma_{\mathbf{1}}^{[1]}\times\sigma_{\mathbf{2}}^{[1]}\right]^{[2]}=(\sigma_{\mathbf{1}}\cdot\mathbf{r})(\sigma_{\mathbf{2}}\cdot\mathbf{r})-\frac{\mathbf{1}}{\mathbf{3}}\sigma_{\mathbf{1}}\cdot\sigma_{\mathbf{2}}\mathbf{r}^{\mathbf{2}}$$

$$\left[\mathbf{p}^{[1]}\times\mathbf{p}^{[1]}\right]^{[2]}\cdot\left[\sigma_{\mathbf{1}}^{[1]}\times\sigma_{\mathbf{2}}^{[1]}\right]^{[2]}=(\sigma_{\mathbf{1}}\cdot\mathbf{p})(\sigma_{\mathbf{2}}\cdot\mathbf{p})-\frac{\mathbf{1}}{\mathbf{3}}\sigma_{\mathbf{1}}\cdot\sigma_{\mathbf{2}}\mathbf{p}^{\mathbf{2}}$$

$$\left[\sigma_{\mathbf{1}}^{[1]}\times\sigma_{\mathbf{2}}^{[1]}\right]^{[2]}\cdot\left[\mathbf{r}^{[1]}\times\mathbf{p}^{[1]}\right]^{[2]}\mathbf{r}\cdot\mathbf{p}$$

南開大學

➢Nucleon-nucleon potential

$$V(1,2) = V_C + V_S(\sigma_1 \cdot \sigma_2) + V_T S_{12}(\mathbf{r}) + V_T S_{12}(\mathbf{p}) + V_{LS}\mathbf{L}\cdot\mathbf{S} + V_Q(\mathbf{L}\cdot\mathbf{S})^2$$

where the operator S_{12} is

$$S_{12} = 3\left(\sigma_1 \cdot \frac{\mathbf{r}}{r}\right)\left(\sigma_2 \cdot \frac{\mathbf{r}}{r}\right) - (\sigma_1 \cdot \sigma_2)$$

V_i is the scalar function of the remaining scalars, r^2 , p^2 and $(\mathbf{r}\cdot\mathbf{p})^2$. Due to

$$(\mathbf{r}\cdot\mathbf{p})^2 = r^2p^2 - L^2$$

r^2 , p^2 and L^2 are independent ones.

南開大學

➢Nucleon-nucleon potential

$$V(1,2) = V_C + V_S(\sigma_1 \cdot \sigma_2) + V_T S_{12}(\mathbf{r}) + V_T S_{12}(\mathbf{p}) + V_{LS}\mathbf{L}\cdot\mathbf{S} + V_Q(\mathbf{L}\cdot\mathbf{S})^2$$

where the operator S_{12} is

$$S_{12} = 3\left(\sigma_1 \cdot \frac{\mathbf{r}}{r}\right)\left(\sigma_2 \cdot \frac{\mathbf{r}}{r}\right) - (\sigma_1 \cdot \sigma_2)$$

V_i is the scalar function of the remaining scalars, r^2 , p^2 and $(\mathbf{r}\cdot\mathbf{p})^2$. Due to

$$(\mathbf{r}\cdot\mathbf{p})^2 = r^2p^2 - L^2$$

r^2 , p^2 and L^2 are independent ones.

Isospin of two nucleon system



南開大學

- **Uncoupled basis**

$$|p_1\rangle|p_2\rangle, \quad |n_1\rangle|n_2\rangle, \quad |p_1\rangle|n_2\rangle, \quad |n_1\rangle|p_2\rangle$$

- **Coupled basis**

$$\begin{array}{lll} & T_z = +1 & |p_1\rangle|p_2\rangle \\ \textbf{Triplet} \quad T=1 & T_z = 0 & \frac{1}{\sqrt{2}}(|p_2\rangle|n_1\rangle + |n_1\rangle|p_2\rangle) \\ & T_z = -1 & |n_1\rangle|n_2\rangle \end{array}$$

$$\textbf{Singlet} \quad T=0 \qquad T_z = 0 \qquad \frac{1}{\sqrt{2}}(|p_1\rangle|n_2\rangle - |n_1\rangle|p_2\rangle)$$

➢Isospin in NN potential

$$V_i = V_{i0} + V_{i1}\tau_1 \cdot \tau_2$$

➢Isospin exchange operator

$$V_t(r)\frac{1}{2}(1 + \tau_1 \cdot \tau_2) = V_t(r)P_r$$

➢Central part of NN potential

$$V_C(r) = V_W(r) + V_r(r)P_r + V_\sigma(r)P_\sigma + V_t(r)P_t$$

Wigner potential **Majorana potential** **Bartlett potential**

南開大學

The good quantum numbers in NN scattering problem

Momentum *k*, **Total angular momentum *J*,**

Total spin *S*, **Total isospin *T***

$$S + T + L = \text{odd}$$

The wave function of NN scattering states

$$|kplSTJM\rangle \Longleftrightarrow {}^{2S+1}L_J$$

The NN interaction in partial wave representation

$$\left\langle \vec{k}' \middle| V \middle| \vec{k} \right\rangle = (4\pi)^2 \sum_{ll'STJm} i^{l+l'} Y^*_{l'm'}(\vec{k}') Y_{lm}(\vec{k}) \langle l' S m_{l'} M_s | JM \rangle$$

$$\langle l S m_l M_s | JM \rangle \langle k' l' STJM | V | klSTJM \rangle$$

Two nucleon states

States	Central	Tensor and spin-orbit
$J=0, S=0, T=1, L=0$	${}^1S_0\,(pp,np,nn)$	-
$J=0, S=1, T=1, L=1$	${}^3P_0\,(pp,np,nn)$	-
$J=1, S=0, T=0, L=1$	${}^1P_1\,(np)$	-
$J=1, S=1, T=1, L=1$	${}^3P_1\,(pp,np,nn)$	-
$J=1, S=1, T=0, L=0,2$	${}^3S_1-{}^3D_1\,(np)$	${}^3S_1-{}^3D_1\,(np)$
$J=2, S=0, T=1, L=2$	${}^1D_2\,(pp,np,nn)$	-
$J=2, S=1, T=0, L=2$	${}^3D_2\,(np)$	-
$J=2, S=1, T=1, L=1,3$	${}^3P_2-{}^3F_2\,(pp,np,nn)$	${}^3P_2-{}^3F_2\,(pp,np,nn)$
$J=3, S=0, T=0, L=3$	${}^1F_3\,(np)$	-
$J=3, S=1, T=1, L=3$	${}^3F_2\,(pp,np,nn)$	-
$J=3, S=1, T=0, L=2,4$	${}^3D_3-{}^3G_3\,(np)$	${}^3D_3-{}^3G_3\,(np)$

Phenomenological interaction

Based upon the standard non-relativistic operator structure

- **Gammel-Thaler potential** Hard core

  J. L. Gammel, and R. M. Thaler, Phys. Rev. 107(1957)291

- **Hamada-Johnston potential** Hard core

  T. Hamada, and I. D. Johnston, Nucl. Phys. 34 (1962)382

- **Reid potential** soft core

  R.V. Reid, Ann. Phys. (N.Y.) 50(1968)411

- **Argonne V14 potential**

  R. B. Wiringa, R. A. Smith, and T. L. Ainsworth, Phys. Rev. C 29(1984)1207

- **Argonne V18 potential**

  R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C 51(1995)38

➢Hamada-Johnston potential form

$$V = V_c(r) + V_t(r) S_{12} + V_{ls}(r) \vec{L}.\vec{S} + V_{ll}(r) L_{12},$$

where,

$$S_{12} = 3(\vec{\sigma}_1.\hat{r})(\vec{\sigma}_2.\hat{r}) - (\vec{\sigma}_1.\vec{\sigma}_2), \quad L_{12} = (\delta_{\ell j} + \vec{\sigma}_1.\vec{\sigma}_2)\, L^2 - (\vec{L}.\vec{S})^2,$$

and

$$\begin{cases} V_c(r) = 0.08 \left(\frac{1}{3} m_{pi}\right) (\vec{\tau}_1.\vec{\tau}_2)\, (\vec{\sigma}_1.\vec{\sigma}_2)\, Y(x)\, [1 + a_c\, Y(x) + b_c\, Y^2(x)], \\ V_t(r) = 0.08 \left(\frac{1}{3} m_{pi}\right) (\vec{\tau}_1.\vec{\tau}_2)\, Z(x)\, [1 + a_t\, Y(x) + b_t\, Y^2(x)], \\ V_{ls}(r) = m_{pi}\, G_{ls}\, Y^2(x)\, [1 + b_{ls} Y(x)], \\ V_{ll}(r) = m_{pi}\, G_{ll} x^{-2} Z(x)\, [1 + a_{ll}\, Y(x) + b_{ll}\, Y^2(x)], \end{cases}$$

$$Y(x) = \frac{e^{-x}}{x}, \quad Z(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right) Y(x).$$

Relativistic interaction

Based upon the quantum field theory

- One boson exchange potential

  R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. 149(1987)1

- CD Bonn potential

  R. Machleidt, Phys. Rev. C 63(2001)024001

- Chiral effective potential

  R. Machleidt, and D. R. Entem, Phys. Rep. 503(2011)1

- Lattice QCD potential

  N. Ishii, S. Aoki, and T. Hatsuda, Phys. Rev. Lett. 99(2007)022001

Chiral NN interaction

➢N²LO chiral potential

S. Weinberg, Nuclear Phys. B 363 (1991) 3

C. Ordonez, L. Ray, U. van Kolck, Phys. Rev. Lett. 72 (1994) 1982

C. Ordonez, L. Ray, U. van Kolck, Phys. Rev. C 53 (1996) 2086

➢N³LO chiral potential

D. R. Entem, R. Machleidt, Phys. Rev. C 66 (2002) 014002

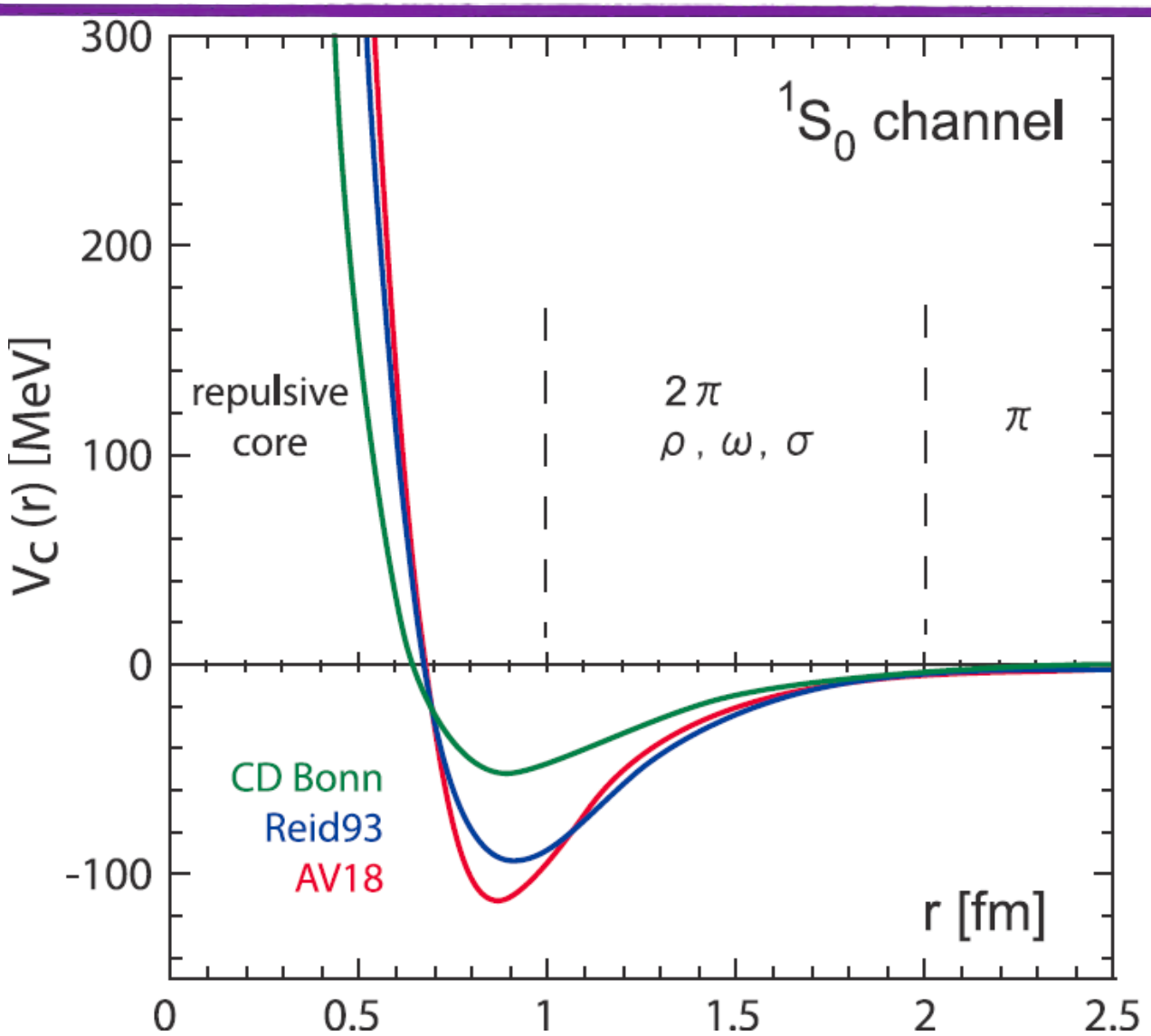
E. Epelbaum, W. Gloeckle, U.-G. Meissner, Nucl. Phys. A 747 (2005) 362

➢N⁴LO chiral potential

D. R. Entem, N. Kaiser, R. Machleidt, and Y. Nosyk, Phys. Rev. C 91 (2015) 014002

E. Epelbaum, H. Krebs, U.-G. Meissner, Phys. Rev. Lett. 115 (2015) 122301

The shape of NN interaction



Argonne V18 potential

- Argonne V18 potential composed by EM and nuclear short, intermediate and 1 pion exchange term

$$V_{ij} = V_{ij}^{EM} + V_{ij}^{\pi} + V_{ij}^{I} + V_{ij}^{S}$$

- Electromagnetic terms include pp, pn and nn
- The pion-exchange term

$$V_{ij}^{\pi} = \left[Y_{\pi}(r_{ij})\vec{\sigma}_i \cdot \vec{\sigma}_j + T_{\pi}(r_{ij}) S_{ij} \right] \vec{\tau}_i \cdot \vec{\tau}_j$$

$$S_{ij} = 3\left(\vec{\sigma}_i \cdot \hat{\vec{r}}_{ij}\right)\left(\vec{\sigma}_j \cdot \hat{\vec{r}}_{ij}\right) - \vec{\sigma}_i \cdot \vec{\sigma}_j$$

Y_{π} is the Yukawa function with cutoff

$$Y_{\pi}(r) = \frac{e^{-\mu r}}{\mu r}\left(1 - e^{-cr^2}\right) \qquad \mu \approx 0.7 \text{ fm}^{-1}$$

T_{π} is the corresponding function to the tensor term

$$T_{\pi}(r) = \left(1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2}\right)\frac{e^{-\mu r}}{\mu r}\left(1 - e^{-cr^2}\right)$$

The basic concept of the Fermi-gas model

The theoretical concept of a Fermi-gas may be applied for systems of weakly interacting fermions, i.e. particles obeying Fermi-Dirac statistics leading to the Pauli exclusion principle ➔

• Simple picture of the nucleus:

— Protons and neutrons are considered as moving freely within the nuclear volume. The binding potential is generated by all nucleons

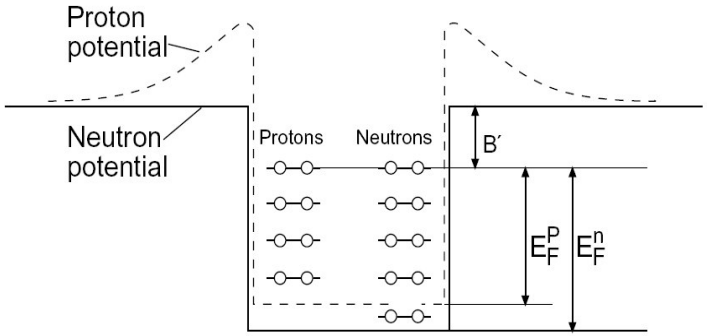
— In a first approximation, these nuclear potential wells are considered as rectangular: it is constant inside the nucleus and stops sharply at its edge

— Neutrons and protons are distinguishable fermions and are therefore situated in two separate potential wells

— Each energy state can be ocupied by two nucleons with different spin projections

— All available energy states are filled by the pairs of nucleons ➔ no free states , no transitions between the states

— The energy of the highest occupied state is the Fermi energy E_F



— The difference B' between the top of the well and the Fermi level is constant for most nuclei and is just the average binding energy per nucleon B'/A = 7–8 MeV.

Nuclear mean potential

南開大學

Single particle equation in mean field

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r})\right)\psi_i(\boldsymbol{r}) = \varepsilon_i\,\psi_i(\boldsymbol{r})$$

Square-well potential

$$V(r) = \begin{cases} -V_0 & \text{for} \quad r \le R \\ \infty & \text{for} \quad r > R \end{cases}$$

Woods-Saxon potential

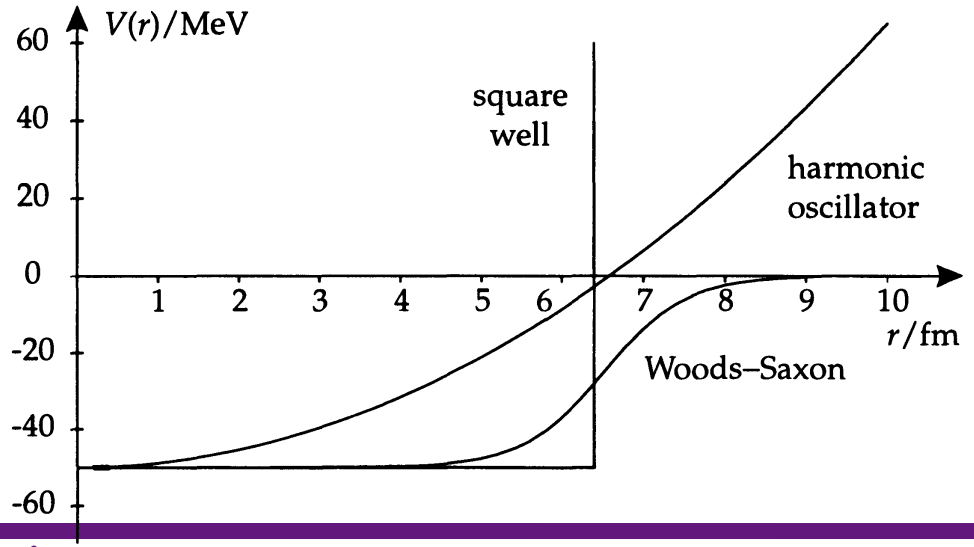
$$V(r) = -\frac{V_0}{1+\exp[(r-R)/a]}$$

where $V_0 \sim$ -50 MeV, R~1.1 fm $A^{1/3}$, $a \sim$ 0.5 fm

Harmonics-oscillator potential

$$V(r) = \tfrac{1}{2}m\omega^2 r^2$$

where $\omega \sim$ 41 MeV $A^{-1/3}$



Nuclear shell model

南開大學

Schroedinger equation with harmonics oscillator potential

$$H_{ho}\psi = (-\frac{\nabla^2}{2m} + \frac{1}{2}m\omega^2 r^2)\psi = E\psi$$

Kinetic energy in spherical coordinate

$$-\frac{1}{2m}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]$$

Wave function

$$\psi(r,\theta,\varphi) = R(r)Y(\theta,\varphi)$$

Angular equation

$$\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} + \lambda\right]Y(\theta,\varphi) = 0$$

Solution of angular equation

$$Y_{lm}(\theta,\varphi) = (-)^m\left[\frac{(2l+1)(l-m)!}{4\pi(l+m)!}\right]^{1/2}P_l^m(\cos\theta)e^{im\varphi} \qquad \lambda = l(l+1)$$

Nuclear shell model

南開大學

Radial Schoedinger equation

$$\left[\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)+2m\left(E-\frac{m\omega^2 r^2}{2}\right)-\frac{\lambda}{r}\right]R(r)=0$$

Variable substitution

$$\frac{1}{2}\left[-\frac{1}{\rho^2}\frac{d}{d\rho}\left(\rho^2\frac{d}{d\rho}\right)+\rho^2+\frac{l(l+1)}{\rho}\right]R(\rho)=\tilde{E}R(\rho)$$

with $\rho = r/b$, $b = \sqrt{\hbar\omega}$ **and** $\tilde{E} = E/\hbar\omega$.

Wave function

$$R_{nl}(r)=\left[\frac{2n!}{b^3\Gamma(n+l+\frac{3}{2})}\right]^{1/2}\left(\frac{r}{b}\right)^l e^{-r^2/2b^2}L_n^{l+1/2}(r^2/b^2)$$

Eigenengery

$$E_{nl}=(2n+l+\frac{3}{2})\hbar\omega$$

Nuclear shell model

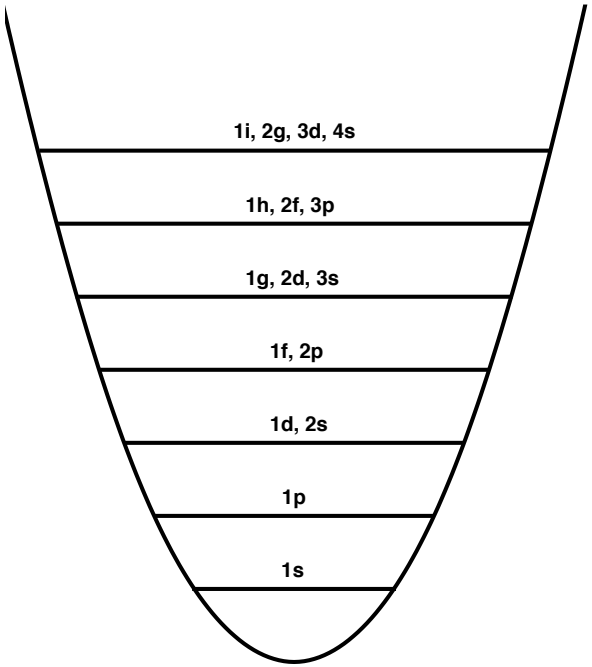
南開大學

Nuclear parameters

$$\hbar\omega = 41A^{-1/3}\ \text{MeV}, \qquad b = 1.005A^{1/6}\ \text{fm}$$

New quantum number

$$E_{nl} = (N + \frac{3}{2})\hbar\omega \qquad N = 2n + l,$$



N	E_N	d_N	$\sum_N d_N$	$n(l)$	parity
0	$\frac{3}{2}\hbar\omega$	2	2	1s	+
1	$\frac{5}{2}\hbar\omega$	6	8	1p	-
2	$\frac{7}{2}\hbar\omega$	12	20	1d, 2s	+
3	$\frac{9}{2}\hbar\omega$	20	40	1f, 2p	-
4	$\frac{11}{2}\hbar\omega$	30	70	1g, 2d, 3s	+
5	$\frac{13}{2}\hbar\omega$	42	112	1h, 2f, 3p	-
6	$\frac{15}{2}\hbar\omega$	56	168	1i, 2g, 3d, 4s	+

The total degeneracy at level *N*

$$(N+1)(N+2)$$

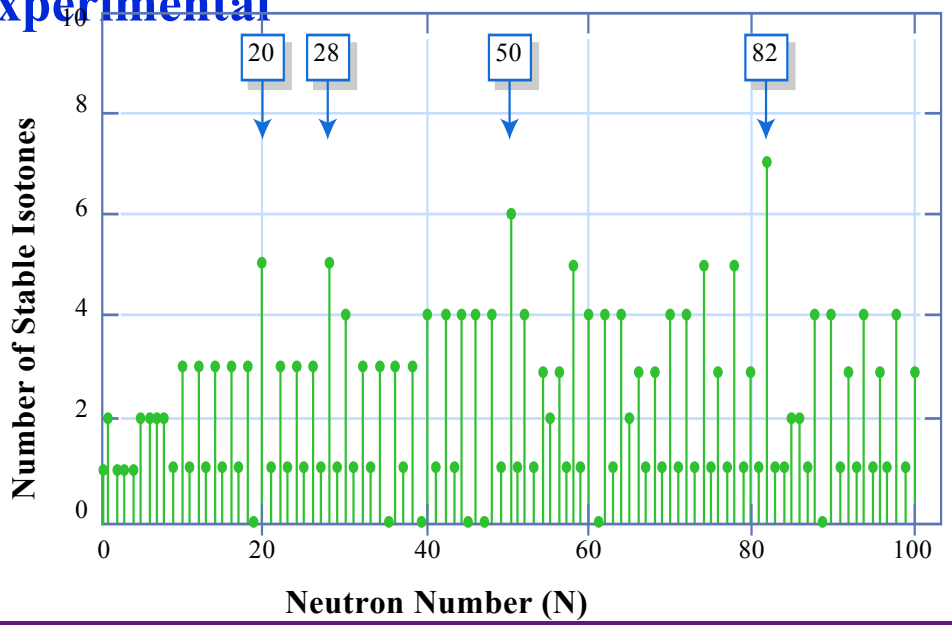
The magic number in harmonics oscillator

$$(N+1)(N+2)(N+3)/3$$

2, 8, 20, 40, 70, 112, ...

The magic number in experimental

2, 8, 20, 50, 82, 126, ...



南開大學

Spin-orbit force

$$V_{so} = \zeta_{zo}\mathbf{l}\cdot\mathbf{s}$$

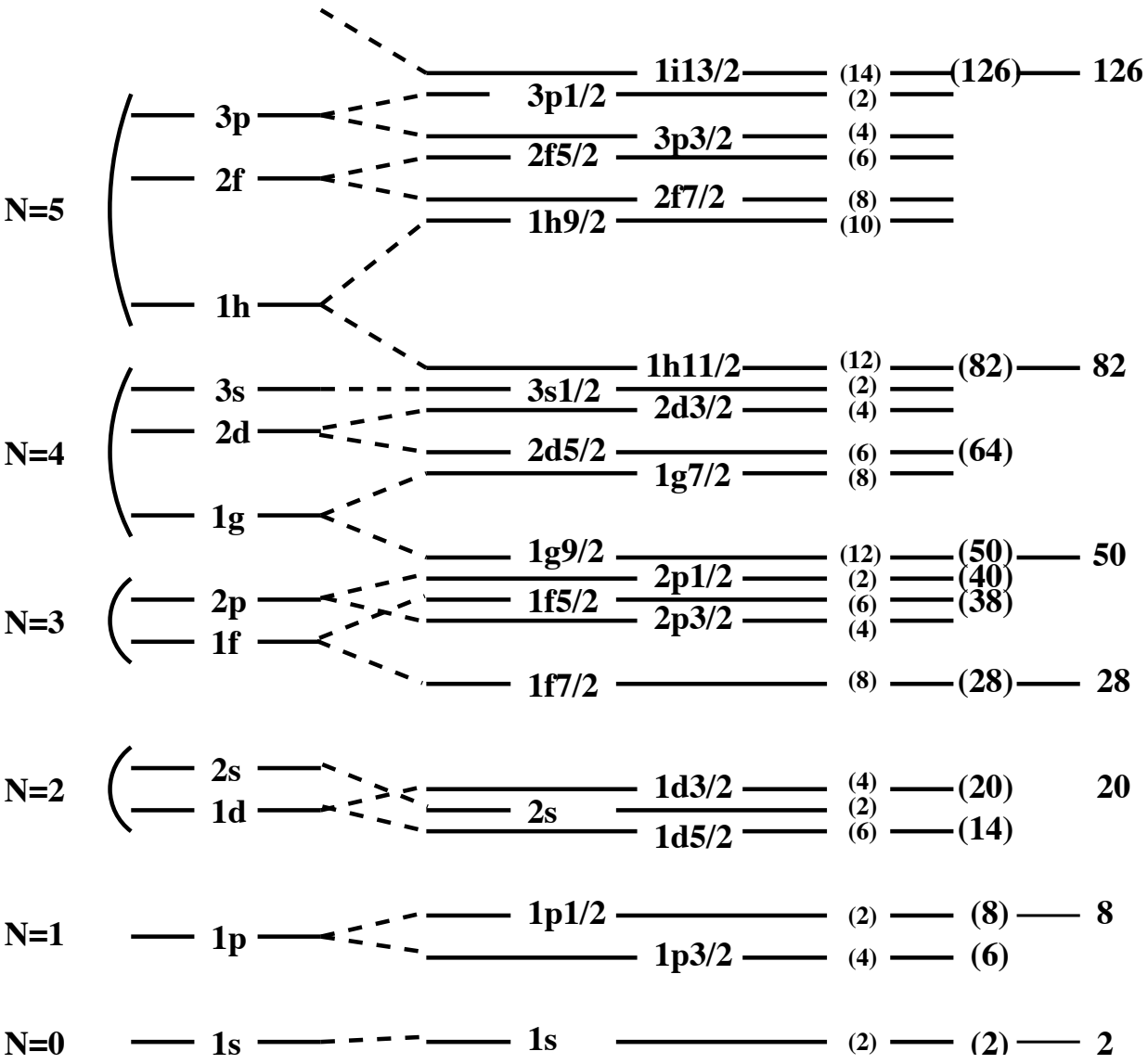
The expecting energy of spin-orbit force

$$\begin{aligned}\langle nljm|V_{so}|nljm\rangle &= \frac{1}{2}\langle\zeta_{so}(r)\rangle_{nl}\left[j(j+1)-l(l+1)-\frac{3}{4}\right] \\ &= \begin{cases} -\frac{1}{2}(l+1)\langle\zeta_{so}(r)\rangle_{nl} & \text{for } j=l-1/2 \\ \frac{1}{2}l\langle\zeta_{so}(r)\rangle_{nl} & \text{for } j=l+1/2 \end{cases}\end{aligned}$$

where

$$\langle\zeta(r)\rangle_{nl} = \int_0^\infty \zeta(r)R_{nl}^2(r)r^2dr$$

Nuclear shell model



$$a + X \rightarrow Y + b \qquad X(a,b)Y$$

reaction	name	example
X(a,γ)Y	*radiative capture*	${}^{A}_{Z}X_N(n,\gamma){}^{A+1}_{Z}X_{N+1}$
X(γ,b)Y	*nuclear photoeffect*	${}^{A}_{Z}X_N(\gamma,p){}^{A-1}_{Z-1}X_N$
X(a,a)X	*nuclear scattering*	${}^{A}_{Z}X_N(\alpha,\alpha){}^{A}_{Z}X_N$, Rutherford scattering
X(a,a)X*	*inelastic scattering*	${}^{A}X(n,n){}^{A}X^*$
	knock out reaction	${}^{A}_{Z}X_N(n,nn){}^{A}_{Z}X_{N-1}$, ${}^{A}_{Z}X_N(n,np){}^{A}_{Z-1}X_N$
	transfer reaction	${}^{A}_{Z}X_N(n,p){}^{A}_{Z-1}X_{N+1}$
	direct reactions	"a" interacts with only one or a few nucleons in X
	compound reactions	$a + X \rightarrow Y^{m}_{t_{1/2}\gg\sim 10^{-23}s} \rightarrow$ fragments
	resonance reaction	n or $p + X \rightarrow Y^{m} \rightarrow$ decay products

- Transfer (strong interaction)
  - $^{15}N(p,\alpha)^{12}C$ σ~0.5 b at E= 2.0 MeV
- Capture (electromagnetic interaction)
  - $^{3}He(\alpha,\gamma)^{7}Be$ σ~10^{-6} b at E= 2.0 MeV
- β-decay or electron capture (weak interaction)
  - $p(p,e^{+}\nu)d$ σ~10^{-20} b at E= 2.0 MeV

- Elastic Scattering – projectile and target stay in their ground state
- Inelastic scattering – projectile and/or target are left in an excited state
- Transfer reaction – one or more nucleons move
- Breakup (nuclear or Coulomb) – projectile is broken into pieces (target is normally ignored
- Charge Exchange – atomic mass, A, of target and projectile remain the same, but the atomic charge, Z, changes
- Knockout – one or more nucleons are removed from a nucleus (could be target or projectile)
- Spallation – a light ion breaks (spalls) pieces of a nucleus
- Deeply inelastic – highly excited states are produced; many nucleons are transferred
- Capture – beam is absorbed and nucleus deexcites
- Fusion (incomplete, fusion-fission, fusion-evaporation) – nuclei stick together

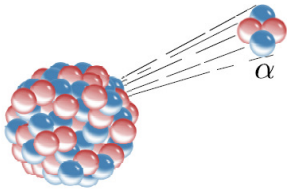


Stability of nuclei

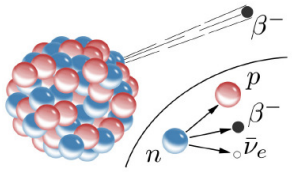
Stability of nuclei

Stable nuclei ← **Stability of nuclei** → **Unstable nuclei**

Decay schemes :

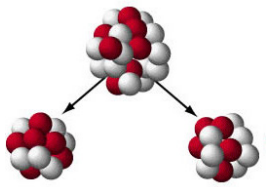


- **α-decay – emission of α-particle (^{4}He):** $^{238}U \rightarrow {}^{234}Th + \alpha$
- **β-decay - emission of electron (β^-) or positron (β^+) by the weak interaction**



β^- decay: the weak interaction converts a neutron (n) into a proton (p) while emitting an electron (e^-) and an electron antineutrino (ν_e): $n \rightarrow p + e^- + \bar{\nu}_e$

β^+ decay: the weak interaction converts a proton (p) into a neutron (n) while emitting a positron (e^+) and an electron neutrino (ν_e): $p \rightarrow n + e^+ + \nu_e$



- **fission - spontaneous decay into two or more lighter nuclei**
- **proton or neutron emission**



Radionuclides

Unstable nuclides are radioactive and are called radionuclides.
Their decay products ('daughter' products) are called radiogenic nuclides.

About 256 stable and about 83 unstable (radioactive) nuclides exist naturally on Earth.

The probability per unit time for a radioactive nucleus to decay is expressed by the decay constant λ. It is related to the lifetime τ and the half life $t_{1/2}$ by:

$$\tau = 1/\lambda \quad \text{and} \quad t_{1/2} = \ln 2/\lambda$$

The measurement of the decay constants of radioactive nuclei is based upon finding the activity (the number of decays per unit time):

$$A = -dN/dt = \lambda N$$

where N is the number of radioactive nuclei in the sample.

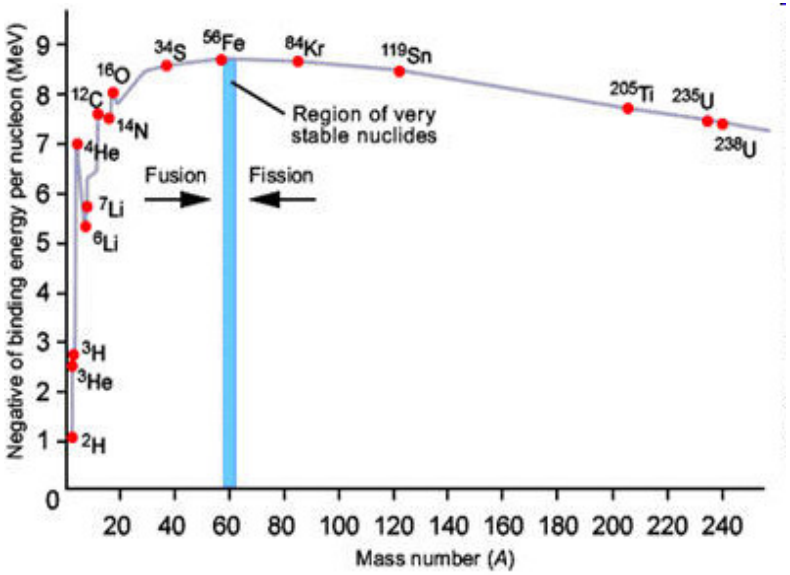
The unit of activity is defined 1 Bq [Becquerel] = 1 decay/s.



Binding energy of nuclei

The mass of the nucleus: $M(Z,N) = m_N \cdot N + m_P \cdot Z - E_B$

E_B is the binding energy per nucleon or mass defect (the strength of the nucleon binding). The mass defect reflects the fact that the total mass of the nucleus is less than the sum of the masses of the individual neutrons and protons that formed it. The difference in mass is equivalent to the energy released in forming the nucleus.

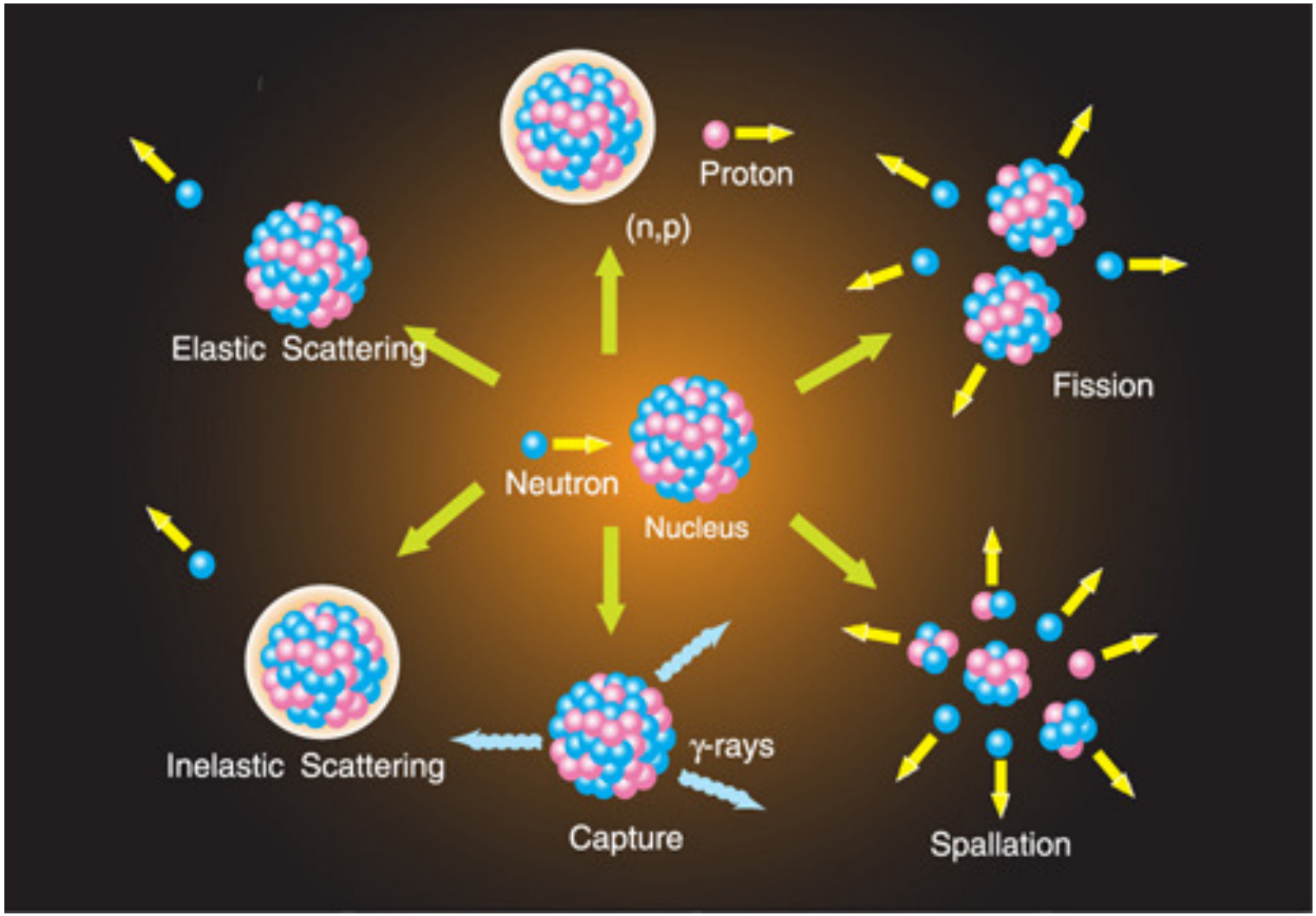


- **The general decrease in binding energy beyond iron (^{58}Fe) is due to the fact that, as nuclei get more massive, the ability of the strong force to counteract the electrostatic repulsion between protons becomes weaker.**
- **The most tightly bound isotopes are ^{62}Ni, ^{58}Fe, and ^{56}Fe, which have binding energies of 8.8 MeV per nucleon.**
- **Elements heavier than these isotopes can liberate energy by nuclear fission; lighter isotopes can yield energy by fusion.**

Fusion - two atomic nuclei fuse together to form a heavier nucleus
Fission - the breaking of a heavy nucleus into two (or more rarely three) lighter nuclei



Nuclear reactions





Nuclear reactions

The proton size: $\Delta L \sim 1$ fm $= 10^{-15}$m $= 10^{-13}$ cm

In order to study experimentally such small objects, one needs a probe of the same or smaller size!

Quantum mechanics ➔ wave-corpuscular duality:

the particle exhibits both wave and particle properties

- **de-Broglie wave-length λ or reduced wave-length** $\lambda = \lambda/2\pi$

$$\lambda = \frac{\hbar}{p} = \frac{\hbar c}{\sqrt{2mc^2 E_{\rm kin} + E_{\rm kin}^2}} \approx \begin{cases} \hbar/\sqrt{2mE_{\rm kin}} & \text{for } E_{\rm kin} \ll mc^2 \\ \hbar c/E_{\rm kin} \approx \hbar c/E & \text{for } E_{\rm kin} \gg mc^2 \end{cases}$$

(nonrelativistic) **(relativistic)**

with the kinetic energy $E_{kin}=E-mc^2$, the total energy $E^2=m^2c^4+p^2c^2$

➔ in order to localize the particle to small size, one needs to accelerate it to hight energy, i.e. to resolve small structures, large beam energies are required.

The largest wavelength that can resolve structures of linear extension *Δx*, is of the same order: $\lambda \lesssim \Delta x$



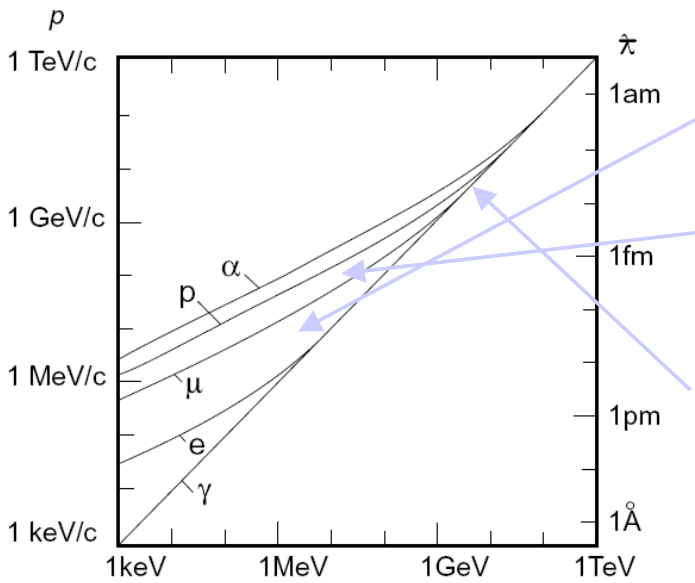
Nuclear reactions

From the Heisenberg's uncertainty principle the corresponding particle momentum is:

$$p \gtrsim \frac{\hbar}{\Delta x} , \qquad pc \gtrsim \frac{\hbar c}{\Delta x} \approx \frac{200\,\mathrm{MeV\,fm}}{\Delta x} .$$

The connection between kinetic energy, momentum and reduced wave-length of photons (γ), electrons (e), muons (μ), protons (p), and ^{4}He nuclei (α).

Atomic diameters are typically a few ˚A (10^{-10} m), nuclear diameters a few fm (10^{-15} m) :



- **Thus to study nuclei, whose radii are of a few fm, beam momenta of the order of 10-100 MeV/*c* are necessary.**
- **Individual nucleons have radii of about 0.8 fm and may be resolved if the momenta are above ≈ 100 MeV/*c*.**
- **To resolve the constituents of a nucleon, the quarks, one has to penetrate deeply into the interior of the nucleon. For this purpose, beam momenta of many GeV/*c* are necessary**

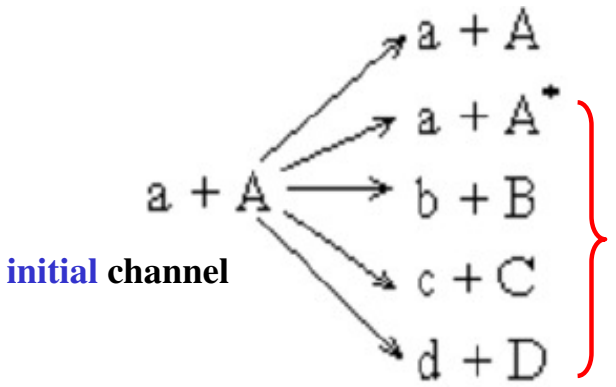


Nuclear reactions

A nuclear reaction is a process in which two nuclei collide to produce products different from the initial particles.

$$a_1 + a_2 \rightarrow b_1 + b_2 + \ldots,$$

initial particles **final particles**



Types nuclear reactions:

elastic reactions:
the initial channel = the final channel

inelastic reactions:
the final channel differs from the initial channel

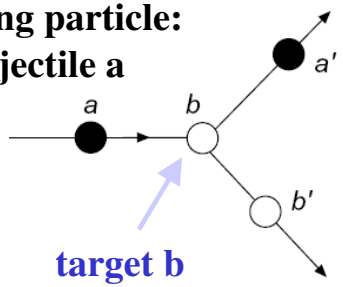
Example: p+^{14}N →

$$p + {}^{14}N \rightarrow {}^{14}N + p$$
$$p + {}^{14}N \rightarrow {}^{14}N^* + p$$
$$p + {}^{14}N \rightarrow {}^{15}O + \gamma$$
$$p + {}^{14}N \rightarrow {}^{14}O + n$$
$$p + {}^{14}N \rightarrow {}^{13}N + p + n$$
$$p + {}^{14}N \rightarrow 8p + 7n$$



Elastic reactions

incoming particle:
projectile a



In an elastic process

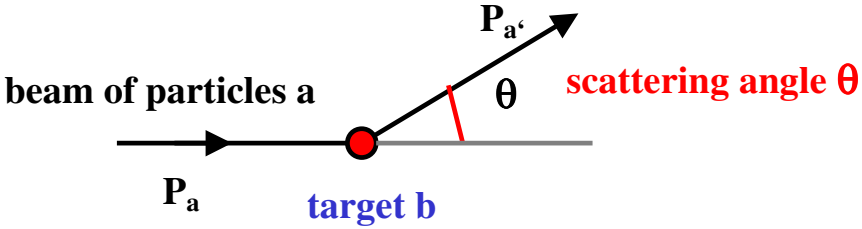
$$a + b \rightarrow a' + b'$$

the same particles are present both before and after the scattering, i.e. the initial and in the final state are identical (including quantum numbers) up to momenta and energy.

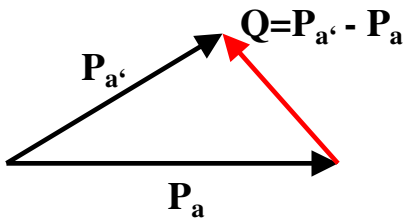
The target b remains in its ground state, absorbing merely the recoil momentum and hence changing its kinetic energy.

The scattering angle and the energy of the projectile particle a and the production angle and energy of target b are unambiguously correlated.

scattered outgoing particle a'



momentum transfer Q:

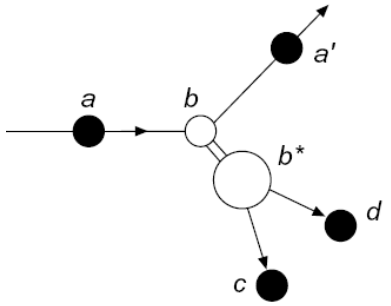




Inelastic reactions

i) In inelastic reactions

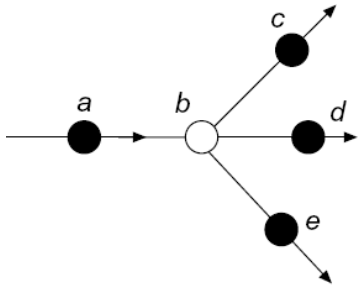
$$a + b \rightarrow a' + b^*$$
$$\hookrightarrow c + d$$



a part of the kinetic energy transferred from a to the target b excites it into a higher energy state b*. The excited state will afterwards return to the ground state by emitting a light particle (e. g. a photon or a π-meson) or it may decay into two or more different particles.

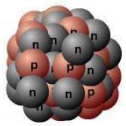
ii) In inelastic reactions

a+b → c+d+e



the beam particle may completely disappear in the reaction. Its total energy then goes into the production of new particles.

- **A measurement of a reaction in which only the scattered particle a is observed (and the other reaction products are not), is called an inclusive measurement.**
- **If all reaction products are detected, one speaks of an exclusive measurement.**



Conservation laws of nuclear reactions

- ✓ **Electric charge conservation law**
- ✓ **Baryon number conservation law**
- ✓ **Energy conservation law**
- ✓ **Momentum conservation law**
- ✓ **Angular momentum conservation law**
- ✓ **Parity conservation law**
- ✓ **Isospin and its projection conservation law**

Example: Consequences of the electric charge and baryon number conservation law:

One can find a missing product X of the reaction (e.g.): $p + {}^7Li \rightarrow {}^4He + X$

$Z_{initial} = Z(p) + Z({}^7Li) = 1 + 3 = 4 = Z_{final} = Z({}^4He) + Z(x) = 2 + Z(x) \rightarrow Z(x) = 2$

$A_{initial} = A(p) + A({}^7Li) = 1 + 7 = 8 = A_{final} = A({}^4He) + A(x) = 4 + A(x) \rightarrow A(x) = 4$

$\rightarrow$ X = α-particle



Energy-momentum conservation

➢ **Energy and momentum conservation :**

$$a+b \rightarrow c+d+\ldots \qquad (1)$$

N_{final} – number of the particles in the final state; $N_{initial}=2$ for (1)

$$E_a + E_b = E_c + E_d + \ldots \Leftrightarrow \sum_{i=1}^{N_{initial}} E_i = \sum_{i=1}^{N_{final}} E_i$$

$$\vec{p}_a + \vec{p}_b = \vec{p}_c + \vec{p}_d + \ldots \Leftrightarrow \sum_{i=1}^{N_{initial}} \vec{p}_i = \sum_{i=1}^{N_{final}} \vec{p}_i \qquad (2)$$

where $E_i^2 = c^2 p_i^2 + c^4 m_i^2$

Kinetic energy: $T_i = E_i - c^2 m_i$

from (1) =>

$$T_a + m_a c^2 + T_b + m_b c^2 = \sum_{i=1}^{N_{final}} (T_i + m_i c^2) \qquad (3)$$

$$T_a + T_b = \sum_{i=1}^{N_{final}} T_i - Q$$

Energy transfer of the reaction:

$$Q = \sum_{i=1}^{N_{initial}} m_i c^2 - \sum_{i=1}^{N_{final}} m_i c^2 \qquad (4)$$

Exothermic reaction: Q>0 : reaction in which energy is released by the system into the surrounding environment.

Endothermic reaction: Q<0: reaction in which energy is absorbed by the system from the surrounding environment

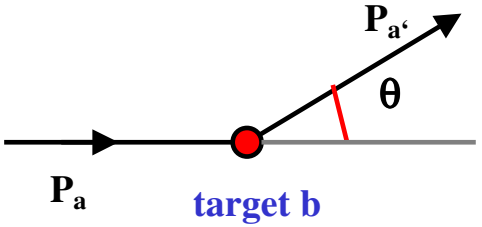


Energy-momentum conservation

Let's find the threshold energy for the reaction – the minimal kinetic energy when the reaction is possible

❑ Consider laboratory frame: $\vec{p}_b = 0, \quad E_b = m_b$

4-momenta $p_a = (E_a, \vec{p}_a), \quad p_b = (m_b, 0)$



Energy and momentum conservation laws ➔ the invariant energy s is conserved

$$s = (p_a + p_b)^2 = \left(\sum_{i=1}^{N_{final}} p_i\right)^2 \qquad (5)$$

In the laboratory frame:

$$s = (p_a + p_b)^2 = (E_a + E_b)^2 - (\vec{p}_a + \vec{p}_b)^2 = E_a^2 + 2E_a m_b c^2 + m_b^2 c^4 - \vec{p}_a^{\,2} \qquad (6)$$
$$= (m_a^2 c^4 + m_b^2 c^4 + 2(T_{kin} + m_a c^2) m_b c^4) = 2T_{kin} m_b c^2 + (m_a + m_b)^2 c^4$$

The threshold (=minimal kinetic) energy of incoming particle a → all goes to the mass production of the final particles which are at rest (i.e. $T_{i\,kin} = 0$ for $i = 1, \ldots N_{final}$)

➔ The threshold (minimal) invariant energy:

$$s_{th} = 2T_{th} m_b c^2 + (m_a + m_b)^2 c^4 = \left(\sum_{i=1}^{N_{final}} m_i\right)^2 c^4 \qquad (7)$$



Energy-momentum conservation

➔ **Threshold energy:**

(in the laboratory frame)

$$T_{th} = \frac{1}{2m_b}\left[\left(\sum_{i=1}^{N_{final}} m_i\right)^2 - (m_a + m_b)^2\right]c^2 \quad (8)$$

$$T_{th} = \frac{1}{2m_b}\left(\sum_{i=1}^{N_{final}} m_i - m_a - m_b\right)\left(\sum_{i=1}^{N_{final}} m_i + m_a + m_b\right)c^2 \quad (9)$$

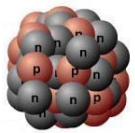
$$T_{th} = |Q|\left(1 + \frac{m_a}{m_b} + \frac{|Q|}{2m_b c^2}\right) \quad (10)$$

where Q is the energy of the reaction – eq.(4)

❑ In the nonrelativistic limit where $Q << 2m_b c^2$

(in the laboratory frame)

$$T_{th} = |Q|\left(1 + \frac{m_a}{m_b}\right) \quad (11)$$

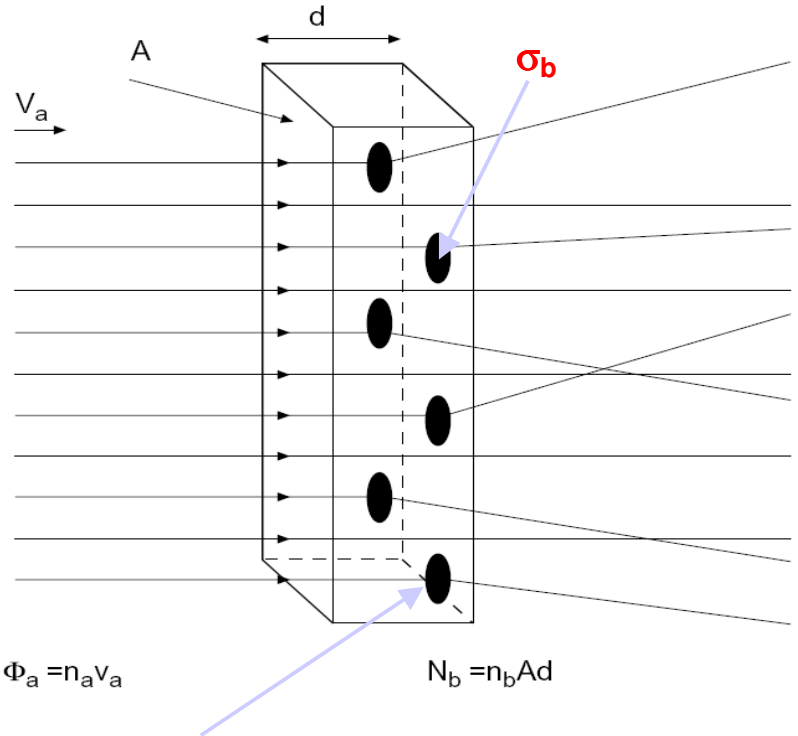


Cross section of the nuclear reaction

The cross section gives the probability of a reaction between the two colliding particles.

Consider an idealised experiment:

- **we bombard the target with a monoenergetic beam of point-like particles a with velocity v_a.**
- **a thin target of thickness d and total aria A with N_b scattering centres b and with a particle density n_b.**
- **each target particle has a cross-sectional area σ_b, which we have to find by experiment!**

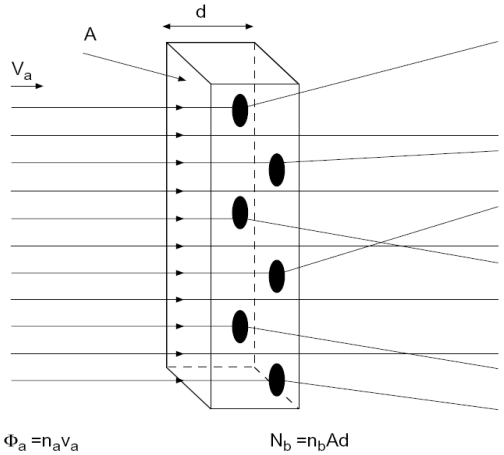


$\Rightarrow$ Some beam particles are scattered by the scattering centres of the target, i. e. they are deflected from their original trajectory. The frequency of this process is a measure of the cross-section area of the scattering particles σ_b

Cross section of the nuclear reaction

The **total reaction rate** (i.e. the total number of reactions per unit time):

$$\dot{N} \equiv \frac{dN}{dt} = \Phi_a N_b \sigma_b \qquad (12)$$



Φ_a is the **flux** - the number of projectiles hitting the target per unit area A and per unit time dt and has dimensions $[(\text{area}\times\text{time})^{-1}]$:

$$\Phi_a = \frac{\dot{N}_a}{A} = n_a v_a \qquad (13)$$

The **total number of target particles** within the beam area is

$$N_b = n_b A\, d \qquad (14)$$

from (12) ➔ σ_b **reaction cross-section:**

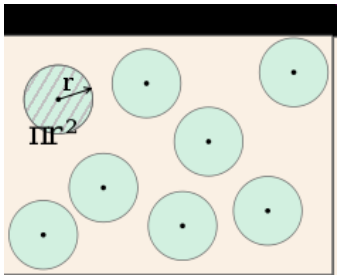
$$\sigma_b = \frac{\dot{N}}{\Phi_a N_b} \qquad (15)$$

$$\sigma_b = \frac{\text{number of reactions per unit time}}{\text{beam particles per unit time} \times \text{scattering centres per unit area}}$$



Cross section of the nuclear reaction

Physical interpretation of σ_b:



1) Geometric reaction cross-section

the area presented by a single scattering centre to the incoming projectile a

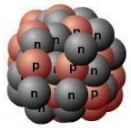
2) Total reaction (or scattering) cross-section

This naive description of the geometric reaction cross section as the effective cross sectional area of the target particles is in many cases a good approximation to the true reaction cross-section (e.g.: as for pp-scattering at high energy)

The reaction probability for two particles, however, generally may be very different from these geometric considerations.

Furthermore, a strong energy dependence is also observed (e.g.: the reaction rate for the capture of thermal neutrons by uranium, for example, varies by several orders of magnitude within a small energy range) – resonant scattering.

⇒ In experiment one measures the reaction cross section which generally is not equal to the geometrical reaction cross section determined by the size of the interacting particles. The reaction cross section depends on the strength of the interaction and the presence of intermediate resonant states.



Cross section of the nuclear reaction

Total reaction (or scattering) cross-section

In analogy to the *total* cross-section, cross-sections for *elastic* reactions σ_{el} and for *inelastic* reactions σ_{inel} may also be defined. The inelastic part can be further divided into different reaction channels.

The *total cross-section* is the sum of these parts:

$$\sigma_{tot} = \sigma_{el} + \sigma_{inel} \tag{16}$$

The cross-section is a physical quantity with dimension of [area], and is independent of the specific experimental design.

A commonly used unit is the *barn*, which is defined as:

$$1 \text{ barn} = 1 \text{ b} = 10^{-28} \text{ m}^2$$

$$1 \text{ millibarn} = 1 \text{ mb} = 10^{-31} \text{ m}^2 \tag{17}$$

Typical total cross-sections at a beam energy of 10 GeV, for example, are

$\sigma_{pp}(10 \text{ GeV}) \approx 40$ mb for proton-proton scattering;

$\sigma_{\nu p}(10 \text{ GeV}) \approx 7 \cdot 10^{-14}$ b = 70 fb for neutrino-proton scattering.



Luminosity of the nuclear reaction

The quantity

$$\mathcal{L} = \Phi_a \cdot N_b \tag{18}$$

is called the *luminosity*.

Like the flux, it has the dimension of [(area×time)$^{-1}$].

From (12): $\dot{N} \equiv \frac{dN}{dt} = \Phi_a N_b \sigma_b$ and (14): $N_b = n_b \cdot d \cdot A$ ➔

$$\mathcal{L} = \Phi_a \cdot N_b = \dot{N}_a \cdot n_b \cdot d = n_a \cdot v_a \cdot N_b \tag{19}$$

⇒ the luminosity is the product of

- the number of incoming beam particles per unit time $\dot{N}_a$ $= \dot{N}_a \cdot n_b \cdot d$
- the target particle density in the scattering material n_b
- and the target's thickness d

Or

- the beam particle density n_a $= n_a \cdot v_a \cdot N_b$
- their velocity v_a
- and the number of target particles N_b exposed to the beam



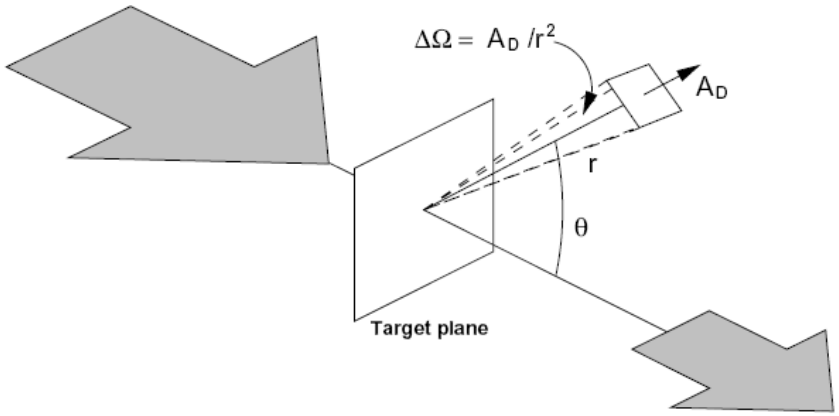
Differential cross section

In practice, only a fraction of all the reactions are measured. A detector of area A_D is placed at a distance r and at an angle θ with respect to the beam direction, covering a solid angle $\Delta\Omega = A_D/r^2$.

The rate of reactions seen by this detector is then proportional to the *differential cross-section* $d\sigma(E, \theta)/d\Omega$:

$$\dot{N}(E,\theta,\Delta\Omega) = \mathcal{L}\cdot\frac{d\sigma(E,\theta)}{d\Omega}\Delta\Omega \qquad (20)$$

If the detector can determine the energy E of the scattered particles then one can measure the *doubly differential* cross-section $d^2\sigma(E,E', \theta)/d\Omega\, dE'$.



The total cross-section σ is then the integral over the total solid angle and over all scattering energies:

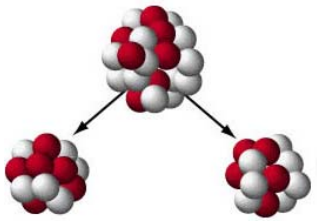
$$\sigma_{\rm tot}(E) = \int_0^{E'_{\rm max}}\int_{4\pi}\frac{d^2\sigma(E,E',\theta)}{d\Omega\, dE'}\, d\Omega\, dE' \qquad (21)$$



Nuclear fission

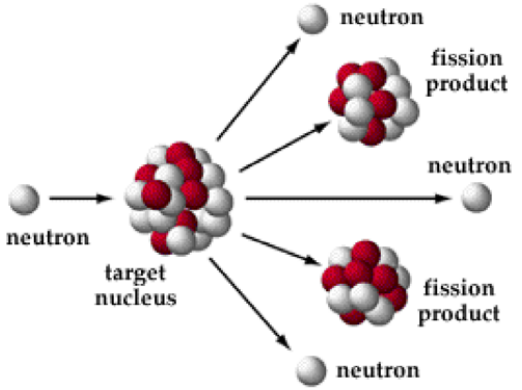
Nuclear fission

- decay into two or more lighter nuclei :



- ❑ **spontaneous** fission (tunneling effect)

- ❑ **induced fission** – due to nuclear reactions, e.g. under neutron bombardment



- Fission is **energetically more favourable** for heavy isotopes

- **Fission products**: the two nuclei produced are most often of comparable size, typically with a mass ratio around 3:2 for common fissile isotopes.